PHY392 Solution Set 1 2014

1) From hydrostatic balance we have

$$\frac{dP}{dr} = -\rho(r)\, g(r)$$

If ρ is constant

$$\frac{dP}{dr} = -\rho \frac{G\, m(r)}{r^2}$$

at radius r, $m(r) = \frac{4}{3}\pi r^3 \rho$

$$\Rightarrow \frac{dP}{dr} = -\rho \frac{G}{r^2}\left(\frac{4}{3}\pi r^3 \rho\right) = \frac{4}{3}\pi G \rho^2 r$$

$$\left.\begin{array}{l} P(r=0) = P_0 \\ P(r=a) = 0 \end{array}\right\} \int_0^{P_0} dP = -\frac{4}{3}\pi G\rho^2 \int_a^0 r\, dr$$

$$\Rightarrow P_0 = \frac{2}{3}\pi G \rho^2 a^2$$

since $g(a) = \frac{G\, m(a)}{a^2} = \frac{G}{a^2}\frac{4}{3}\pi a^3 \rho$

$$= \frac{4}{3}\pi G a \rho$$

$$\Rightarrow \boxed{P_0 = \frac{1}{2} g(a) \rho a}$$

for $\rho \simeq 2000\ kg\, m^{-3}$ $\quad g(a) = 9.8\ m s^{-2}$

and $a \simeq 6.37 \times 10^6\ m$

$$P_0 = \frac{1}{2}(9.8)\, 2000\, (6.37 \times 10^6\ m)$$

$$\boxed{P_0 = 6 \times 10^{10}\ Pa = 6 \times 10^5\ atm}$$

2) On Venus $P_s \simeq 92$ bar

1 bar $= 1000$ mb $= 10^5$ Pa

1 atm $= 1.013$ bar

$\Rightarrow P_s = 92 \times 10^5\ Pa$ $\quad$ for $g = 8.87\ m/s^2$

mass/unit area $= \dfrac{P_s}{g} = \dfrac{92 \times 10^5\ Pa}{8.8\ m/s^2}$

$$= \boxed{1.04 \times 10^6\ kg/m^2}$$

For Earth $\dfrac{P_s}{g} = \dfrac{1.013 \times 10^5\ Pa}{9.8\ m/s^2}$

$$= \underline{1.03 \times 10^4\ kg/m^2}$$

a hundred times smaller

On Venus the atmosphere is 96% CO_2 and 3.5% N_2, so the average molecular weight

$$= 0.96(44) + 0.035(28) = 43.22$$

The mass fraction for $N_2 = 0.035\left(\frac{28}{43.22}\right)$

$$\Rightarrow N_2 \text{ mass} = 0.035\left(\frac{28}{43.22}\right)\left(1.04 \times 10^6 \text{ kg m}^{-2}\right)$$

$$\boxed{N_2 \text{ mass} = 2.3 \times 10^4 \text{ kg m}^{-2}}$$

For Earth, the average molecular weight is 28.97 and N_2 = 78% of atmosphere,

$$\Rightarrow N_2 \text{ mass} = 0.78\left(\frac{28}{28.97}\right) 1.03 \times 10^4 \text{ kg m}^{-2}$$

$$= \underline{7.8 \times 10^3 \text{ kg m}^{-2}}$$

Venus has about 3 times as much N_2 as Earth, even though Earth's atmosphere is 78% N_2.

If the initial composition of the atmospheres was similar, Earth must have lost its nitrogen or sequestered it in the crust or interior

3) To ventilate the town we require a lapse rate Γ_c such that $\Gamma_c \geq \Gamma_d$ so that vertical motions are not inhibited.

a) We need $-\frac{dT_e}{dz} = 10°C/km$ between $z = 0 - 0.5$ km

$\Rightarrow T(z=0) = 5°C \quad \Rightarrow \Delta T(z=0) = 10°C$

$T(z=0.5) = 0°C \quad \Rightarrow \Delta T(z=0.5) = 0$

$$\boxed{\Delta T(z) = a(z_T - z)}$$

$a = 20°C/km$ and $z_T = 0.5$ km

b) $dQ = c_p dT$ per unit mass

heat required per unit area:

$$Q = \rho \int_0^{z_T} \Delta T(z)\, dz$$

$$Q = a \rho c_p \int_0^{z_T} (z_T - z)\, dz$$

$$Q = \frac{a}{2} \rho c_p z_T^2 \qquad a = 2 \times 10^{-2}\ K/m$$

$$Q = \frac{2 \times 10^{-2}\ K/m}{2} \left(1\ kg\ m^{-3}\right)\left(1 \times 10^{3}\ J\ kg^{-1}\ K^{-1}\right)(500\ m)^2$$

$$\boxed{Q = 2.5 \times 10^{6}\ J\ m^{-2}}$$

c) $$Q = \int_{t_0}^{t_1} F(t)\, dt = \int_{t_0}^{t_1} F_{max} \cos\left[\frac{2\pi (t - t_{noon})}{\Delta t}\right] dt$$

$$Q = F_{max} \frac{\Delta t}{2\pi}\left[\sin\left(\frac{2\pi (t_1 - t_{noon})}{\Delta t}\right) - \sin\left(\frac{2\pi (t_0 - t_{noon})}{\Delta t}\right)\right]$$

$$t_1 = t_{noon} - \frac{\Delta t}{2\pi} \sin^{-1}\left[\frac{2\pi Q}{F_{max}\, \Delta t} + \sin\left(\frac{2\pi (t_0 - t_{noon})}{\Delta t}\right)\right]$$

$$= t_{noon} - \frac{\Delta t}{2\pi} \sin^{-1}\left[\frac{2\pi Q}{F_{max} \Delta t} + \sin\left(-\frac{\pi}{2}\right)\right]$$

$$= t_{noon} - \frac{\Delta t}{2\pi} \sin^{-1}\left[\frac{2\pi (2.5 \times 10^{6})}{300 (86400)} - 1\right]$$

$$t_1 = 12\,hr - \frac{24\,hr}{2\pi} \sin^{-1}\left[-0.39\right]$$

$$= 12\,hr - 1.53\,hr \simeq 10.47\,hr$$

$$\boxed{t_1 \simeq 10:30\ am}$$

4) Since the planet has a global ocean, water will evaporate into the atmosphere until the system is in equilibrium. With a surface temp of $T_s = 280K$, the saturation vapour pressure will be 1000 Pa (10 mb) (see Figure 2.5).

If we assume a dry adiabat (i.e. we somehow supress condensation)

$$\frac{d\ln T}{d\ln p} = \frac{R_{H_2O}}{c_p}$$

$$\Rightarrow \quad T(p) = T_s \left(\frac{p}{p_s}\right)^{R_{H_2O}/c_p}$$

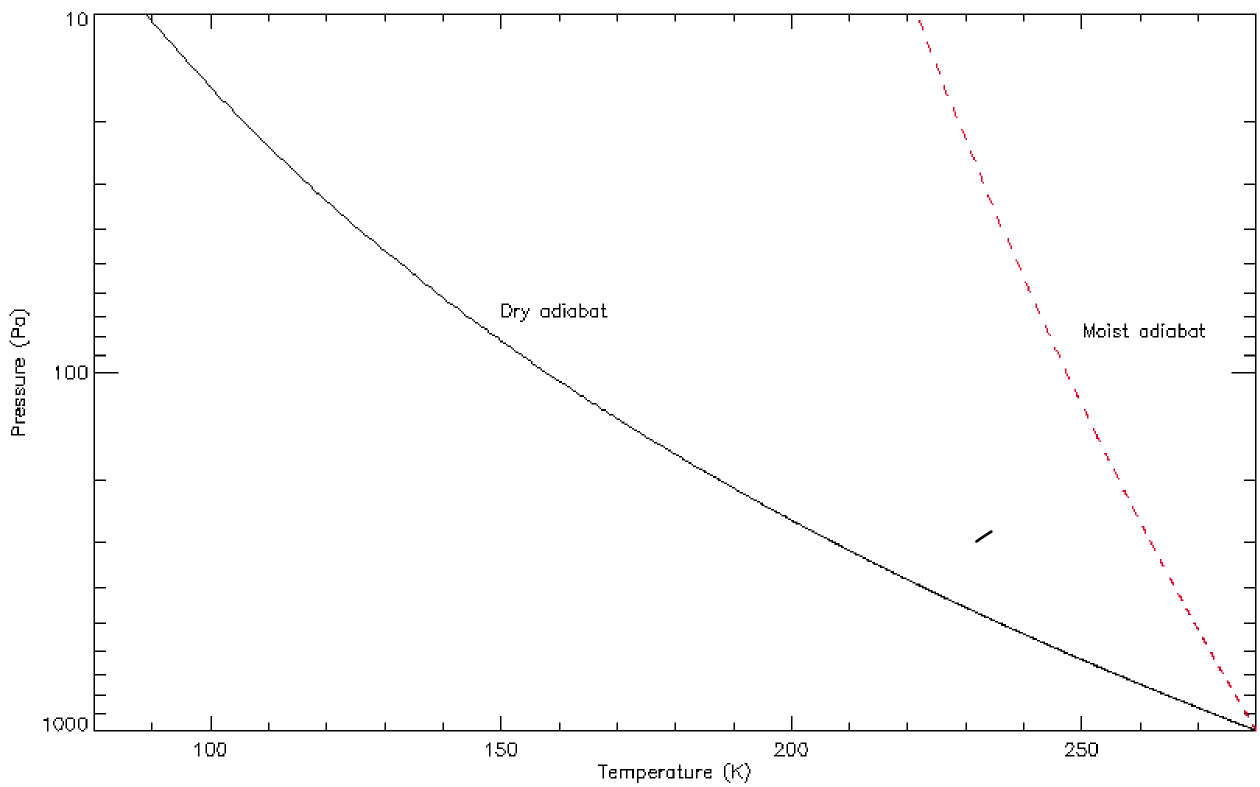
$$c_p = c_v + R_{H_2O} \Rightarrow T(p) = T_s \left(\frac{p}{p_s}\right)^{(1-1/\gamma)}$$

with $T_s = 280\,K$, $p_s = 10\,mb$, $\gamma = c_p/c_v$

Alternatively, we can estimate R_{H_2O} as:

$$R = \frac{R^*}{M_a} = 287 \ J/kg K$$

$$\Rightarrow R_{H_2O} = R \frac{M_a}{M_{H_2O}} \approx 287 \left(\frac{28.97}{18} \right) = 462 \ J kg^{-1} K^{-1}$$



If we use the moist adiabat, eq (2.27) then

$$T(P) = \frac{T_s}{1 - \frac{R_{H_2O} T}{L} \ln(P/P_s)}$$

The plot shows the moist adiabat with

$L = 22.55 \times 10^5 \ J \ kg^{-1}$

$T_s = 280 K$, $R_{H_2O} = 462 \ J \ kg^{-1} K^{-1}$

$P_s = 10 \ mb$

In this case, the rate of decrease of T with decreasing P is much smaller, as expected.

No points lost for not including the moist adiabat

Depth of the atmosphere.

From the barometric law

$$\rho(z) = \rho_0 \, e^{-z/H}$$

where $H = \frac{RT}{g} = \frac{(462 \ J \ kg^{-1} K^{-1})(280 \ K)}{1.3 \ m \ s^{-2}}$

$$\Rightarrow \boxed{H \simeq 1.0 \times 10^5 \ m = 100 \ km}$$

H = The approximate depth which contains most of the mass of the atmosphere.

5) a) As the parcel descends, it will warm at the dry adiabatic rate

$$T_p = T_e(z) + \Gamma \Delta z$$

but $\Delta z = w \Delta t$

$$\Rightarrow \quad T_p = T_e(z) + \frac{g}{c_p} w \Delta t$$

to determine how much warmer is the parcel, we need to know the change in the environmental temperature

$$T_e(z - \Delta z) = T_e(z) - \frac{dT_e}{dz} \Delta z$$

$$= T_e(z) - \frac{dT_e}{dz} w \Delta t$$

$$\Delta T = T_p - T_e(z - \Delta z)$$

$$= \left(T_e(z) + \frac{g}{c_p} w \Delta t\right) - \left(T_e(z) - \frac{dT_e}{dz} w \Delta t\right)$$

$$= \left(\frac{g}{c_p} + \frac{dT_e}{dz}\right) w \Delta t$$

$$\boxed{\Delta T = w \Lambda_e \Delta t} \qquad \boxed{\Lambda_e = \frac{dT_e}{dz} + \frac{g}{c_p}}$$

5) If T_p is alway equal to T_e,
the parcel must lose ΔQ energy
per unit volume in time Δt

$$\Delta Q = \rho c_p \Delta T$$
$$= \rho c_p w \Lambda_e \Delta t$$
$$\frac{dQ}{dt} = \rho c_p w \Lambda_e$$

Intergrate WRT z to get energy per
unit horizontal area

$$J_{cool} = \int_0^{\infty} \frac{dQ}{dt} dz \qquad 0$$

$$\Rightarrow J_{cool} = \int_0^{\infty} \rho c_p w \Lambda_e dz = -\int_{P_s}^{0} \frac{c_p}{g} w \Lambda_e dP$$

Since $\frac{dP}{g} = -\rho dz$

$$\Rightarrow \boxed{\int_0^{\infty} \rho c_p w \Lambda_e dz = \frac{c_p}{g} \int_0^{P_s} w \Lambda_e dP}$$

c) If $J_{cool} = 20\ W/m^2$

let $J_{cool} = \frac{c_p \Lambda_e p_s \bar{w}}{g}$

where $\bar{w} = \frac{1}{p_s} \int_0^{p_s} w\, dp$

$$\bar{w} = \frac{g J_{cool}}{c_p \Lambda_e p_s} \qquad p_s = 1000\ hPa = 10^5\ Pa$$

$$\Lambda_e = \frac{dT_e}{dz} + \frac{g}{c_p} = -7\ K/km + \frac{9.8\ m/s^2}{1005\ J\,kg^{-1}K^{-1}}$$

$$= -7 \times 10^{-3}\ K/m + \frac{9.8}{1005}$$

$\Lambda_e = 2.8 \times 10^{-3}\ K/m$

$$\Rightarrow \bar{w} = \frac{(9.8\ m\,s^{-2})(20\ W\,m^{-2})}{(1005\ J\,kg^{-1}K^{-1})(2.8\times10^{-3}\ K\,m^{-1})(1\times10^5\ Pa)}$$

$$= 6.97 \times 10^{-4}\ m/s$$

$$\boxed{\bar{w} = 0.697\ mm/s}$$