Instability of a coastal current, nonlinear evolution and vortex formation

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Density-driven coastal currents, which are bounded by a coast and a surface density front are ubiquitous in the ocean, playing essential role in the mesoscale variability of the ocean. Their highly unstable nature is well known from observations, laboratory and numerical experiments. In the present work we revisit the linear stability problem for such currents both in reduced gravity, and fully baroclinic 2-layer models, and study nonlinear evolution of the instability with the help of recent high-resolution finite-volume numerical methods. We show how nonlinear saturation of the ageostrophic instabilities leads to reorganisation of the mean flow and emergence of coherent vortices.

Linear instabilities of the coastal fronts, and their fully nonlinear evolution are first studied within the reduced-gravity rotating shallow-water model. By using the collocation method, we benchmark the classical linear stability results on zero PV fronts ([3]), and generalize them to non-zero PV fronts. In both cases, we find the instabilities due to the resonance of frontal and coastal waves trapped in the current, and identify the most unstable long-wave mode. We then study the nonlinear evolution of the unstable modes with the help of a high-resolution well-balanced finite-volume numerical scheme for shallow-water equations ([1]). The simulations are initialized with the unstable modes obtained from the linear stability analysis. We found that the most unstable mode is growing to finite amplitude, and then saturates ("breaks") leading to the formation of coherent structures in form of vortices and vortex filaments.

These results are then generalized to the two-layer rotating shallow-water model along the same lines. We perform a complete linear stability analysis of the baroclinic coastal currents for various depth and density ratios, and study the nonlinear evolution of the unstable modes with the help of the recent efficient two-layer generalization of the one-layer finite-volume scheme ([2]), which allows to treat outcropping and loss of hyperbolicity associated to Kelvin-Helmholtz (KH) instabilities. The previous single-layer results are recovered in the limit of large depth ratios. For depth ratios of order one, new baroclinic long-wave instabilities come into play, due to the resonances among vortical (Rossby-like) and frontal, or coastal trapped waves. Coherent baroclinic vortices form as these instabilities saturate, and lead to a complete reorganisation of the initial current. For even smaller depth ratios short-wave instabilities of KH type with large growth rates occur (e.g; [4]). We show that at nonlinear stage they produce rapidly growing short-wave meanders with enhanced dissipation, giving way to secondary large-scale instabilities and vortex formation afterwards.

References

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