Chapter 5
Force and Motion - I

5-1 Change in the velocity of a body (and therefore its acc.) $\xleftrightarrow{\text{is related}}$ to the int. of the body with its enviroment.

The force laws

Enviroment → Force → Body (mass) → Acceleration

The laws of motion

5-2 Newton's First law;

Before Newton formulated his mechanics, it was thought that some <u>influence</u> or <u>force</u> was needed to keep a body moving at <u>const. velocity</u>!

Ex. - Imagin a block to be pushed with a const. velocity on a surface with <u>friction</u>.

Newton's First law:

Consider a body on which <u>a net force</u> zero acts. If the body is at rest, it will remain at rest if the body is moving with const. velocity, it will continue to do so.

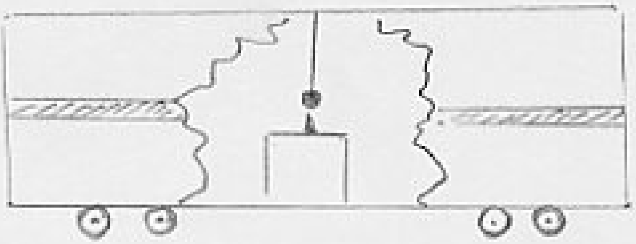
Alternatively:

Newton's First law;

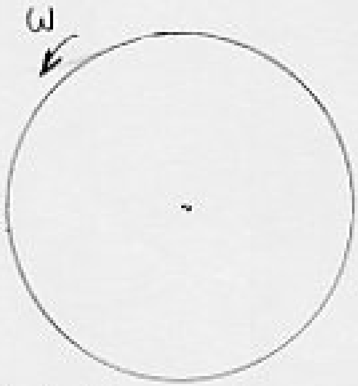
If the net force acting on a body is zero, it is possible to find reference frames in which the body has no acc. (inertial reference frames).

How to check a ref. frame is inertial?

The ball will be deviated from its initial position if the system is not inertial (i.e. railroad car has $a \neq 0$)



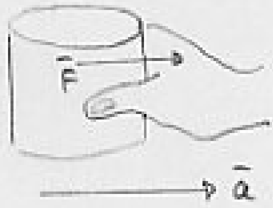
Ex. - Earth is non-inertial because of its rotation, but if we are not considering large-scale motions such as winds and ocean currents, we can make approx. and consider it as an inertial ref frame.



Rotating system are non-inertial

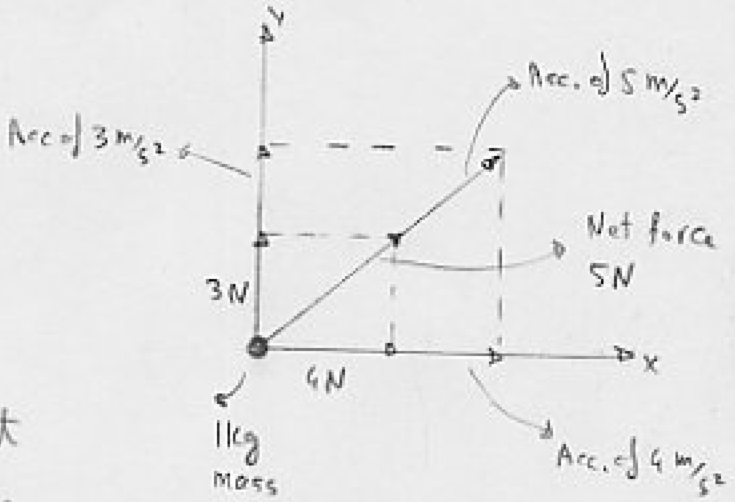
5-3 Force;

Def. - If we pull a standard body (mass = 1 kg) so that it experiences an acc. of $1\,m/s^2$, we declare that we are exerting a force on the standard body whose magnitude is 1 N.



Acc. is a vector. Is force a vector?

We have to show it obeys the laws of vector addition.

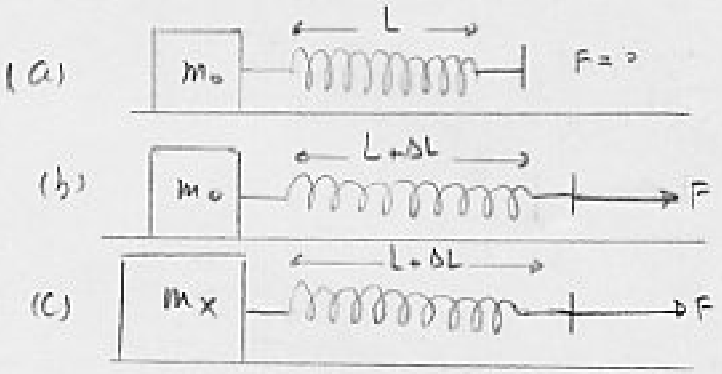


Experiment confirms that F is a vector

5-4 Mass

Everyday experience tells us that a given force will produce different accelerations in different bodies.

Experiment:



a) $m_0 = 1\,kg$ standard mass

$a = 0\ (\rightarrow F = 0)$ spring length $= L$

b) $m_0 = 1\,kg$

$a = a_0\ (\rightarrow F = F_0)$ " " $= L + \Delta L$ { we attach the spring in such a way to produce a_0 acc.

c $m = m_x$

$a = a_x\ (F = F_0)$ " " = " { we attach the spring in such a way to produce the same displacement $L+\Delta L$ (the same force)

$$\rightarrow \frac{m_x}{m_0} = \frac{a_0}{a_x} \quad \text{(define)}$$

$$m_x = m_0 \frac{a_0}{a_x}$$

For example $m_x = m_0 \dfrac{a_0}{a_x} = (1\,kg)\dfrac{1\,m/s^2}{0.25\,m/s^2} = 4\,kg$

The First Test;

b) $a_0 \xrightarrow{\text{is increased}} a_0' = 5\,m/s^2$ (i.e. $F_0 = 1N \rightarrow F_0' = 5N$)

c) $F = 5N$ (the same $L+\Delta L$) $\longrightarrow a_x' = 1.25\,m/s^2$

$\rightarrow m_x = m_0 \dfrac{a_0'}{a_x'} = (1\,kg)\dfrac{5\,m/s^2}{1.25\,m/s^2} = 4\,kg$ (the same as before)

The Second Test:

Suppose we have found the mass of body Y, using the spring method; (using standard mass m_0)

say $m_Y = 6\,kg$

Now we compare body X and body Y directly;

Using the spring method; we obtain again;

$$m_Y = m_X \frac{a''_X}{a''_Y} \underset{\substack{\uparrow \\ \text{say}}}{=} (4\,kg)\,\frac{2.4\,m/s^2}{1.6\,m/s^2} = 6\,kg \quad \text{(the same result)}$$

What is Mass?

Def. - The mass of a body is the characteristic that relates the force on the body to the resulting acc.

5-5 Newton's Second law;

$$\underbrace{\sum F}_{\substack{\text{Net force} \\ \text{(external forces)}}} = ma \qquad \text{Newton's Second law}$$

free-body diagram

$$\rightarrow \sum F_x = ma_x \quad \sum F_y = ma_y \quad \sum F_z = ma_z$$

Conclusion: Newtons First law is a special case of the Newton's Second law ($F=0$)

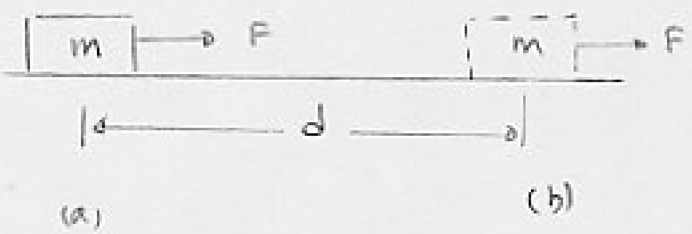
$$F = 0 \longrightarrow a = 0$$

This doesnot trivialize Newton's First law; its role in defining the set of ref. frames in which Newton's mechanics holds, justifies its status as a separate law.

Unit: $1\,N = (1\,kg)(1\,m/s^2)$

Sample prob. 5-1

$m = 240\,kg \quad d = 2.3\,m$



Surface: frictionless

$V_0 = 0 \quad , F = 130\,N$

$V = ?$

Sol.

$$a_x = \frac{F_x}{m} = \frac{130\,N}{240\,kg} = 0.542\,m/s^2$$

$$V^2 = V_0^2 + 2a(x - x_0) \longrightarrow V = \sqrt{2ad} = 1.6\,m/s$$

Sample prob. 5-2

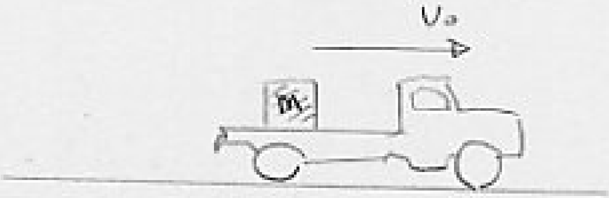
In prob 5-2, we want to reverse the dir. of the velocity in 4.5 Sec. $F = ?$

Sol.

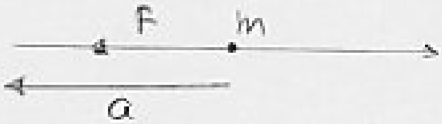
$$V = V_0 + at \qquad a = \frac{V - V_0}{t} = \frac{(-1.6) - (1.6)}{4.5} = 0.711\,m/s^2$$

$$F_x = ma_x = (240)(-0.711) = -171\,N$$

Sample prob. 5-3



A crate of mass $m = 360\,kg$ rests on the bed of a truck that is moving at a speed of $V_0 = 120\,km/h$.
The driver applies the brakes and slows to a speed of $V = 62\,km/h$ in 17 sec.



$F = ?$ on crate (assume the crate does not slide.)

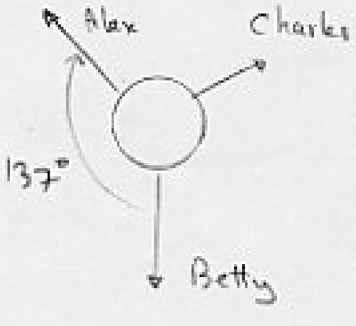
Sol.

$$V = V_0 + at \qquad a = \frac{V - V_0}{t} = \frac{(62\,km/h) - (120\,km/h)}{17\,s}$$

$$= \left(-3.41\,\frac{km}{h \cdot s}\right)\left(\frac{1\,h}{3600\,s}\right)\left(\frac{1000\,m}{1\,km}\right) = -0.947\,m/s^2$$

$$F = ma = (360\,kg)(-0.947\,m/s^2) = -340\,N$$

Sample prob 5-4



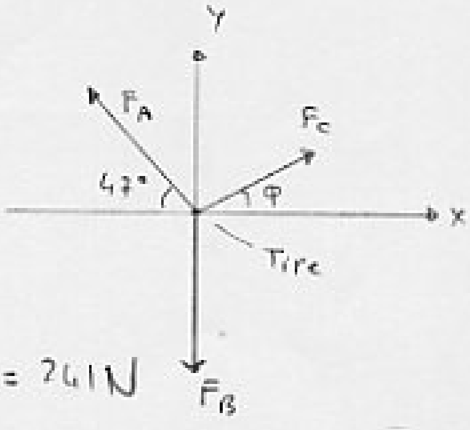
$F_A = 220\,N \quad F_C = 170\,N \quad F_B = ?$ (the tire to be stationary)

Sol.

$$\Sigma F = \bar{F}_A + \bar{F}_B + \bar{F}_C = 0$$

$$\Sigma F_x = F_C \cos\varphi - F_A \cos 47 = 0 \quad (1)$$

$$\Sigma F_y = F_C \sin\varphi + F_A \sin 47 - F_B = 0 \quad (2)$$



$(1) \rightarrow \quad 170 \cos\varphi - 220 \cos 47 = 0 \quad \rightarrow \varphi = 28°$

$(2) \quad F_B = F_C \sin\varphi + F_A \sin 47 = 170 \sin 28 + 220 \sin 47 = 241\,N$

5-6 Some Particular Forces;

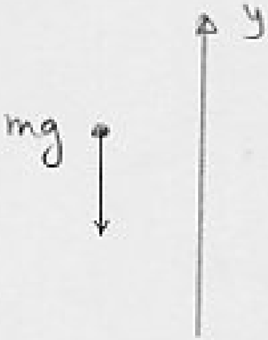
Weight;

The weight W of a body is a force that pulls it directly toward a nearby astronomical body (say Earth). This force is primarily due to gravitational attraction.

Consider a body of mass m located at a point with a free-fall acc. of g.

$W = mg$ the mag. of weight

$\bar{W} = -mg\hat{j} = -W\hat{j}$



$\bar{W} = m\bar{g}$

$\bar{W}$ depends on the location of the body

$W_b = 71\,N$ weight of a ball on the Earth

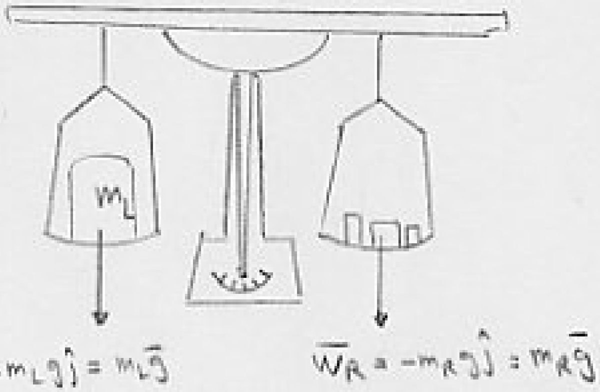
$W'_b = 12\,N$ " " the same " " " Moon

$m_b = 7.2\,kg$ both on the Earth and on the Moon

intrinsic property of the ball

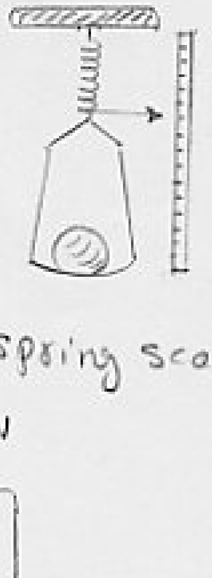
Weight is measured from the inertial ref. frame.
If it is instead measured from a noninertial frame, the measurement gives apparent weight.

The equal-arm balance measures the unknown mass m_L comparing with the standard mass m_R. If g is known we can obtain the weight.



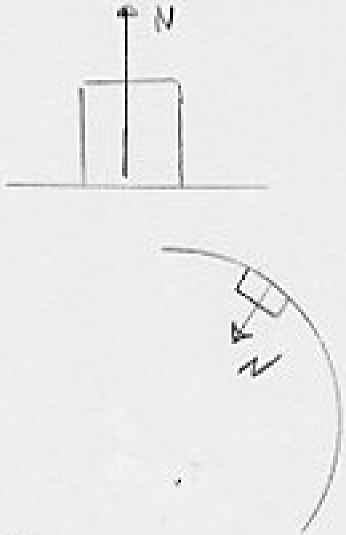
Equal-arm balance

Spring scale measures directly the weight (if it is calibrated).


spring scale

Normal Force:

When a body is pressed against a surface, the body experiences a force that is perpendicular to the surface. This is called normal force.

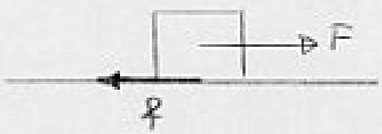


For a body at rest on a horizontal surface

$$\Sigma F_y = N - mg = ma_y \quad , a_y = 0 \rightarrow N = mg$$

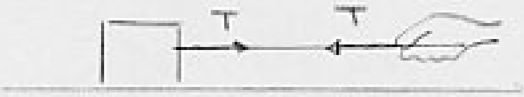
Friction

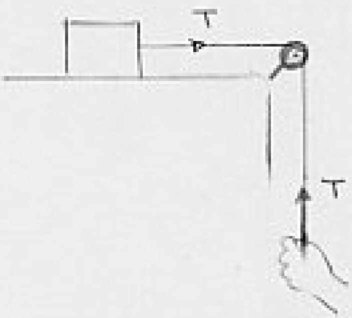
The frictional force is a force which opposes the motion of the object.



Tension

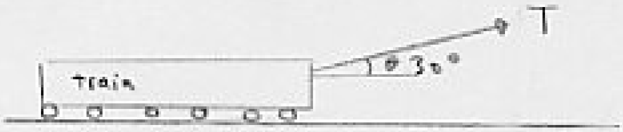
When a cord is attached to a body and pulled taut, the cord is said to be under tension. It pulls the body with a force T.





Sample prob. 5-5

A train is pulled by a man of mass $m = 80\,kg$ at an angle of $\theta = 30°$.

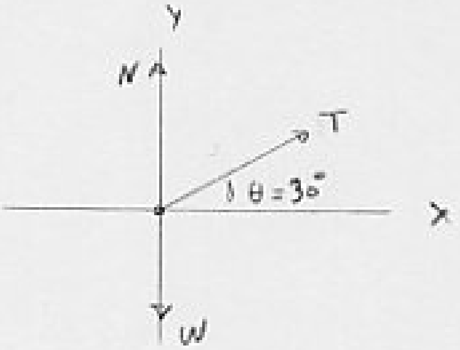


$W_{Train} = Mg = 7 \times 10^5\ N$

$d = 1\,m$ displacement

If $T = 2.5\ W_{Man}$

$V_f = ?$



Sol.

$\Sigma F_x = T\cos\theta = M a_x$

$T = 2.5(80)(9.8) = 1960\,N$

$$W_T = Mg \qquad M = \frac{W_T}{g} = \frac{7\times10^5}{9.8} = 7.143\times10^4\,kg$$

$$a_x = \frac{T\cos\theta}{M} = \frac{1960\cos 30}{7.143\times10^4} = 2.376\times10^{-2}\ m/s^2$$

$$V_x^2 = V_{0x}^2 + 2a_x(x-x_0) \qquad V_x = \sqrt{0+(2)(2.376\times10^{-2})(1)} = 0.22\ m/s$$

5-7 Newton's Third law

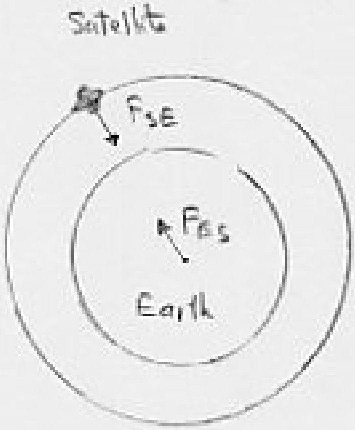
$$F_{AB} = -F_{BA} \qquad \text{Newton's third law}$$

One is called <u>action</u> force

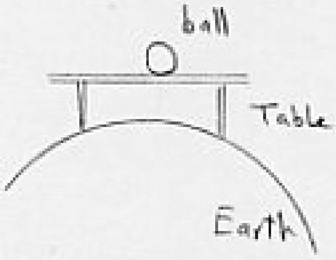
the other is called <u>reaction</u> force

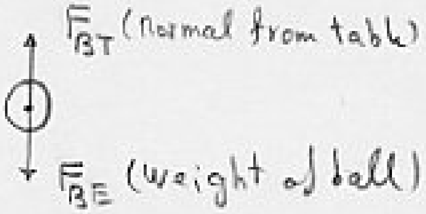
The act on <u>different</u> bodies.

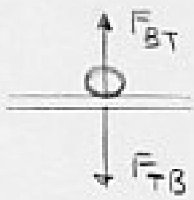
Ex. - An Orbiting Satellite



A ball Resting on a Table (at rest)

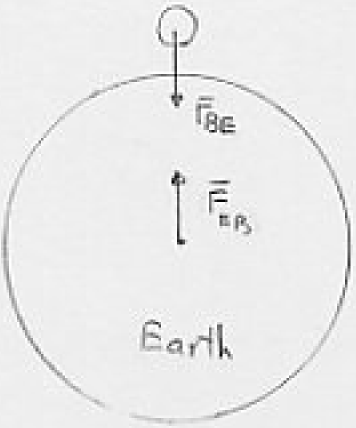






$\bar{F}_{BE} = -\bar{F}_{EB}$ action – reaction

$\bar{F}_{BT} = -\bar{F}_{TB}$ 〃 〃



If we cancel the table the ball would accelerate because $\bar{F}_{BE}$ is the only force acting on the ball

Sample prob. 5-6

$M = 3.3\ \text{kg} \quad m = 2.1\ \text{kg}$

$a_m = ? \quad a_M = ? \quad T = ?$

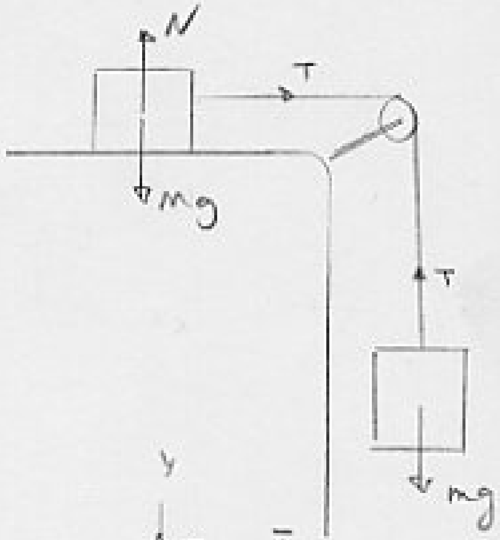
<u>Sol.</u> $\qquad \Sigma \bar{F} = m\bar{a}$

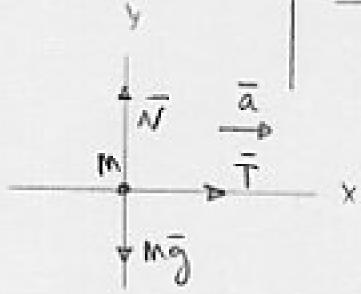
$$\begin{cases} \Sigma F_x = ma_x \rightarrow T = Ma \\ \Sigma F_y = ma_y \rightarrow T - mg = -ma \end{cases}$$

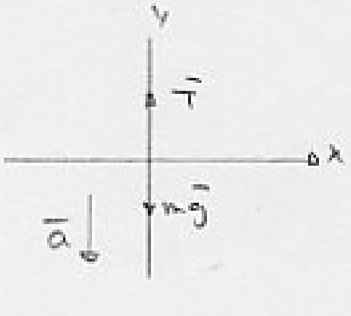
$$\rightarrow a = \frac{m}{M+m} g \qquad T = \frac{Mm}{M+m} g$$

$$a = \frac{2.1}{3.3+2.1}(9.8) = 3.8\ \text{m/s}^2$$

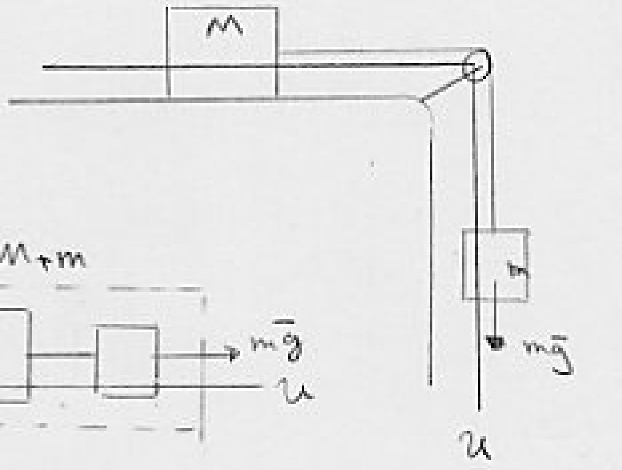
$$T = \frac{(3.3)(2.1)}{3.3+2.1}(9.8) = 13\,\text{N}$$







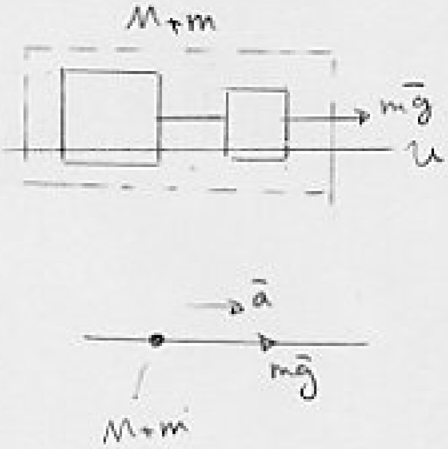
Alternative way:



$\sum F_u = (M+m)\,a_u$

$mg = (M+m)\,a$

$a = \dfrac{m}{M+m}\,g$

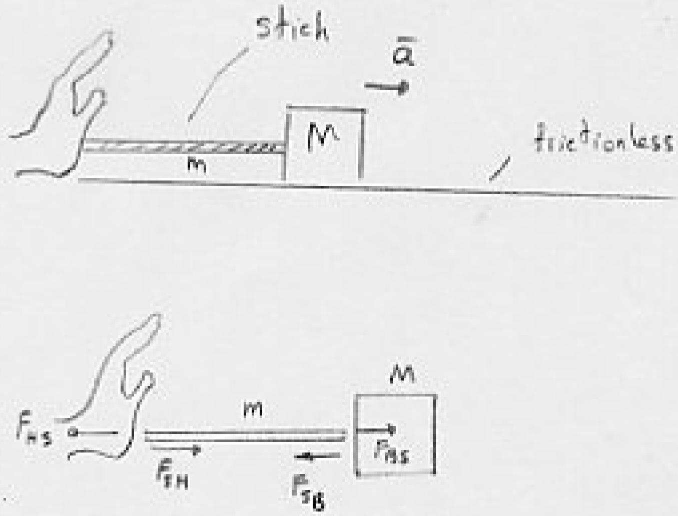


Now apply Newton's second law to either m or M

$\rightarrow$ $T = Ma$

or $mg - T = ma$

Sample prob 5-7



$M = 33\,\text{kg}$ $m = 3.2\,\text{kg}$

$d = 77\,\text{cm}$ displacement

$\Delta t = 1.7\,\text{s}$

$a = \text{const.}$

a) Action and reaction forces?

$\bar{F}_{HS} = -\bar{F}_{SH}$

$\bar{F}_{SB} = -\bar{F}_{BS}$

b) $F_{SH} = ?$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

$v_0 = 0 \quad x - x_0 = d \quad \rightarrow a = \frac{2d}{t^2} = \frac{2(0.77)}{(1.7)^2} = 0.533 \ m/s^2$

$$F_{SH} = (M + m)a = (33 + 3.2)(0.533) = 19.3 \ N$$

c) $F_{BS} = ?$

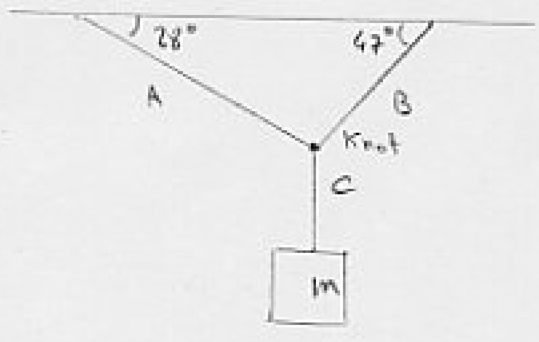
$$F_{BS} = Ma = (33)(0.533) = 17.6 \ N$$

d) Net force on stick?

$$F = F_{SH} - F_{SB} = 19.3 - 17.6 = 1.7 \ N$$

or $F = ma = (3.2)(0.533) = 1.7 \ N$

Sample prob. 5-8



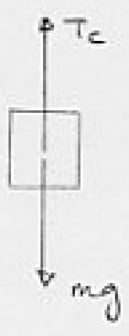
$m = 15 \ kg \quad T_A = ? \ T_B = ? \ T_C = ?$

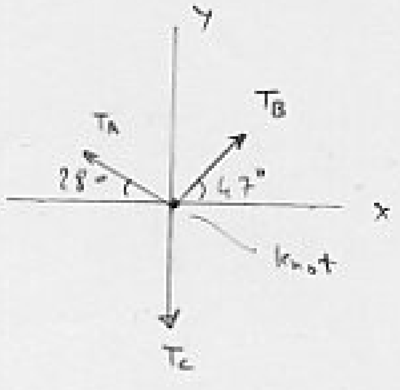
Sol.

Newton's second law for the block;

$$\sum F_y = T_C - mg = 0$$

$$T_C = mg = (15)(9.8) = 147 \ N$$





Newton's second law at the knot

$$\sum \bar{F} = \bar{T}_A + \bar{T}_B + \bar{T}_C = 0$$

$$\sum F_y = T_A \sin 28^\circ + T_B \sin 47 - T_C = 0$$

$$\sum F_x = -T_A \cos 28^\circ + T_B \cos 47^\circ = 0$$

$$\rightarrow \quad T_A = 104\ N \qquad T_B = 134\ N$$

Sample prob. 5-9

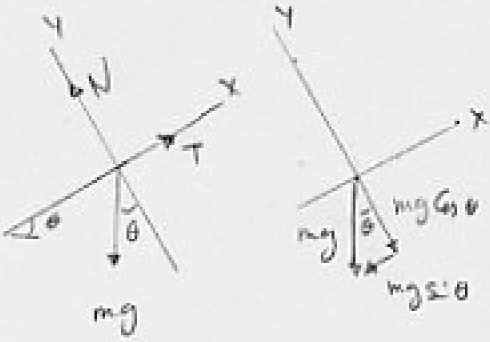
$\theta = 27^\circ$ $\quad T = ?$

$m = 15\ kg$

Sol.

$$\sum \bar{F} = T + N + mg = 0$$

$$\begin{cases} \sum F_x = T - mg \sin\theta = 0 \\ \sum F_y = N - mg \cos\theta = 0 \end{cases}$$



$$\rightarrow \quad T = mg \sin\theta = (15)(9.8)\sin 27$$
$$= 67\ N$$

$$N = mg\cos\theta = 131\ N$$

Sample prob. 5-10

Suppose you cut the cord holding the block in prob 5-9.

$a = ?$

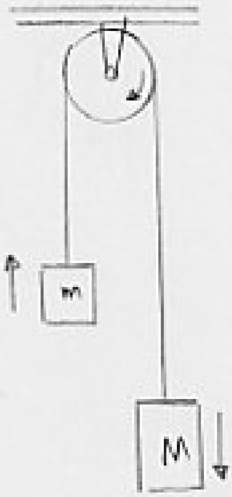
Sol. $\quad \sum F_x = 0 - mg\sin\theta = ma$

$$a = -g\sin\theta = -9.8 \sin 27^\circ = -4.4\ m/s^2$$

Sample prob 5-11

$a = ?$ $T = ?$

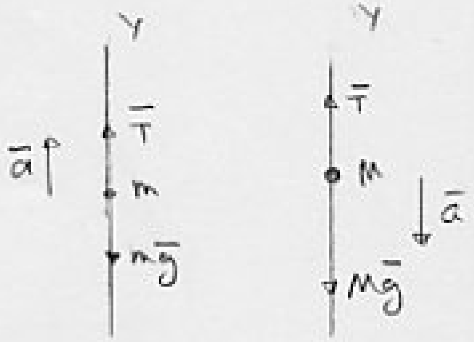
Sol.



$\sum F_y = ma$

$$\begin{cases} T - mg = ma & \text{Newton's second law for } m \\ T - Mg = -Ma & \text{" " " " } M \end{cases}$$

$$\rightarrow a = \frac{M-m}{M+m} g \qquad T = \frac{2mM}{M+m} g$$



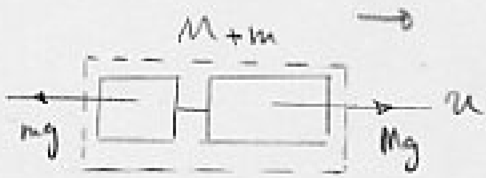
$$\text{or} \quad T = \frac{M+M}{M+m} mg \rightarrow T > mg$$

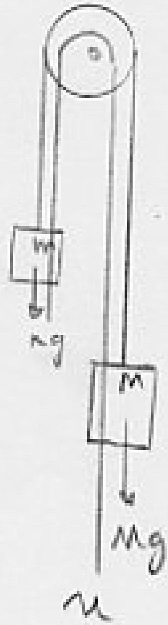
$$\text{or} \quad T = \frac{m+m}{M+m} Mg \rightarrow T < Mg$$

Alternative way; (unconventional axis)

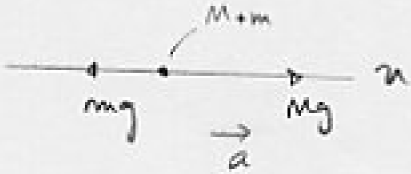
Sol.

$\sum F_u = Mg - mg = (M+m)a$





$$a = \frac{M-m}{M+m} g$$



Apply Newton's Second law to either block

$T - mg = ma$

or $-T + Mg = Ma$

sample prob 5-12

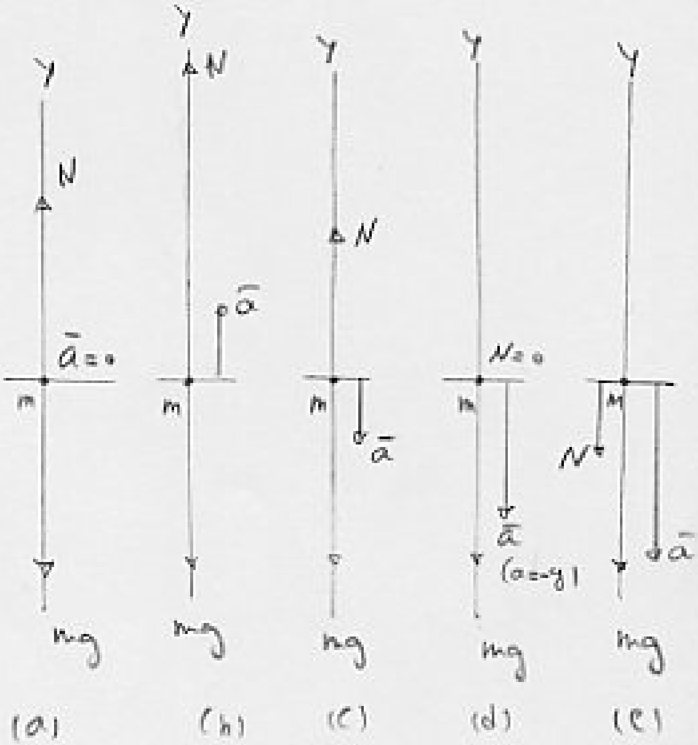
A passenger of mass $m = 72.2$ kg stands on a platform scale in an elevator. What is the scale reading for the acc. values given in the Fig.?

b) $a = +3.2\ m/s^2$

c) $a = -3.2$ "

d) $a = -g = -9.8$ "

e) $a = -12$ "



Sol.

Applying Newton's second law,

$$N - mg = ma$$

$$N = m(g+a)$$

a) $N = m(g+a) = (72.2)(9.8+0) = 708\ N$ true weight

b) $N =$ " $= (72.2)(9.8+3.2) = 939\ N$ apparent "

c) $N =$ " $= (72.2)(9.8-3.2) = 477\ N$ " "

d) $N =$ " $= (72.2)(9.8-9.8) = 0$

The passenger in his accelerating frame, concludes that he is weightless. The same thing happens in the aircraft, the elevator or spacecraft and its occupant are each in free fall with the same acc.

e) $N = m(g+a) = (72.2)(9.8-12) = -159\ N$