8 Baroclinic Instability

The previous sections have examined in detail the dynamics behind quas-geostrophic motions in the atmosphere. That is motions that are of large enough scale and long enough timescale for rotation to be important and for the Rossby number to be small. The dynamics of Rossby waves has been examined. These are wave motions that require a background gradient of potential vorticity for their propagation. The mechanism of production of large scale Rossby waves by flow over topography has been discussed together with the propagation of such waves vertically into the stratosphere. This section will now focus on the mechanism responsible for the smaller scale baroclinic eddies that are ubiquitous throughout the mid-latitude troposphere. In understanding the process that leads to these motions we will also gain an understanding of why the atmosphere is full of synoptic scale motions of a similar size. There are typically 5 or 6 synoptical scale eddies around a latitude circle at any one time. These eddies have characteristic length scales close to the Rossby radius of deformation L_D.

These eddies are also quasi-geostrophic motions that grow by feeding on the available potential energy associated with the meridional temperature gradient in mid-latitudes - the process of **Baroclinic Instability**.

8.1 Introduction

Baroclinic instability is relevant in situations where there is

- Rotation
- Stratification
- A horizontal temperature gradient

Consider the process of geostrophic adjustment examined in Section 4.4. An initially unbalanced situation consisting of a step-function in the depth of a shallow water layer is allowed to adjust to equilibrium, first in the absence of rotation and then with a constant rotation f_o. In the absence of rotation the shallow water layer equilibrates to a situation with zero height anomaly. All the initial available potential energy is converted to kinetic energy. In contrast, the presence of rotation inhibits the adjustment of the fluid. Adjustment only occurs over a small region around the initial step function until a geostrophically balanced state is reached. The presence of rotation inhibits the conversion of potential energy to kinetic energy. As a result, the geostrophically balanced state still has potential energy associated with it.

The geostrophically balanced state in the simpler shallow water system can be seen to be analogous to the situation that occurs in the mid-latitudes. The differential solar heating between the equator and poles means that the tropics are warmer and the poles are cooler. Therefore there is a negative temperature gradient between the equator and poles as can be seen on pressure surfaces in Fig. 1 (a). When a horizontal temperature gradient occurs on a pressure surface the flow is baroclinic and there are vertical wind shears through thermal wind balance (See Fig. 1 (b)).

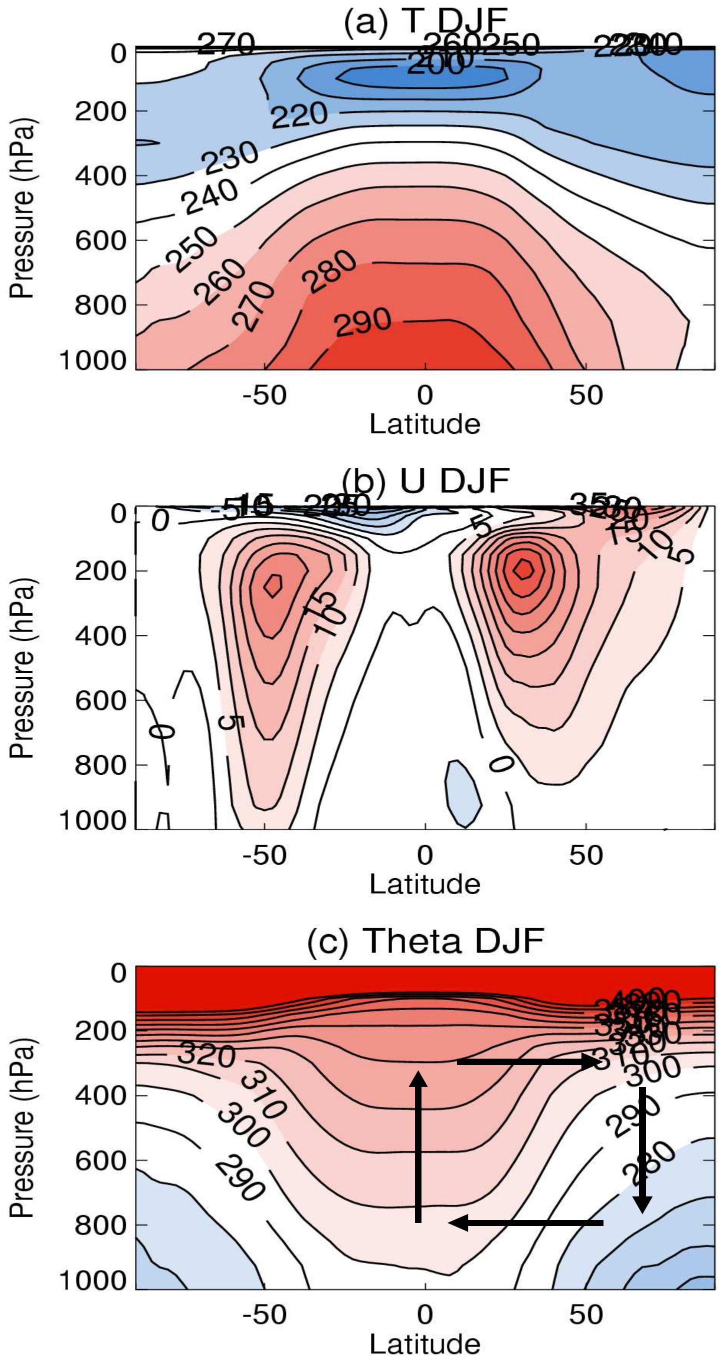

Figure 1: (a) Climatology of zonal mean temperature in DJF on pressure levels from ERA-40. (b) as (a) but zonal mean zonal wind. (c) as (a) but potential temperature. The arrows illustrate, schematically, the overturning circulation that would be set up in the absence of rotation. This would act to flatten the potential temperature contours and reduce the baroclinicity in mid-latitudes.

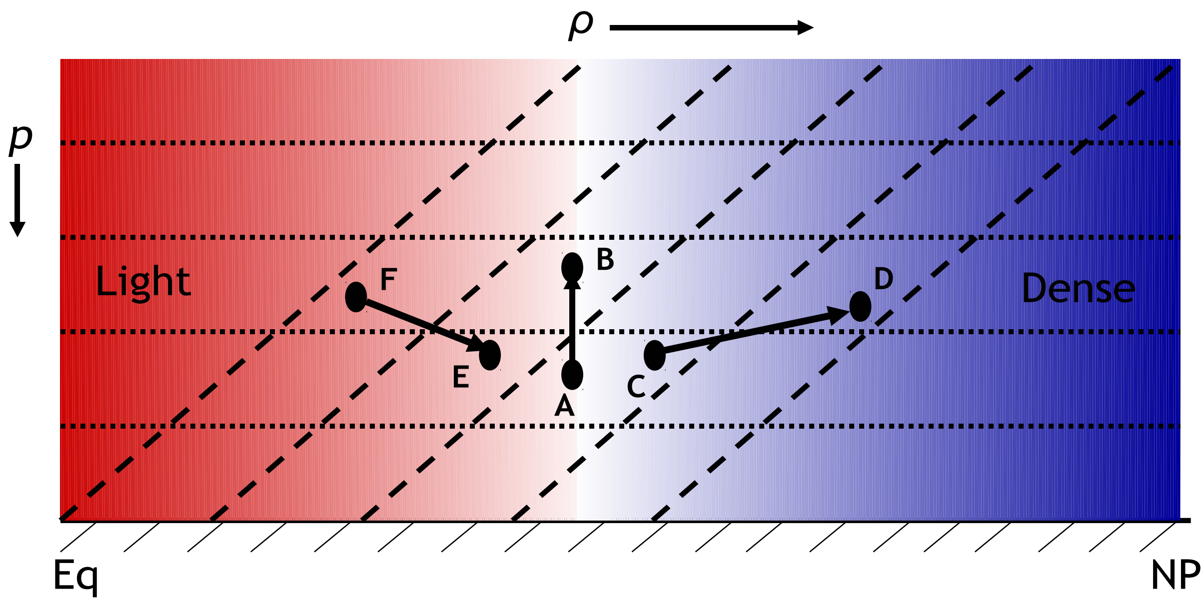

Figure 2: Schematic illustration of sloping convection. Surfaces of constant pressure are illustrated by the dotted lines and surfaces of constant density by the dashed lines.

Now, remembering the relationship between the geopotential of a pressure surface and the integral of temperature below that surface:

$$\Phi(p) = \Phi(p_s) + \int_p^{p_s} RT dlnp'$$

it can be seen that a particular pressure surface in the tropics will exist at higher geometric height than that same pressure surface at high latitudes i.e. there is a horizontal pressure gradient on geometric height surfaces. This is analogous to the shallow water geostrophically adjusted situation above. So, the situation of an equator to pole temperature gradient on the rotating Earth has potential energy associated with it. Potential energy that may be extracted by the baroclinically growing disturbances.

If the Earth was not rotating this situation would not be maintained. An equator to pole overturning circulation would be set up as depicted in Fig. 1 (c). Potential temperature, being a materially conserved quantity, would be advected by this overturning circulation and it can be seen that this would act to flatten the potential temperature gradients in Fig. 1 (c) until eventually potential temperature (and so temperature itself) is constant on pressure surfaces. In this situation there would no longer be any potential energy available and there would no longer be baroclinicity.

The energetics behind Baroclinic Instability can be illustrated through Fig. 2. This depicts the situation of a warm equator and a cold pole, with pressure surfaces illustrated by the dotted lines and surfaces of constant density depicted by the dashed lines.

The warm air, by the ideal gas law, is lighter and the cold air is denser. Consider the exchange of the parcels between A and B. The denser parcel A is displaced upward and is now surrounded by air of lower density and vice-versa for B. These air parcels will therefore experience a restoring force back to their original positions via the stratification. In contrast, parcels C and D can be exchanged and now the light parcel is surrounded by even denser fluid and so is buoyant and may continue and vice versa for the heavier parcel. Moreover, the center of gravity has been lowered since the heaver parcel has moved lower and the lighter parcel has moved higher. Potential energy has been extracted from the system and converted to kinetic energy of the air parcels.

In the final situation, the exchange of parcels E and F, although the parcels are buoyant relative to their surroundings in their new positions, there has been no lowering of the centre of gravity of the system. The opposite has happened, the lighter parcel is lower and the heavier parcel is higher. So, this motion requires an input of energy and so will not happen spontaneously.

Convection in the sloping sense of the C and D parcels is both allowed by the buoyancy considerations and also releases potential energy and so it's possible that such motions can occur spontaneously and will continue to grow. The motion however, cannot increase indefinately because the effects of rotation will inhibit the poleward motion and induce a circulation.

8.2 The Eady Model

In order to understand the process of baroclinic instability, the Eady model will be used. This was formulated by Eady in 1949. The situation considered is depicted in Fig. 3 and can be summarized as follows.

- The motion is on an f-plane.
- The stratification is uniform i.e. N^2 is constant. This is a reasonable approximation in the troposphere.
- The atmosphere is Boussinesq i.e. density variations are ignored except in the static stability.
- The motion is between two, flat, rigid horizontal surfaces. The upper surface may be considered to be the tropopause with the increase in static stability inhibits vertical motion.
- log-pressure coordinates are used.
- There is a uniform vertical wind shear $u_o = \Lambda z$. By thermal wind balance this must be associated with a horizontal temperature gradient.

The quasi-geostrophic PV equation that will govern the flow is

$$\frac{Dq}{Dt} = 0 \qquad\qquad q = \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{f_o^2}{N^2}\frac{\partial^2}{\partial z^2}\right]\psi \tag{1}$$

This is the same as the Q-G PV equation derived in Section 7 but since the situation is on an f-plane the term associated with the coriolis parameter can be omitted and given that the situation is considered to be Boussinesq the density terms have cancelled.

This may be written in terms of the Rossby radius of deformation for a stratified atmosphere $L_D = NH/f_o$ as

$$q = \nabla^2\psi + \frac{H^2}{L_D^2}\frac{\partial^2\psi}{\partial z^2} \tag{2}$$

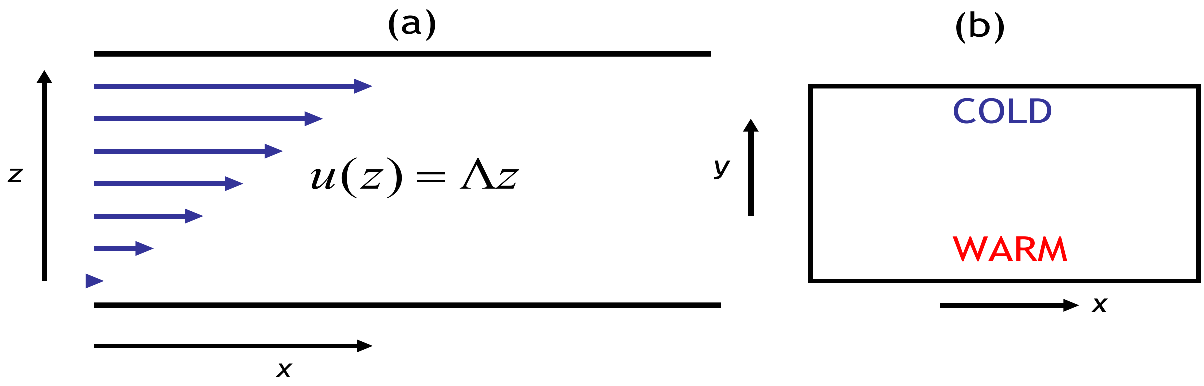

Figure 3: Setup of the Eady model (a) in the x-z plane and (b) in the horizontal.

The background uniform zonal wind shear means the basic state stream function can be written as $\overline{\psi} = -\Lambda yz$. Therefore

$$\overline{q} = \nabla^2\overline{\psi} + \frac{H^2}{L_D^2}\frac{\partial^2\overline{\psi}}{\partial z}^2 = 0$$

So, the potential vorticity of air parcels is initially zero and so it must remain zero for all time.

Note that in this situation there is no variation of the coriolis parameter with latitude and so there is no background gradient of potential vorticity. Therefore, Rossby wave motion cannot simply be induced by meridional displacement of air parcels in the interior as discussed in Section 4.2.4 because there is no background potential vorticity gradient to provide the restoring force. (Note that the more complex situation where β is included is formulated in the Charney model not discussed here.)

The wave motion in the Eady model comes from the boundary conditions.

8.3 Boundary conditions: Eady Edge waves

While there is no background potential vorticity gradient, wave motion is possible at the boundary. Wave motions can exist on a rigid boundary in the presence of a temperature gradient along that boundary.

The boundary condition at $z = 0$ and $z = H$ is that the vertical velocity w must be zero. Therefore, considering the thermodynamic equation

$$\frac{D_g\theta}{Dt} + w\frac{d\theta_o}{dz} = 0$$

Away from the boundary, fluid parcels will move along isentropic surfaces. The advection of potential temperature by the geostrophic wind will be balanced by the advection associated with the vertical velocity so that potential temperature will be materially conserved following the fluid motion. However, on the boundary there can be no vertical motion and so, in the presence of a horizontal temperature gradient, a horizontal displacement of air parcels along the boundary will result in a potential temperature anomaly on the boundary. This temperature anomaly on the boundary will induce a circulation.

The form of the waves on the boundaries can be found by solving the thermodynamic equation in the absence of a vertical velocity. Remembering that

$$\theta = \frac{H}{R} exp\left(\frac{\kappa z}{H}\right)\frac{\partial \phi}{\partial z}$$

and, remembering that the stream function is related to the geopotential by $\phi = f_o \psi$, this gives the boundary condition

$$\frac{D_g}{Dt}\left(\frac{\partial \psi}{\partial z}\right) = 0 \qquad \text{at } z = 0 \text{ and } H \tag{3}$$

Consider the stream function to consist of a basic state and a small zonally asymmetric perturbation i.e. $\psi = \overline{\psi} + \psi'$ and linearise to give

$$\left(\frac{\partial}{\partial t} + u_o \frac{\partial}{\partial x}\right)\frac{\partial \psi'}{\partial z} + v' \frac{\partial}{\partial y}\frac{\partial \overline{\psi}}{\partial z} = 0$$

Remembering that $v' = \partial \psi'/\partial x$, $u_o = \Lambda z$ and $\partial^2 \overline{\psi}/\partial y \partial z = -\Lambda$ gives

$$\left(\frac{\partial}{\partial t} + \Lambda z \frac{\partial}{\partial x}\right)\frac{\partial \psi'}{\partial z} - \frac{\partial \psi'}{\partial x}\Lambda = 0 \tag{4}$$

We now search for wave solutions propagating on the boundary of the form $\psi' = Re(\psi_o)(z) exp(i(kx + ly - \omega t))$. (Note that here we're assuming that the wave is periodic in both the x and y directions. Perhaps a more realistic situation is for the waves to be bounded in the y direction in which case a solution could be of the form $\psi' = \psi_o sin(l(y/Y)2\pi) exp(i(kx - \omega t))$, where Y is the width of the domain in the y direction. In that case the solution would vanish at the lateral boundaries. But, this makes little difference to the key results so the doubly periodic boundary conditions will be used).

Substitution of this into 4 gives

$$(\omega - k\Lambda z)\frac{\partial \psi_o(z)}{\partial z} + k\psi_o(z)\Lambda = 0 \text{ at } z = 0 \text{ and } z = H \tag{5}$$

Considering each boundary seperately and assuming that the other boundary is absent (or is very far away) these boundary conditions can be solved for the amplitude of the wave on each boundary as a function of z. The solution can then be plugged into PV conservation (Eq. 2) to find the dispersion relation for each of the boundary waves.

- **At z =0:** The solution of 4 gives

$$\psi_o(z) = \psi_o(0) exp\left(-\frac{k\Lambda}{\omega} z\right) \tag{6}$$

so that the wave solution is given by

$$\psi'(z) = \psi_o(0) exp\left(-\frac{k\Lambda}{\omega} z\right) exp(i(kx + ly - \omega t)).$$

On insertion into PV conservation this gives the following dispersion relation

$$\omega = \frac{k\Lambda f_o}{N(k^2 + l^2)^{1/2}}$$

From this it can be seen that the wave will have an Eastward phase speed relative to the mean flow which, at the lower boundary, is zero. Also, the amplitude of the wave decreases exponentially away from the boundary and will have fallen by a factor e at the level where the zonal wind (Λz) is equal to the phase speed (ω/k) of the wave. This level is known as the **steering level**.

- **At z=H:** The solution of 4 gives

$$\psi_o(z) = \psi_o(0)exp\left(-\frac{k\Lambda}{\omega - k\Lambda H}z\right) \tag{7}$$

On substitution of the wave solution into PV conservation the following dispersion relation is obtained

$$\omega = -\frac{k\Lambda f_o}{N(k^2+l^2)^{1/2}} + k\Lambda H$$

The zonal wind speed at the upper boundary is equal to ΛH and so this represents a wave that has a westward phase speed relative to the mean flow.

So, even though this is a situation where there is no potential vorticity and no background gradient of potential vorticity. The fact that there is a meridional temperature gradient means that on imposing the boundary conditions, a displacement of an air parcel in the meridional direction will induce a circulation on the boundary which will result in an Eastward/Westward propagating wave on the Lower/Upper boundary and the amplitudes of these waves will decay exponentially away from the boundary.

An example of temperature anomalies that could exist by displacement of air parcels on the boundary together with the circulation they induce is depicted in Fig. 4. On the lower boundary a poleward displacement of air parcels will induce a warm anomaly which will be associated with a cyclonic circulation. The opposite is true equatorward displacement.

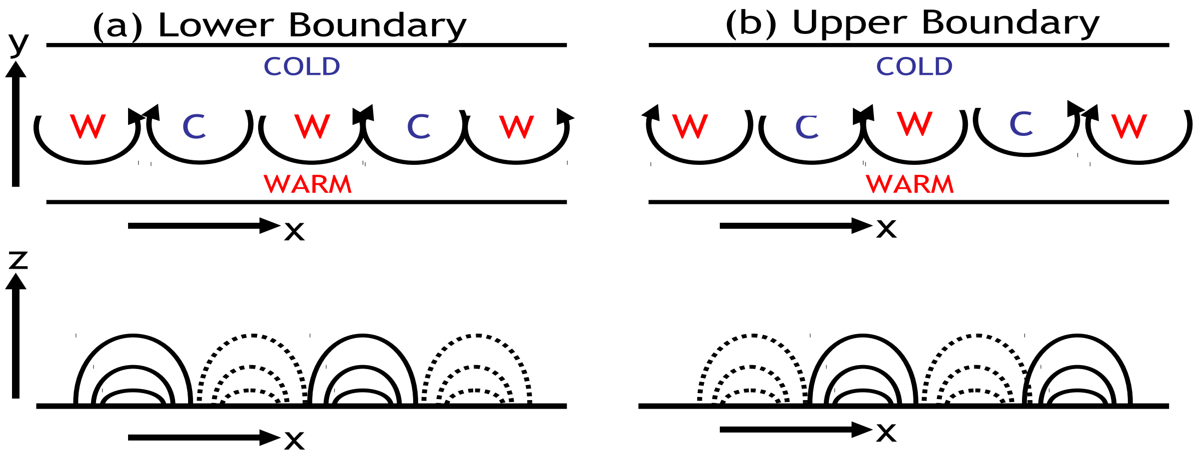

Figure 4: Illustration of the circulation associated with the temperature anomalies on (a) the lower and (b) the upper boundaries. These temperature anomalies would be induced by a meridional displacement of air parcels on the boundaries. The lower panel shows the vertical profile of meridional velocity associated with the temperature anomalies. The solid lines indicate velocity in the positive y direction and the dotted indicated velocity in the negative y direction.

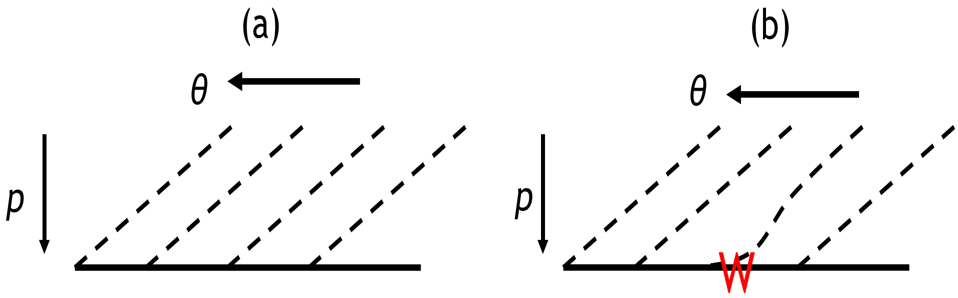

Figure 5: An illustration of the change in potential temperature contours in an infinitesimal region near the boundary, associated with a warm temperature anomaly at the boundary.

It can be seen that the circulalation induced will act to cause the temperature anomalies to propagate toward the East.

One way of thinking about this physically is depicted in Fig. 5. Warming at the surface will act to alter the location of potential temperature surfaces right at the boundary. This will act to increase $|\partial p/\partial\theta|$. By conservation of potential vorticity this must be associated with an increase in the vorticity (i.e. a cyclonic circulation).

The lower panels of Fig. 4 depict an x-z cross section of the meridional velocity at the boundary. Here, the fall off of the amplitude with height is demonstrated. It can also be seen that there will be no net transport of heat poleward since the v and T anomalies are 90^o out of phase. This edge wave doesn't alter the temperature structure of the atmosphere. It doesn't flatten the potential temperature surfaces or extract energy from the sytem. It is stable.

The opposite occurs at the upper boundary and the wave will propagate to the west. The process of baroclinic instability in the Eady model relies on the presence of these boundary waves and their interaction.

8.4 Solving for unstable growing modes in the Eady problem

The interior PV equation together with the boundary conditions provide 3 equations which can be solved for the wave solutions in the Eady problem. In this section a "normal mode" analysis will be performed to find the most unstable waves. The wave solution is proportional to $exp(-ikct)$. Therefore, in order to have an unstable solution i.e. a solution whose amplitude grows exponentially with time, the phase speed c must be imaginary. The solutions for which the phase speed is imaginary will be found and the normal mode analysis involves finding the solutions that have the largest magnitude of imaginary phase speed, which will be those that grow fastest and come to dominated the system.

In the interior, potential vorticity can be considered to consist of the basic state potential vorticity and a small perturbation i.e. $q = \overline{q} + q'$. Remembering that $\overline{q} = 0$ everywhere and linearising PV conservation about the basic state gives

$$\frac{\partial q'}{\partial t} + u_o \frac{\partial q'}{\partial x} = 0 \tag{8}$$

This is the same as the PV conservation equation used to examine Rossby waves in section 4.2.4 but here there is no background gradient of potential vorticity. Remembering that

$u_o = \Lambda z$ and plugging in the expression for the perturbation PV in terms of the stream function gives

$$\left(\frac{\partial}{\partial t} + \Lambda z \frac{\partial}{\partial x}\right)\left(\nabla^2 \psi' + \frac{H^2}{L_D^2}\frac{\partial^2 \psi'}{\partial z^2}\right) = 0 \tag{9}$$

Again seeking a solution for a wave of horizontal wavenumbers k and l as was done for the boundary conditions ($\psi'(x, y, z, t) = Re(\psi_o)exp(i(kx + ly - \omega t))$ gives

$$(\Lambda z - c)\left(-(k^2 + l^2)\psi_o + \frac{H^2}{L_D^2}\frac{\partial^2 \psi_o}{\partial z^2}\right) = 0 \tag{10}$$

For $\Lambda z \neq c$ this gives

$$-(k^2 + l^2)\psi_o + \frac{H^2}{L_D^2}\frac{\partial^2 \psi_o}{\partial z} = 0 \rightarrow \frac{\partial^2 \psi_o}{\partial z^2} - \frac{\mu^2}{H^2}\psi_o = 0 \tag{11}$$

where this has been written in terms of the parameter $\mu^2 = L_D^2(k^2 + l^2)$. Since a real amplitude is required this equation has solutions of the form

$$\psi_o = Acosh\left(\frac{\mu}{H}z\right) + Bsinh\left(\frac{\mu}{H}z\right) \tag{12}$$

So this parameter (μ) which depends in the horizontal length scale of the wave determines the vertical structure of the solutions.

This solution can then be inserted into the boundary conditions at $z = 0$ and $z = H$ to give

$$A[\Lambda H] + B[\mu c] = 0$$

$$A[(c - \Lambda H)\mu sinh(\mu) + \Lambda H cosh(\mu)] + B[(c - \Lambda H)\mu cosh(\mu) + \Lambda H sinh(\mu)] = 0$$

These boundary conditions can be written as a matrix equation

$$\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{13}$$

where x_1 and x_2 are the coefficients of A and B in the equation for $z = 0$ and x_3 and x_4 are the coefficients of A and B in the equation for $z = H$.

In order to have non-zero amplitudes A and B, the determinant of the x matrix must be zero. If it is non zero then it's inverse can be formed and the trivial result of $A = B = 0$ is obtained, i.e. there is no wave amplitude.

Forming the determinant of the x matrix and setting it equal to zero gives

$$(\Lambda H)[(c - \Lambda H)\mu cosh(\mu) + \Lambda H sinh(\mu)] - (\mu c)[(c - \Lambda H)\mu sinh(\mu) + \Lambda H cosh(\mu)] = 0$$

which can be re-written as

$$c^2 - \Lambda H c + (\Lambda H)^2(\mu^{-1}coth(\mu) - \mu^{-2}) = 0$$

This is a quadratic equation for the phase speed c which can be solved to give

$$c = \frac{\Lambda H}{2} \pm \frac{\Lambda H}{\mu}\left[\left(\frac{\mu}{2} - coth\frac{\mu}{2}\right)\left(\frac{\mu}{2} - tanh\frac{\mu}{2}\right)\right]^{\frac{1}{2}} \tag{14}$$

Exponentially growing solutions require an imaginary phase speed. So, the quantity in the square root must be negative for instability and exponentially growing amplitudes. So we require

$$\left(\frac{\mu}{2} - coth\frac{\mu}{2}\right)\left(\frac{\mu}{2} - tanh\frac{\mu}{2}\right) < 0$$

Noting that $\tanh x < x$ for all x the only way to get an imaginary phase speed is to have

$$\frac{\mu}{2} < coth\left(\frac{\mu}{2}\right)$$

This is satisfied if

$$\mu = L_D(k^2 + l^2)^{1/2} < \mu_c = 2.399 \qquad (15)$$

8.4.1 Growth rates of unstable Eady modes

So, there are only certain horizontal scales of waves that may grow exponentially and the scale of these waves will depend on the rotation rate, the depth of the layer and the static stability which all appear in the expression for the Rossby radius of deformation.

The growth rate (σ) for the unstable growing modes can be determined from $\sigma = kc_i$ where c_i is the magnitude of the imaginary component of the phase speed. This comes from the fact that the stream function amplitude is proportional to $exp(-ikct)$. Therefore the amplitude will grow at a rate $exp(\sigma t)$ where $\sigma = kc_i$. The growth rate is therefore given by

$$\sigma = kc_i = \frac{k\Lambda H}{2}\left[\left|\left(\frac{\mu}{2} - coth\frac{\mu}{2}\right)\left(\frac{\mu}{2} - tanh\frac{\mu}{2}\right)\right|\right]^{\frac{1}{2}} \qquad (16)$$

So, the growth rate depends on the quantity μ which depends on the horizontal and meridional wavenumbers and the rossby radius of deformation. This growth rate as a function of the horizontal wavenumbers is plotted in Fig. 6. The gray region represents scales for which there is no imaginary component of the phase speed. The waves that exist in this region are neutrally stable propagating waves. They do not have amplitudes which grow with time and these waves are said to be beyond the **short wave cut-off**. For exponential growth of the disturbances the disturbance has to have a sufficiently large horizontal length scale.

It can be seen that for a given zonal wavenumber (k) the most unstable growing mode is that for which the meridional wavenumber (l) is zero i.e. there are no variations in the stream function amplitude in the meridional direction.

The horizontal length scale for which the growth rate is a maximum is $l = 0$ and $k = 1.61/L_D$. Since this mode grows exponentially at the most rapid rate it is this mode that will come to dominate the system.

Considering typical values for the atmosphere ($N^2 \sim 10^{-4}$, H=10km, and f=10^{-4}) this gives a horizontal wavelength ($\lambda = 2\pi/k$) of around 4000km which corresponds to a characteristic length scale of the high or low pressure centre (L) of around 1000km which is very close to the characteristic length scale observed in the atmosphere.

The growth rate for the most unstable mode can be found by inserting the wavenumber k into Eq. 6 to give

$$\sigma_{max} = 0.31\frac{f_o}{NH}\Delta U \qquad \Delta U = \Lambda H \qquad (17)$$

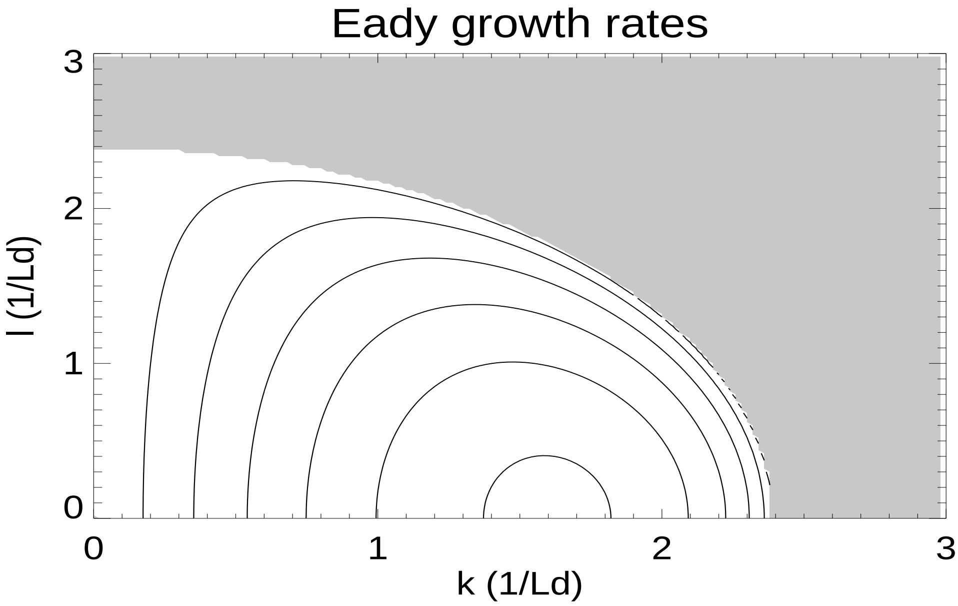

Figure 6: A plot of the variations of Eady growth rates with zonal and meridional wavenumbers. The wavenumbers are in units L_D^{-1} and grey regions represent regions beyond the short wave cut off.

This maximum Eady growth rate gives a measure of the baroclinicity and the importance of baroclinic instability. It can be seen to depend on the vertical wind shear, the static stability and the rotation rate.

8.4.2 Characteristic structure of unstable Eady modes

The vertical structure of the baroclinically unstable waves is given by Eq. 12

$$\psi_o = Acosh\left(\frac{\mu}{H}z\right) + Bsinh\left(\frac{\mu}{H}z\right)$$

The boundary condition at $z = 0$ i.e. $a[\Lambda H] + B[\mu c] = 0$ can be used to write B in terms of A which gives

$$\psi_o = Acosh\left(\frac{\mu}{H}z\right) - \frac{[\Lambda H]}{[\mu c]}Asinh\left(\frac{\mu}{H}\right) \tag{18}$$

where c is given by Eq. 14 and can be written in terms of a real component c_r and an imaginary component c_i, $c = c_r + ic_i$. This gives

$$\psi = Acosh\left(\frac{\mu}{H}z\right) - \frac{\Lambda H}{\mu}\frac{1}{c_r^2 + c_i^2}Asinh\left(\frac{\mu}{H}z\right)(c_r + ic_i)$$

But, $\psi' = Re\left(\psi_o exp\left(i\left(kx + ly\right)\right)\right)$, so finding the real component of this quantity gives the structure of the stream function for the growing mode as a function of x and z. This is shown in Fig. **??**. It can be seen that the stream function has a westward phase tilt with height. The structure of the perturbation velocity $v' = \frac{\partial \psi'}{\partial y}$ is shown in Fig. 7 (b) and the structure of the perturbation temperature $T' \propto \frac{\partial \psi'}{\partial z}$ is shown in Fig. 7 (c). It works out that the zonally averaged poleward temperature $[v'T']$ flux is positive, as must be the

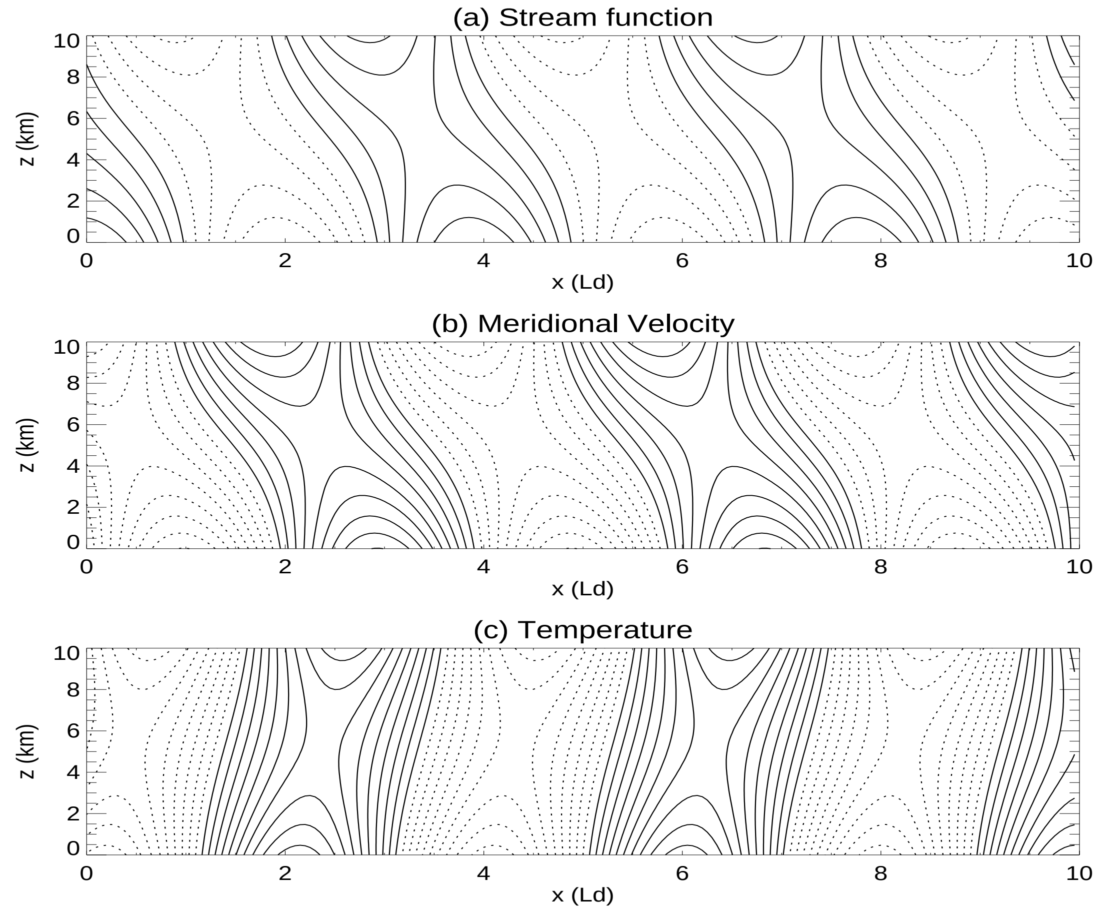

Figure 7: The structure of various fields for the most unstable Eady mode in the x-z plane. (a) Stream function, (b) meridional velocity and (c) Temperature.

case for a growing disturbance. It must transfer heat poleward to reduce the equator to pole temperature gradient and reduce the available potential energy. Heat is transferred poleward whenever the disturbance amplitude has a westward tilt with height (as in the discussion of vertically propagating Rossby waves).

8.4.3 Interpreting the unstable Eady modes physically

The interpretation of baroclinic instability in terms of sloping convection has already been discussed. When horizontal temperature gradients exist on pressure surfaces it is possible that a sloping displacement of air parcels lowers the centre of gravity of the system while at the same time the buoyancy effects can allow the parcels to continue to move. There is thus a release of the available potential energy and a gain of kinetic energy by the disturbance. However, this does not provide any insight in to the typical length scales of growing disturbances or their characteristic structures. In fact, through the sloping convection argument any scale of wave could be unstable, whereas the Eady model suggests otherwise. In order to understand why the unstable growing modes are the way they are, the dynamics of the wave motions that can be supported by the sytem have to be considered.

One way to interpret the unstable growing mode in the Eady model is through the interaction of two phase-locked edge waves. We have learned in Section 8.3 that when a horizontal temperature gradient exists on a boundary, wave motions can exist, whose amplitude decays exponentially with height according to e.g. for the lower boundary

$$\psi_o(z) = \psi_o(0)exp\left(-\frac{\Lambda z}{c}\right) \qquad c = \frac{\Lambda f_o}{N(k^2+l^2)^{1/2}}$$

This gives

$$\psi_o(z) = \psi_o(0)exp\left(-\frac{\mu}{H}z\right)$$

The quantity μ is proportional to the horizontal wavenumbers and this means the stream function amplitude for higher horizontal wavenumbers will fall off more rapidly with height than for lower wavenumbers. For sufficiently low wavenumbers it's possible that there is still sufficient amplitude of the lower boundary wave at the upper boundary and vice versa that the boundary waves can interact with each other and enhance their amplitudes. Consider Fig. 8 which depicts this schematically. Suppose the boundary waves are phase locked so that they don't move relative to one another and that the upper level cyclonic anomaly lies to the west of the lower level cyclonic anomaly. The circulation induced by the lower level wave will result in advection of cold air equatorward at the location of the cool anomaly of the upper wave. Similarly the cyclonic anomaly associated with the cool perturbation on the upper boundary will result in advection of warm air poleward at the location of the warm anomaly on the lower boundary. In this

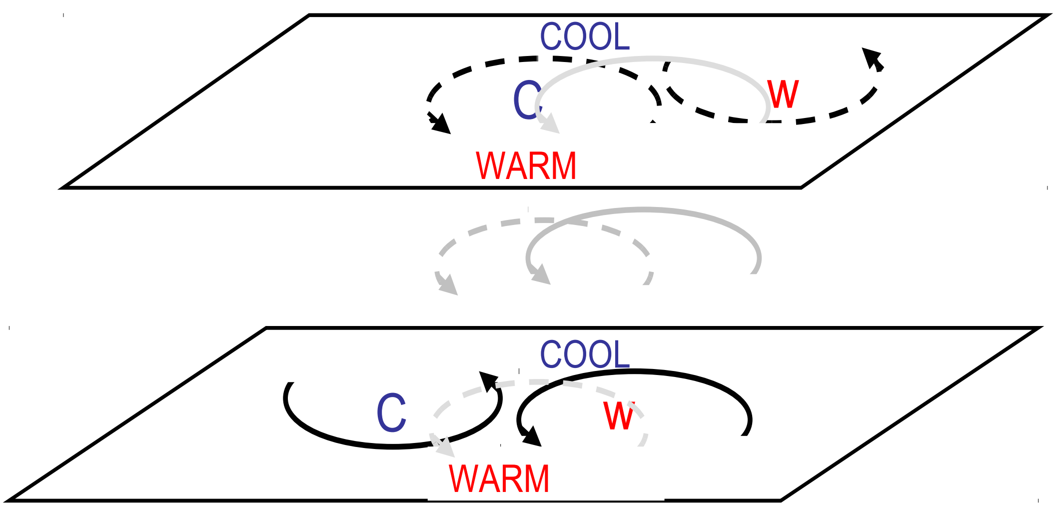

Figure 8: Illustration of the growth of phase locked eady edge waves. If the cyclonic circulation on the upper level is westward of the cyclonic circulation on the lower level and the boundaries are sufficiently close to each other then the circulation induced by the lower level wave on the upper level will act to advect cold air equatorward and so enhance the temperature anomaly on the upper level. Similarly the circulation induced by the upper level cold perturbation will act to advect warm air poleward at the location of the lower level warm anomaly and so it will feed back on the lower level warm anomaly. Together the two waves can interact to make each other grow.

way the circulation associated with the wave at the opposite boundary can cause the perturbation to grow in amplitude.

For waves beyond the short wave cut-off, the amplitude decays too rapidly with height so that the two edge waves cannot interact. The most unstable Eady modes are the waves for which this phase locking and interaction between edge waves can occur. The sloping convection picture of baroclinic instability comes in because if the horizontal and vertical temperature fluxes $[v'T']$ and $[w'T']$ are calculated for the most unstable mode, it will be found that it is poleward and slightly upward i.e. the transfer of air parcels is occuring in the slantwise direction relative to the horizontal temperature gradient.

So, the most unstable waves are the waves for which this phase locking and interaction of the boundary waves occurs and these correspond to displacements of air parcels in the slant wise direction which must also occur by energetic considerations for the disturbance to grow by the conversion of potential energy to kinetic energy.

8.5 Conditions for instability

We have only discussed the Eady model in detail, but there are other models of baroclinic instability. A more complex one is the Charney model which includes the β-effect. Unlike the Eady model, there is no upper boundary. But, the presence of the β-effect means that free Rossby waves may occur in the interior and so the unstable modes in the Charney model can be thought to be associated with the interaction between a wave the propagates on the horizontal temperature gradient at the boundary and one that propagates on the potential vorticity gradient in the interior. There are some differences between the results for the Charney model and the Eady model but overall they give very similar answers for the horizontal length scales and growth rates of the mode unstable modes and these answers agree rather well with the real atmosphere.

The waves must be counter-propagating (with respect to the mean flow) in order for phase-locking to occur and so in the case of the Charney model the potential vorticity gradient in the interior must be positive, so that the upper wave will propagate westward relative to the flow, while the lower wave will propagate Eastward. In the atmosphere there is a negative horizontal tempereature gradient at the surface and the potential vorticity gradient for the most part is dominated by the β-effect and so is positive. This means that the mid-latitude atmosphere provides good conditions for the growth of baroclinic waves.

There are several possibilities of was in which phase locking of waves can occur and this provides us with various different criterea for which baroclinic instability is possible. These are

- 1. The PV gradient changes sign in the interior.
- 2. The meridional temperature gradient at the lower boundary has the same sign as the meridional temperature gradient at the upper boundary.
- 3. The meridional temprerature gradient at the lower boundary has the opposite sign to the interior PV gradient.
- 4. The meridional temperature gradieent at the upper boundary has the same sign as the interior PV gradient.

If one of these criteria is satisfied then baroclinic instability is possible. In the Eady model, (2) is satisfied whereas in the Charney model (3) is satisfied.

8.6 Shortcomings of the simple models of baroclinic instability.

While there are several things that these simple models of baroclinic instability explain very well e.g. the horizontal length scales of the most unstable modes, the growth rates of the most unstable modes and the presence of poleward heat fluxes, there are several aspects of the baroclinic instability process that they fail to capture.

- **Momentum fluxes** - Baroclinic waves in the atmosphere are associated with a meridional flux of momentum. As the eddies grow and dissipate they flux momentum back into their latitude of origin. This helps to maintain the horizontal temperature gradient and the vertical wind shear of the jet, which the growth of the eddies acts to reduce. However, in order for a momentum flux, there has to be a horizontal tilt of the eddies and the Eady and Charney models do not include this. In order to include this a zonal wind profile that varies with both height and latitude is necessary.

- **The non-linear stages of baroclinic lifecycles** - In solving for the Eady modes we used the linearised potential vorticity equation. But, the solution was an exponentially growing disturbance. So, at some point the amplitude is going to get so large that it is no longer appropriate to consider the amplitude as begin small and neglect the non-linear terms. The wave will become such large amplitude that it will start to influence the mean flow. See e.g. Hoskins and Simmons (1978) - The life cycles of some nonlinear baroclinic waves, JAS, 35, 414-432.

- **The episodic growth and decay of weather systems** - Much like the previous point, the simple models of baroclinic instability tell us what modes will grow but it doesn't tell us what happens when they do grow e.g. will they be able to continue to grow, or will they alter the temperature gradient so much that the atmosphere is no longer baroclinic. What actually happens in the real atmosphere is that a baroclinic wave will continue to grow until its source of energy will exhausted. Then dissipative processes destroy the eddy energy and eventually the horizontal temperature gradient is restored, the flow is once again baroclinically unstable, and the process begins again.