PHY392 Solution Set 4 2014

1) a) At position 1 $\tau_\nu = 4.0$
position 2 $\tau_\nu = 0.2$

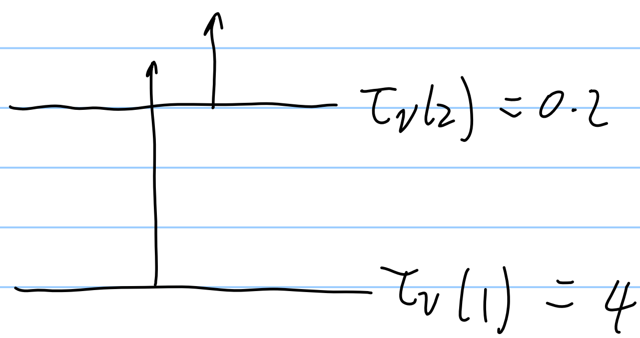
$$I_{\nu,1} = I_\nu(0)\, e^{-\tau_\nu(1)} \quad , \quad I_{\nu,2} = I_\nu(0)\, e^{-\tau_\nu(2)}$$

$$\Delta I_\nu = I_\nu(0)\left(e^{-\tau_\nu(1)} - e^{-\tau_\nu(2)}\right)$$

$$= I_\nu(0)\left(e^{-4.0} - e^{-0.2}\right) = -0.80\, I_\nu(0)$$

$\Rightarrow$ $\boxed{\text{80\% absorbed}}$

b)



$I_\nu = I(0)\, T_\nu$

T_ν = transmittance

fraction outgoing from the top layer

$$= T_\nu(2) = e^{-0.2}$$

fraction outgoing from the bottom layer

$$= T_\nu(1) = e^{-4.0}$$

From the whole layer $e^{-0.2} - e^{-4.0} \Rightarrow \boxed{80\%}$

c) $$\tau_\nu = \int_z^\infty K_\nu \rho \, dz$$ where ρ = density of the absorbing gas.

$$\rho = Q \rho_{air}$$

$$\Rightarrow \tau_\nu = \int_z^\infty K_\nu Q \rho_{air} \, dz$$

For an isothermal atmosphere

$P(z) = P_0 e^{-z/H}$ and $\rho_{air}(z) = \rho_0 e^{-z/H}$

So we can replace ρ_{air} with the expression for its altitude dependence

$$\tau_\nu = \int_z^\infty K_\nu Q \rho_0 e^{-z/H} \, dz$$

$$\Rightarrow \tau_\nu = \frac{K_\nu Q \rho_0}{H} e^{-z/H}$$

so $$z = -H \ln \left(\frac{H \tau_\nu}{K_\nu Q \rho_0} \right)$$

$$z_1 = -H \ln\left(\frac{4H}{K_\nu Q \rho_0}\right) \qquad z_2 = -H\left(\frac{0.2H}{K_\nu Q \rho_0}\right)$$

$$\Delta z = z_2 - z_1 = H \ln\left(\frac{4}{0.2}\right)$$

$$\Rightarrow \boxed{\Delta z = 3.0\,H}$$

2) a) $$T_\nu(\infty, z) = \exp\left[-\int_z^\infty \frac{K_\nu \rho\, dz'}{\cos\theta}\right]$$

$$= \exp\left[-\frac{K_\nu Q}{\mu}\int_z^\infty \rho_{air}\, dz'\right]$$

with $\mu = \cos\theta$

Since K_ν and Q are constant.

We have $dP = -\rho_{air}\, g\, dz$

$\Rightarrow$

$$T_\nu(\infty, z) = \exp\left[\frac{K_\nu Q}{\mu}\int_{P(z)}^{0} \frac{dP}{g}\right]$$

$$\Rightarrow \boxed{T_\nu(\infty, z) = \exp\left[-\frac{K_\nu Q P(z)}{g\mu}\right]}$$

Note we did not specify overhead ($\theta = 0$) geometry, but solution neglecting $\cos\theta$ will be accepted.

b) $W_\nu = \frac{dT_\nu}{dy}$ with $y = -\ln(P)$

$$dy = -\frac{dP}{P}$$

$$\Rightarrow W_\nu = -P\frac{dT}{dP} = -P\frac{d}{dP}\left(\exp\left[\frac{-K_\nu Q P(z)}{\mu g}\right]\right)$$

$$= \frac{P K_\nu Q}{\mu g}\exp\left(\frac{-K_\nu Q P(z)}{\mu g}\right)$$

let $C = \frac{K_\nu Q}{\mu g}$

$$\Rightarrow \boxed{W_\nu = C P(z)\, e^{-CP(z)}}$$

c) We obtain the level at which W_ν is a maximum by

$$\frac{d}{dz}\left(W_\nu\right) = 0 = c\, e^{-c\rho(z)} + c\rho(z)(-c)\, e^{-c\rho(z)}$$

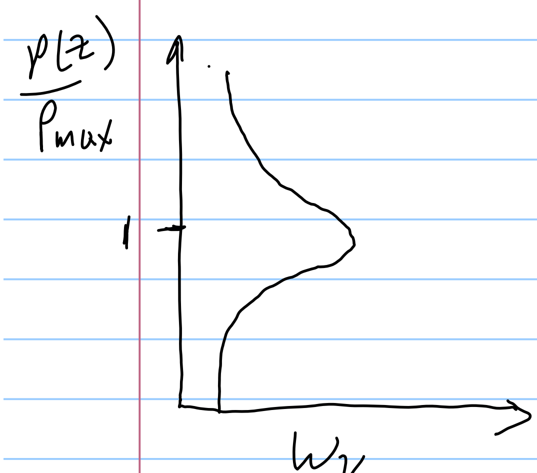
$$\Rightarrow \quad 1 - c\rho(z) = 0$$

$$c\rho(z) = 1$$

$$\boxed{\rho_{max} = 1/c = \frac{\mu g}{K_\nu Q}}$$

d) Since $c = 1/\rho_{max}$

$$\Rightarrow \quad W_\nu = \frac{\rho(z)}{\rho_{max}}\, e^{-\rho(z)/\rho_{max}}$$



The weighting function gives the contribution of emission from each layer in the atmosphere, at wavenumber ν, to the intensity at the top of the atmosphere.

3) $K_\nu^L(\nu) = \frac{S}{\pi} \frac{\alpha_L}{(\nu-\nu_0)^2 + \alpha_L^2}$ for Lorentz broadening

$K_\nu^D(\nu) = \frac{S}{\sqrt{\pi}\,\alpha_D} \exp\left[-\frac{(\nu-\nu_0)^2}{\alpha_D^2}\right]$ for Doppler

at $\nu = \nu_0$, with $\alpha_L = \alpha_L^0 \frac{p}{p_0}\sqrt{\frac{T_0}{T}}$

and $\alpha_D = \sqrt{\frac{2kT}{m}}\frac{\nu_0}{c}$ (from lecture 15),

$K_\nu^L = \frac{S}{\pi\alpha_L} = \frac{S}{\pi\alpha_L^0}\frac{p_0}{p}\sqrt{\frac{T}{T_0}}$

$K_\nu^D = \frac{S}{\sqrt{\pi}\,\alpha_D}$

$\Rightarrow \frac{S}{\pi\alpha_L^0}\frac{p_0}{p}\sqrt{\frac{T}{T_0}} = \frac{S}{\sqrt{\pi}\,\alpha_D}$

$\Rightarrow p = \frac{p_0}{\sqrt{\pi}}\sqrt{\frac{T}{T_0}}\frac{\alpha_D}{\alpha_L^0}$

with α_L^0 in s^{-1} we need α_D in s^{-1},

$\sqrt{\frac{2kT}{m}}$ has units of m/s

so $\alpha_D = \sqrt{\frac{2kT}{m}} \frac{\nu_0}{c}$ has units of m^{-1}

so we need $c\,\alpha_D$

$\nu_0 = 667\ cm^{-1} = 66700\ m^{-1}$

Mass of CO_2 molecule $\simeq 7.3 \times 10^{-26}$ kg

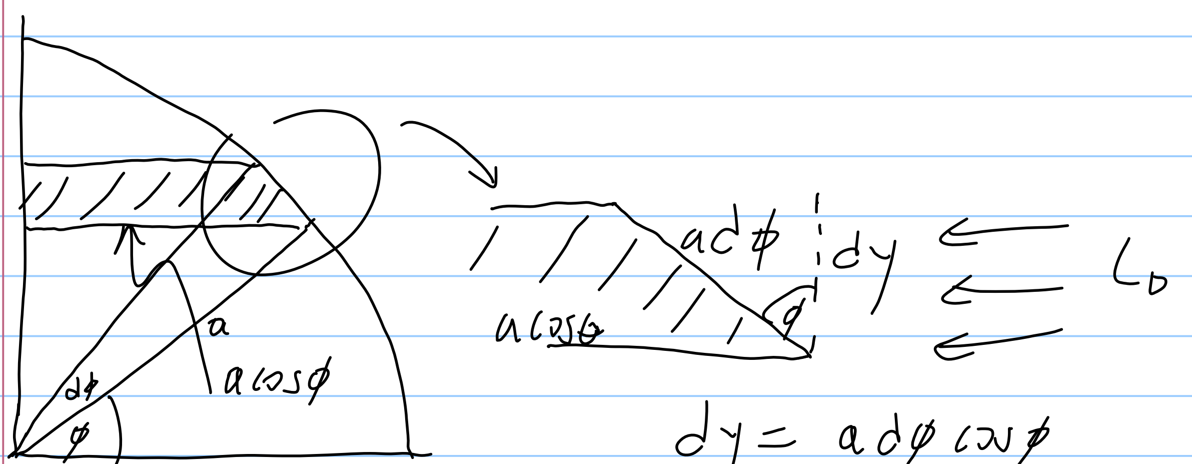
$$\Rightarrow\ p = \frac{p_0}{\sqrt{\pi}} \sqrt{\frac{T}{T_0}} \sqrt{\frac{2kT}{m}} \frac{\nu_0}{\alpha_L^0}$$

$$= \frac{1000\ hPa}{\sqrt{\pi}} \left(\frac{2\,(1.381 \times 10^{-23})\,255}{7.3 \times 10^{-26}} \right)^{1/2} \frac{66700}{3 \times 10^{9}}$$

$$= 1000\ hPa\ \left(3.90 \times 10^{-3}\right)$$

$$\boxed{p \simeq 3.90\ hPa}$$

4)

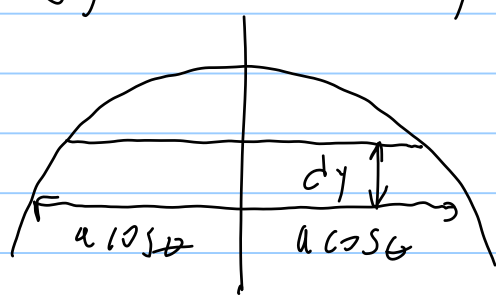


$$dy = a\,d\phi\cos\phi$$

area of band between ϕ and $\phi + d\phi$

$$\approx (2\pi a\cos\phi)\,a\,d\phi = 2\pi a^2\cos\phi\,d\phi$$

energy intercepted by the planet = (area) L_0



$$\text{area} = 2(\text{radius})\,dy = 2a\cos\theta\,dy$$

$$= 2a^2\cos^2\phi\,d\phi$$

flux per unit area

$$\boxed{F = \frac{L_0\,2a^2\cos^2\phi\,d\phi}{2\pi a^2\cos\phi\,d\phi} = \frac{L_0\cos\phi}{\pi}}$$

a) For a single slab atmosphere

In each band

at the top of the atmosphere $\quad \dfrac{S_0 \cos\phi}{\pi}\left(1 - A_p\right) = \sigma T_a^4$

at the surface $\quad \dfrac{S_0 \cos\phi}{\pi}\left(1 - A_p\right) + \sigma T_a^4 = \sigma T_s^4$

$$\Rightarrow \quad T_s = \left[\frac{2 S_0 \cos\phi \left(1 - A_p\right)}{\pi \sigma}\right]^{1/4} = \left[\frac{8\cos\phi}{\pi} \frac{\left(1 - A_p\right) S_0}{4\sigma}\right]^{1/4}$$

$$\Rightarrow \quad \boxed{T_s = \left(\frac{8\cos\phi}{\pi}\right)^{1/4} T_e}$$

where

$$T_e = \left[\frac{\left(1 - A_p\right) S_0}{4\sigma}\right]^{1/4}$$

b) $T(0°) = \left(\dfrac{8}{\pi}\right)^{1/4} 255\,K \Rightarrow \boxed{T(0°) = 322\,K}$

$T(30°) = \left[\dfrac{8\cos(30)}{\pi}\right]^{1/4} 255\,K \Rightarrow \boxed{T(30°) = 311\,K}$

$$T(60°N) = \left[\frac{8\cos(60)}{\pi}\right]^{1/4} 255K$$

$$\boxed{T(60°) = 271\ K}$$

From the figure shown:

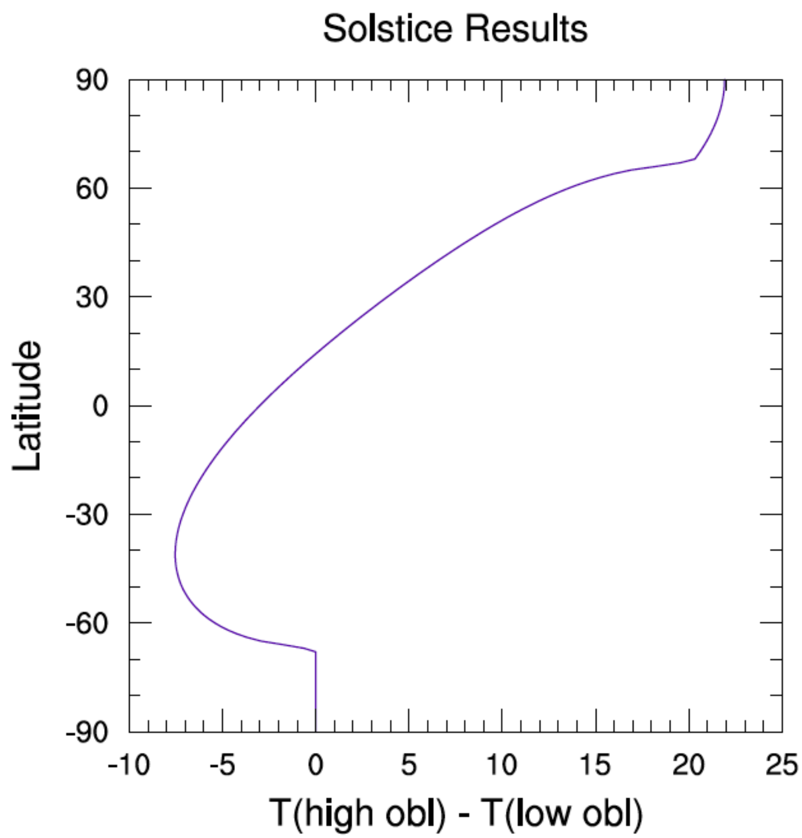
$T(0°) \approx 27°C = 300K$

$T(30°) \approx 20°C = 293K$

$T(60°) \approx 0C = 273K$

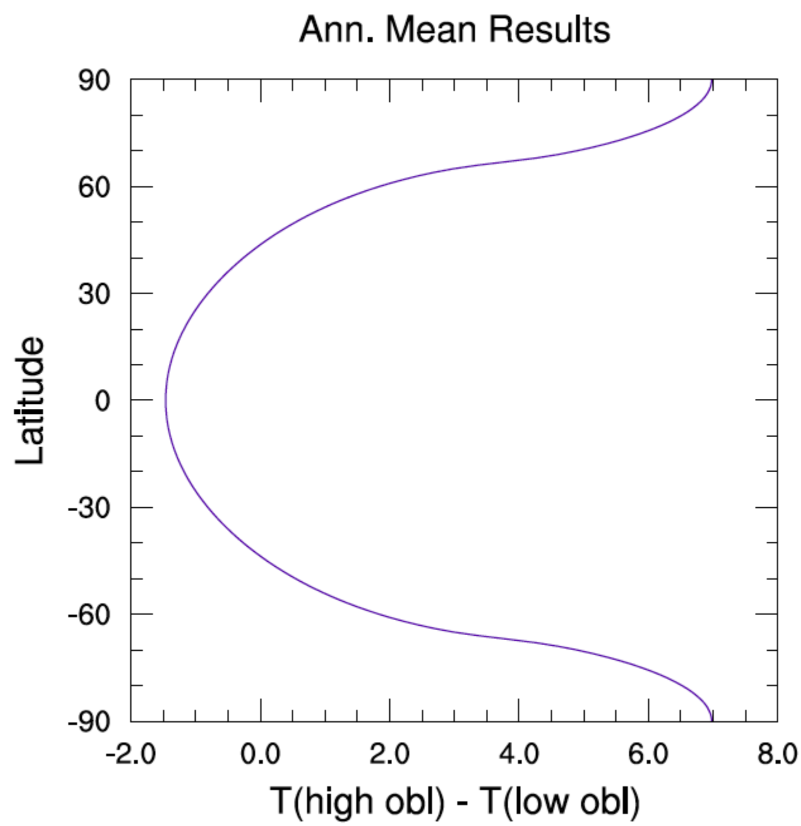
Observed temperatures are low due to the motions of the atmosphere, such as vertical convection and poleward heat transport.

5)



There is a 20K difference at the summer pole in the high obliquity case compared to the low obliquity case. Because there is little sunlight at the winter pole the temperature impact there is zero.

In the mid-latitudes in the winter hemisphere there is modest cooling.



Annually averaged, both poles warm by about 7K and the tropics cool slightly. What we are seeing is a redistribution of the solar insolation ie. the global mean insolation doesn't change with changing obliquity, only the latitudinal distribution of the flux.

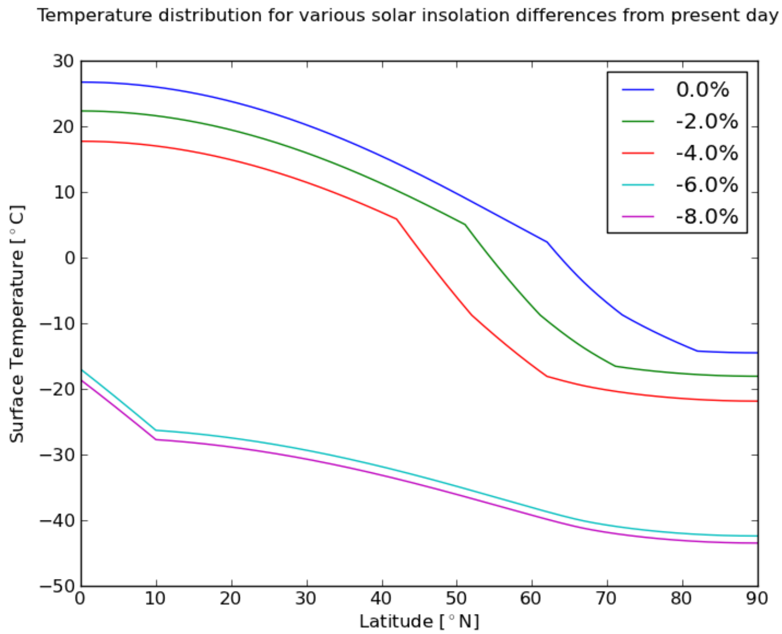


Figure 1: Surface temperature distribution for various values of solar constant

delQp [%]	Global mean temperature [°C]	Sea ice boundary [°N]
0	16.4	72
-2	11.2	61
-4	5.6	52
-6	-30.2	0
-8	-31.6	0

Table 1: Calculated global mean temperature and sea ice boundary for various values of **delQp**.

6) The Budyko model

(a) The model gives a fairly realistic temperature distribution for **delQp**=0%. The temperature is warmest at the equator and decreases towards the pole where there is lower solar insolation and a higher albedo due to the presence of sea ice. The global mean temperature is about 16°C, which is roughly similar to current conditions on Earth.

As **delQp** is decreased, the temperature at all latitudes decreases, as we would expect (see Figure 1 and Table 1). However, the decrease is not smooth: between -4% and -6% there is a much larger change in the temperature distribution than between any other values **delQp**. The reason for this sudden jump is that at **delQp**=-6.0% the Earth has entered a "snowball Earth" state in which the entire surface is covered with ice.

(b) Increasing or decreasing the albedo over either the ocean or sea ice has the effect of, respectively, decreasing or increasing the surface temperature. The surface temperature distribution is more sensitive to changes in the ocean's albedo than changes in the ice's albedo, primarily because the ocean covers much more surface area than the sea ice. See Figure 2.

~~(c) See attached code for how to properly incorporate the `for` loop and create the appropriate figures.~~

(d) As seen in Figure 3, at just less than -6% reduction in solar insolation there is a dramatic jump where the Earth becomes entirely ice-covered and the global mean temperature correspondingly decreases. This a substantially larger decrease than in [1] where the value at which the jump occured was around -1.6%. It is similar to the value found in Question 2a (around -6.4%).

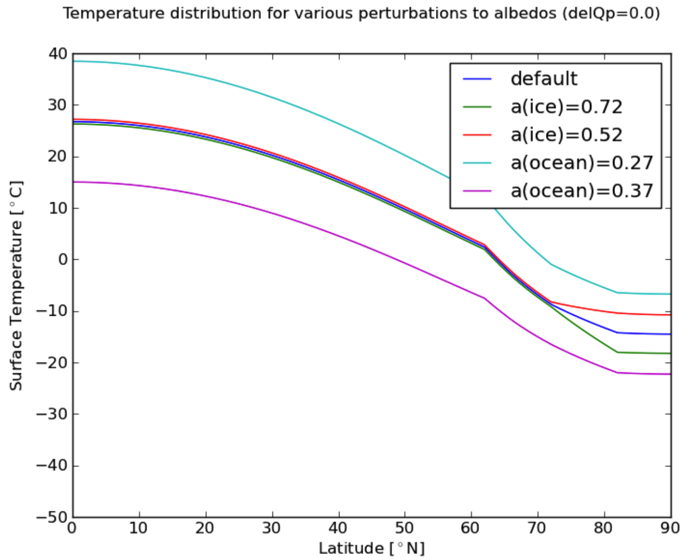


Figure 2: Surface temperature distribution for various perturbations to the surface albedo.

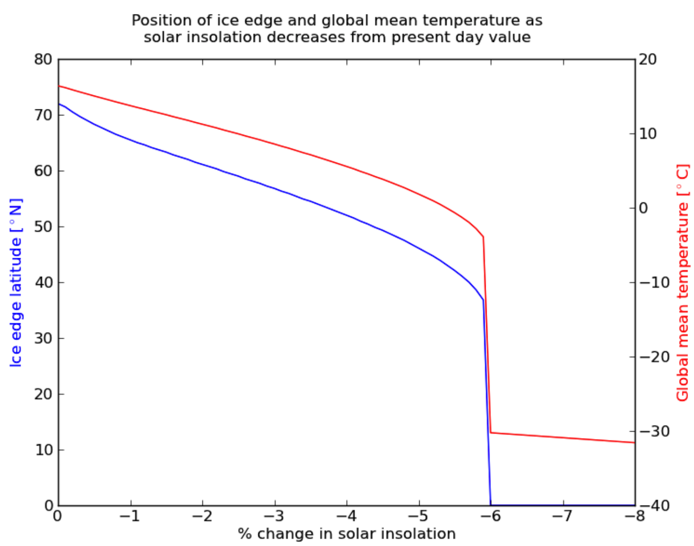


Figure 3: Extent of sea ice (blue) and global mean temperature (red) as a function of **delQp**.

(e) See Figures 4 and 5. As one would expect, by increasing the sea ice albedo, the shift to a snowball Earth state happens sooner (i.e. for a smaller decrease in solar insolation). The opposite is also true: decreasing the sea ice albedo means the shift doesn't happen until there is a larger decrease in insolation.

The effect of changing the albedo of the ocean is reversed. When we increase the albedo of the ocean, even though the global mean temperature is lower for **delQp**=0, the shift to complete sea ice cover doesn't occur until **delQp**=-7.5%. If the albedo of the ocean is decreased, the shift happens sooner.

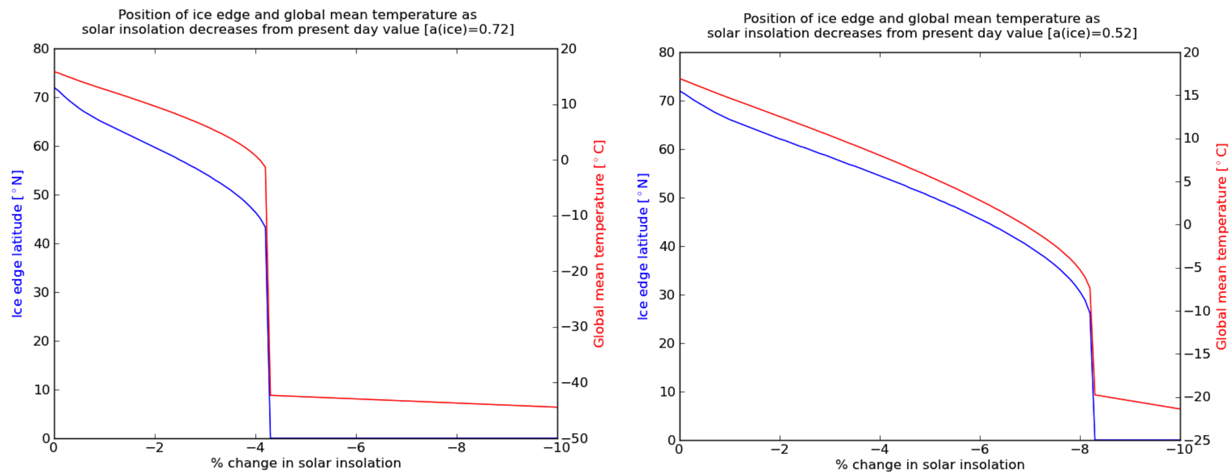


Figure 4: Extend of sea ice and global mean temperature as a function of **delQp** for an ice albedo of 0.72 (left) and 0.52 (right) as opposed to the default value of 0.62.

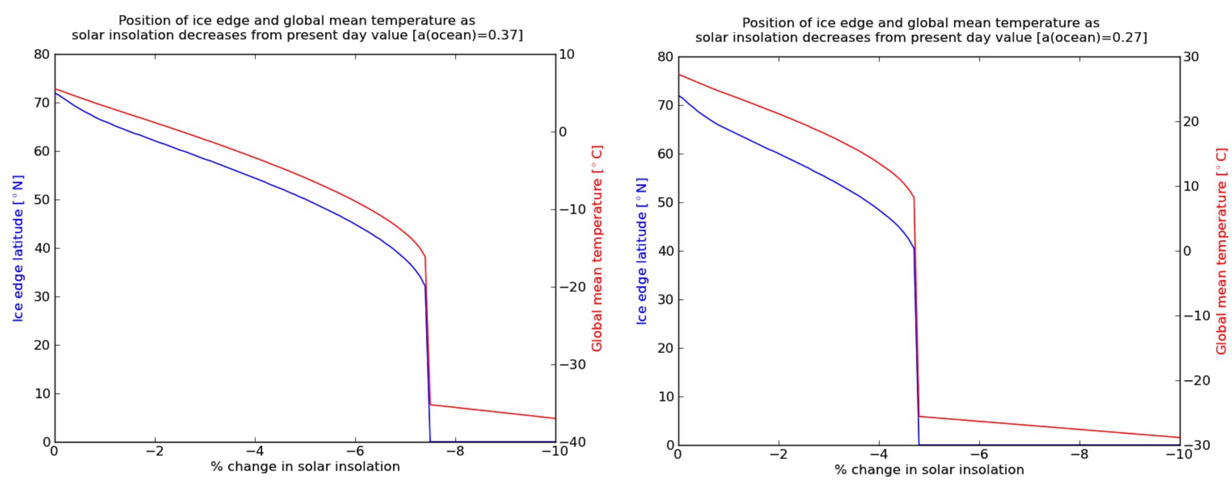


Figure 5: Extend of sea ice and global mean temperature as a function of **delQp** for an ocean albedo of 0.37 (left) and 0.27 (right) as opposed to the default value of 0.32.