Ageostrophic instabilities of a front in a stratified rotating fluid

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Introduction

Motivation: Some fundamental aspects of meso-scale dynamics of atmospheric fronts remain poorly understood, especially the generation of gravity waves and front destabilisation leading to secondary cyclogenesis.

Context: In addition to the classical baroclinic instability, other instabilities of the front such as symmetric, Kelvin-Helmholtz and **Rossby-Kelvin instabilities** may appear.

Aims of the study:

1. Linear stability analysis first carried out for a **2-layer rotating shallow water model** using the **collocation method** to extend known results [3].
2. Linear and nonlinear analysis for **continuously stratified flows** using idealized simulations with a **mesoscale meteorological model (WRF)**.

Linear stability analysis of the frontal configuration in the two-layer fluid

Overview of the model

- Two-layer rotating shallow water model on the f-plane with a vertical shear flow (Fig. 1).
- The domain is a vertically bounded channel of width $2\,Y_{max}$ and height H_0. The lateral boundary conditions are $v_j(\pm Y_{max}) = 0$.
- Linearized equations for the perturbation in non-dimensional form:

$$\begin{cases} \partial_t u_j + Ro\partial_x u_j - v_j = -\partial_x \pi_j \,, \\ \partial_t v_j + Ro\partial_x v_j + u_j = -\gamma \partial_y \pi_j \,, \\ \partial_t h + Ro\partial_x h = \pm(Ro(H_j \partial_x u_j + \gamma \partial_y (H_j v_j))) \,, \end{cases} \quad (1)$$

where $Ro = U_1/(fR_d)$ is the Rossby number, $R_d = (g' H_1/2)^{\frac{1}{2}}/f$ is the Rossby deformation radius and $\gamma = R_d/Y_{max}$.

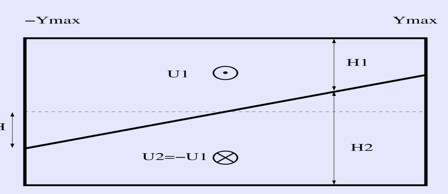

Fig. 1. Schematic diagram of the flow in the 2-layer shallow water model.

Instabilities and growth rates

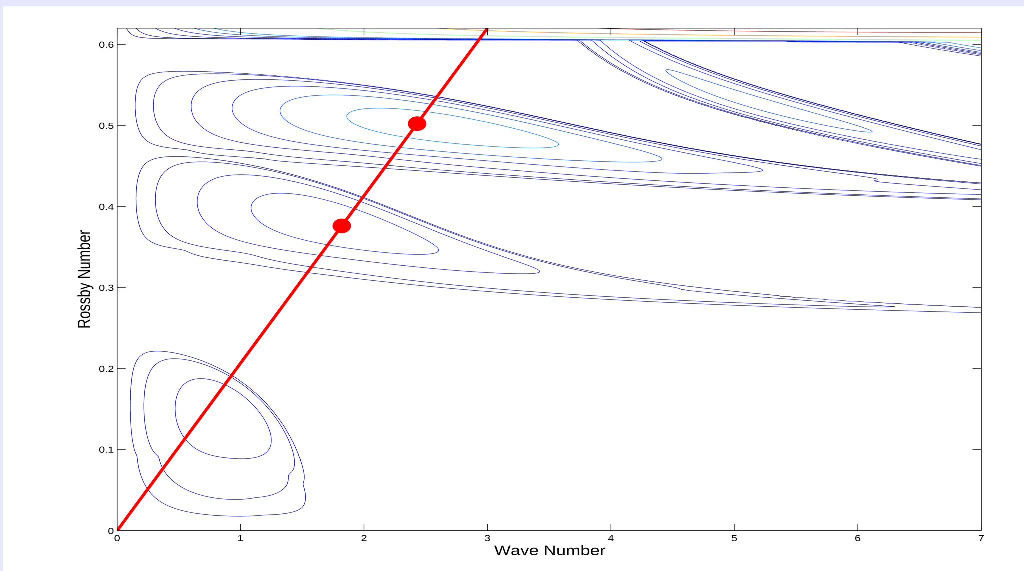

Fig. 2. Growth rate of most unstables modes in (Ro, k)-space for $H_2 = H_1/0.7$. Contours displazed are 0.01, 0.02 and further interval 0.02.

Fig. 2. and 3. shows as an example the growth rates of unstable modes for a non-symmetric configuration ($H_2 = H_1/0.7$):

- For small Ro and k, one finds baroclinic instability.
- For a stronger shear ($Ro \approx 0.6$), Kelvin-Helmoltz instability occurs.
- For intermediate values of the Rossby number, one finds two regions of instability ($Ro \approx 0.4$ and $Ro \approx 0.5$) which correspond to Rossby-Kelvin instability extending results of [3].

These modes exist due to the interaction of a Rossby wave in one layer and a Kelvin wave in the other as shown in Fig. 4.

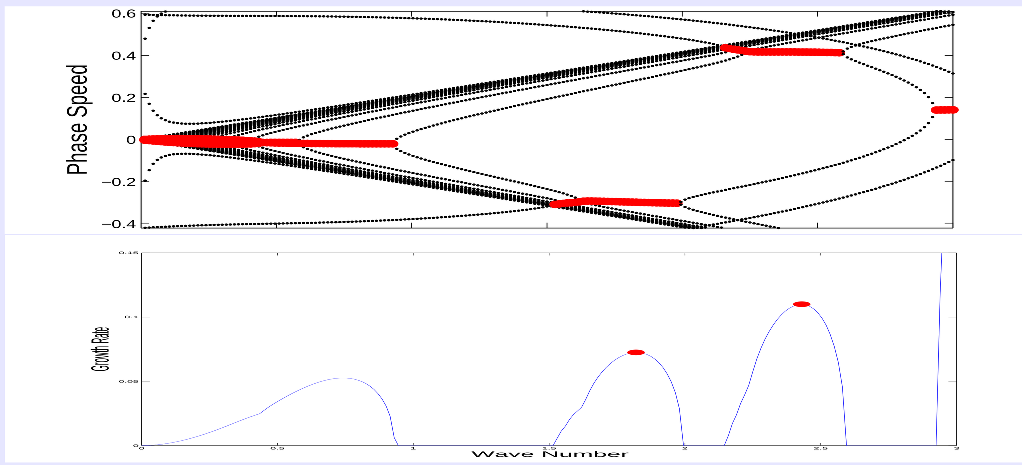

Fig. 3. Dispersion diagram (upper panel) and growth rate (lower panel) of the modes along the section $Ro = k/5$. Red segments on the upper panel correspond to the unstables modes.

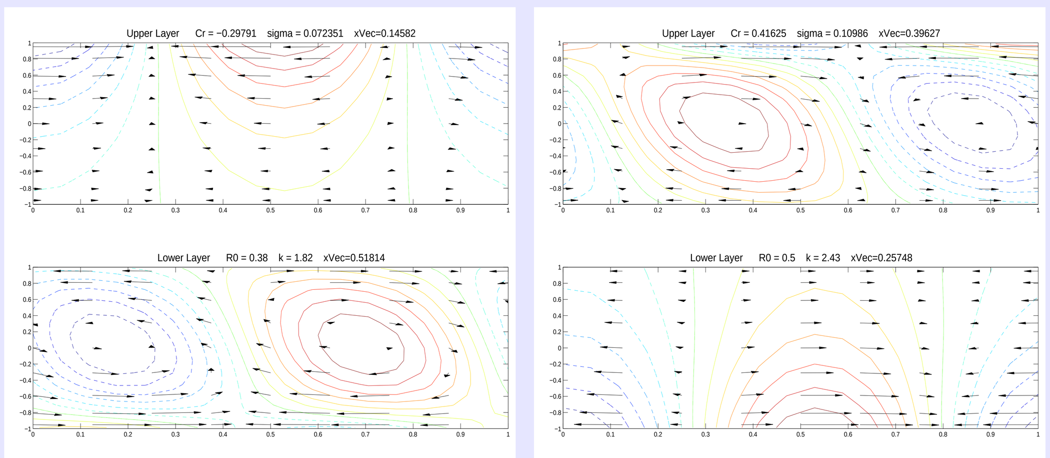

Fig. 4. Pressure and velocity fields of the Rossby-Kelvin instability in the upper layer and the lower layer at the maximum growth rate for $H_2 = H_1/0.7$. Left: $k = 1.8$ and $Ro = 0.4$ and right: $k = 2.4$ and $Ro = 0.5$, as shown by the red points in Fig. 3.

Stability analysis in a continuously stratified fluid

Model setup

- Meteorological mesoscale model: Weather Research and Forecast (WRF).
- Periodic channel on the f-plane bounded by lateral walls in y, with a flat bottom without a boundary layer.
- The basic state is defined by including a stantard stratification and a potential temperature jump along the front (Fig. 5):

$$z_{fr}(y) = z_0 + S\ y,\ \theta = \theta_0 + \Theta_z z + \frac{\theta_{ju}}{2}\left(1 + \tanh(\frac{z - z_{fr}}{z_{ju}})\right) \quad (2)$$

where z_0 is the average height of the front, S the slope, θ_{ju} the potential temperature jump, z_{ju} the thickness of the frontal zone, and Θ_z describes the basic stratification.

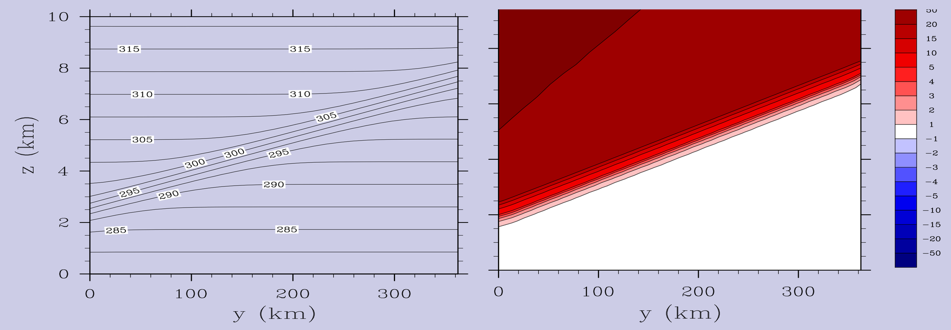

Fig. 5. Initial distribution of potential temperature (Left) and corresponding geostrophically balanced zonal wind (Right) for (z,y) in km.

Unstable modes

Breeding cycles were carried out for the identification of fastest growing unstable mode, if any. The different modes of instability are reproduced, with growth rates that are very close to those obtained with the 2-layer model. In particular, the Rossby-Kelvin instability is also present in the continously stratified case (Fig. 6):

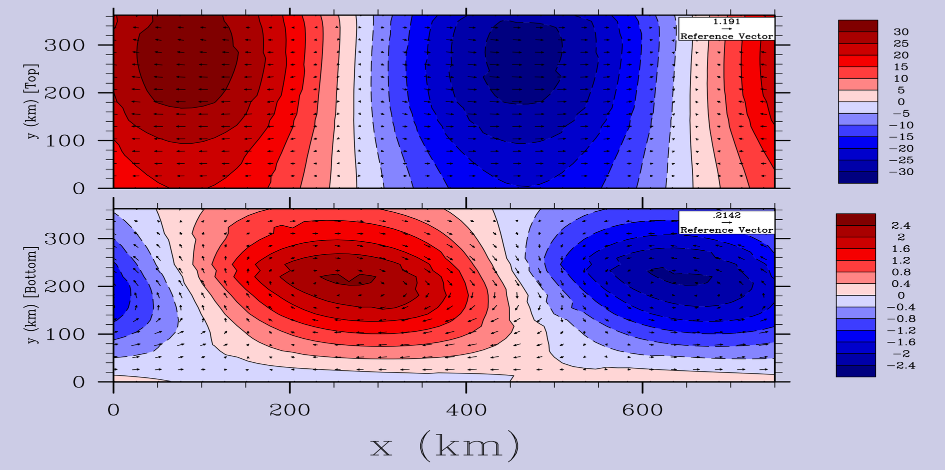

Fig. 6. Pressure and velocity field vertically averaged below and above the front for a Rossby-Kelvin instability ($Ro = 0.5$ and $k = 2.4$).

As for the 2-layer model (Fig. 4), a Rossby wave (geostrophic wind around pressure extrema) is identified clearly below the front, and a Kelvin wave (wind parallel to the boundaries, pressure extrema at the lateral boundaries) is identified above the front.

Growth rate of the unstable modes

The growth rate was evaluated from the evolution of the kinetic energy of the perturbation (Fig. 8) along the line $Ro = k/5$. Non-dimensional growth rates (Fig. 7) are remarkably close to those found for the two-layer model for the case $H_1 = H_2$ in [3].

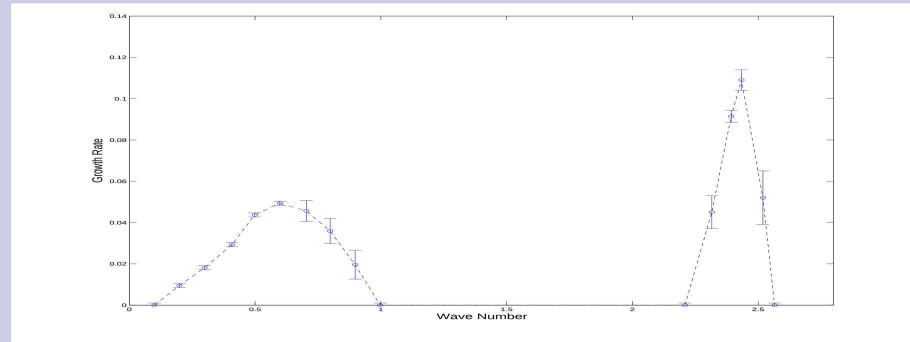

Fig. 7. Growth rates of the different instabilities of a front in the continuously stratified fluid along the line $Ro = k/5$. The peak at low k corresponds to classical baroclinic instability. The peak for larger k corresponds to Rossby-Kelvin instability.

Nonlinear evolution of the instability

The instability shown in Fig. 6 grows for about 5 days (Fig. 8), then saturates. The nonlinear effect of the instability on the zonal mean flow is shown in Fig. 9 (broadening of the frontal zone and modification of the flow in the upper layer near the boundary).

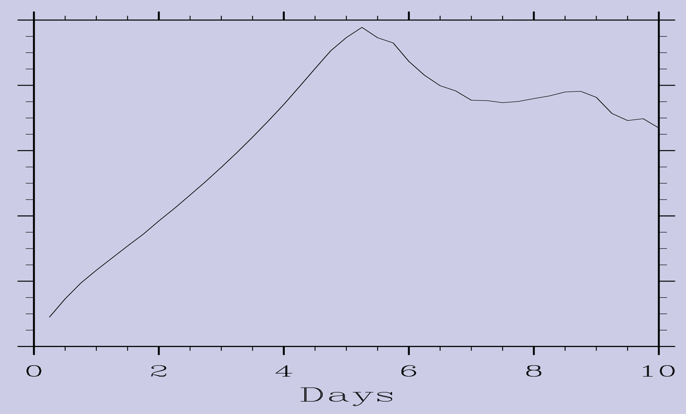

Fig. 8. Logarithm of the Kinetic energy for the Rossby-Kelvin instability ($Ro = 0.5$ and $k = 2.4$).

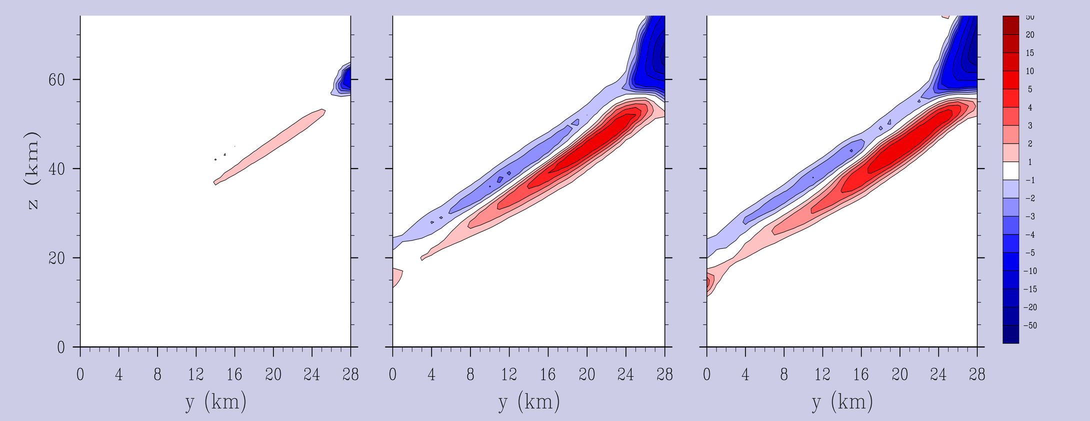

Fig. 9. Zonal mean flow deviation during the growing of the the Rossby-Kelvin instability for days 4, 5 and 6 (from left to right).

Discussion

The linear stability of a front in a rotating stratified fluid has been investigated both for a 2-layer fluid and a continuously stratified fluid. Rossby-Kelvin modes occur in both cases, with quite similar structures. Growth rates are also quite comparable, in spite of important differences between the two models (background stratification, compressibility, upper boundary condition).

This work confirms **existence of the Rossby-Kelvin instability in a frontal region of continuously stratified fluid**. The simulations also allow investigation of the **saturation of this mode**, showing the modification of the frontal zone. Further study of the nonlinear behaviour of the instability is in progress.

References

[1] **R. Plougonven and D.J. Muraki and C. Snyder.** A baroclinic instability that couples balanced motions and gravity waves. J. Atmos. Sci., 62:1545-1559, 2005. [2] **F. Poulin and G. Flierl.** The nonlinear evolution of barotropically unstable jets. J. Fluid Mech., 2173-2192, 2003. [3] **P. Ripa.** General stability conditions for a multi-layer model. J. Fluid Mech., 119-137, 1991. [4] **S. Sakai.** Rossby-Kelvin instability : a new type of ageostrophic instability caused by a resonance between Rossby waves and gravity waves. J. Fluid Mech., 149-176, 1989. [5] **W.C. Skamarock and J.B. Klemp and J. Dudhia and D.O. Gill and D.M. Barker and W. Wang and J.G. Powers.** A description of the Advanced Research WRF Version 2. NCAR Technical Note, 2005.