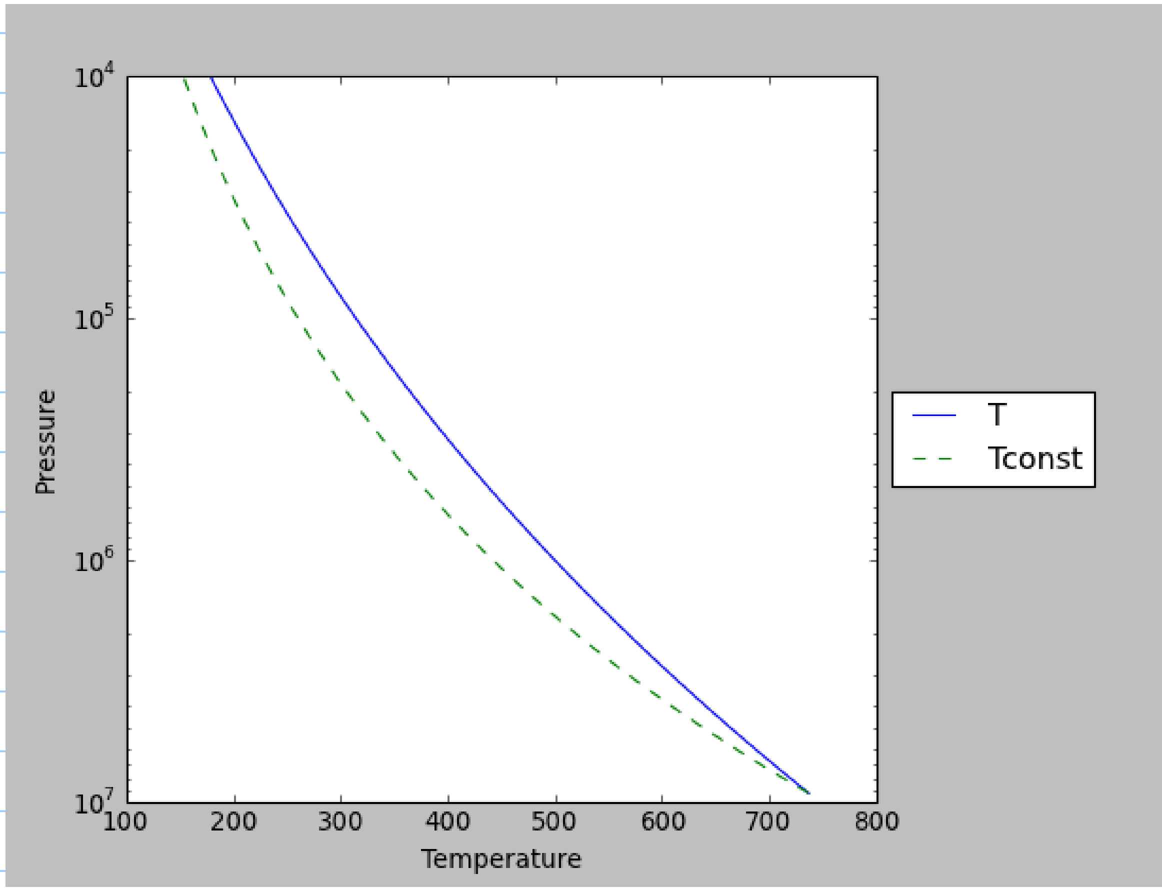
PHY392 Solution Set 3 2014

1)



1 bar = 10^5 Pa

At 10 bars our temperature-dependent calculation yields a temperature of about 500 K, which is consistent with the Magellan data in Figure 2.2. Assuming a constant c_p gives an underestimate in T ($T \approx 450$ K).

At 1 bar we have T = 325 K, whereas T = 250 K with constant c_p. From Magellan we have T = 340 K

At 0.1 bar we have T = 175 K, whereas

$T = 150K$ with constant c_p. Observations suggest $T = 225K$. A possible source of error in our calculation could be the assumption that $p = \rho T$ holds.

2) for $n = \alpha T_s^{\beta}$

$$T_s^4 = (\alpha T_s^{\beta} + 1) Te^4$$

$$T_s^4 - \alpha Te^4 T_s^{\beta} - Te^4 = 0$$

a) for $\beta = 4$

$$\frac{T_s^4 - Te^4}{Te^4 T_s^4} = \alpha$$

$$\frac{1}{Te^4}\left[1 - \left(\frac{Te}{T_s}\right)^4\right] = \alpha$$

for $T_s = 288K$ and $Te = 255K$

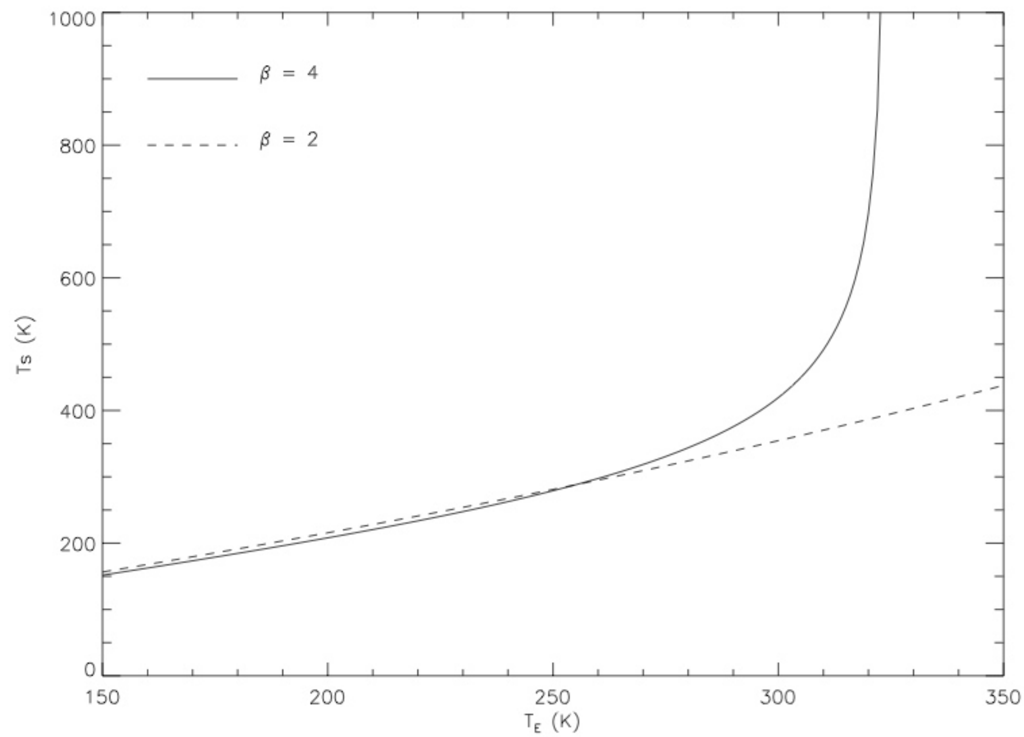
$$\Rightarrow \boxed{\alpha = 9.1 \times 10^{-11} K^{-4}}$$

For $\beta = 2$

$$T_s^4 - \alpha Te\, T_s^2 - T_e^4 = 0$$

$$\alpha = \frac{1}{T_s^2}\left[\left(\frac{T_s}{Te}\right)^4 - 1\right] = \boxed{7.56 \times 10^{-6}}$$

b)



I would characterize the case of $\beta = 4$ as an example of a runaway greenhouse effect as the surface temperature diverges for a a finite T_e.

Furthermore, from the Clausius-Clapeyron relationship the saturation vapour pressure of water increases exponentially with temperature so we expect an exponential increase in n

c) If the albedo of Venus were $A_p = 0.15$

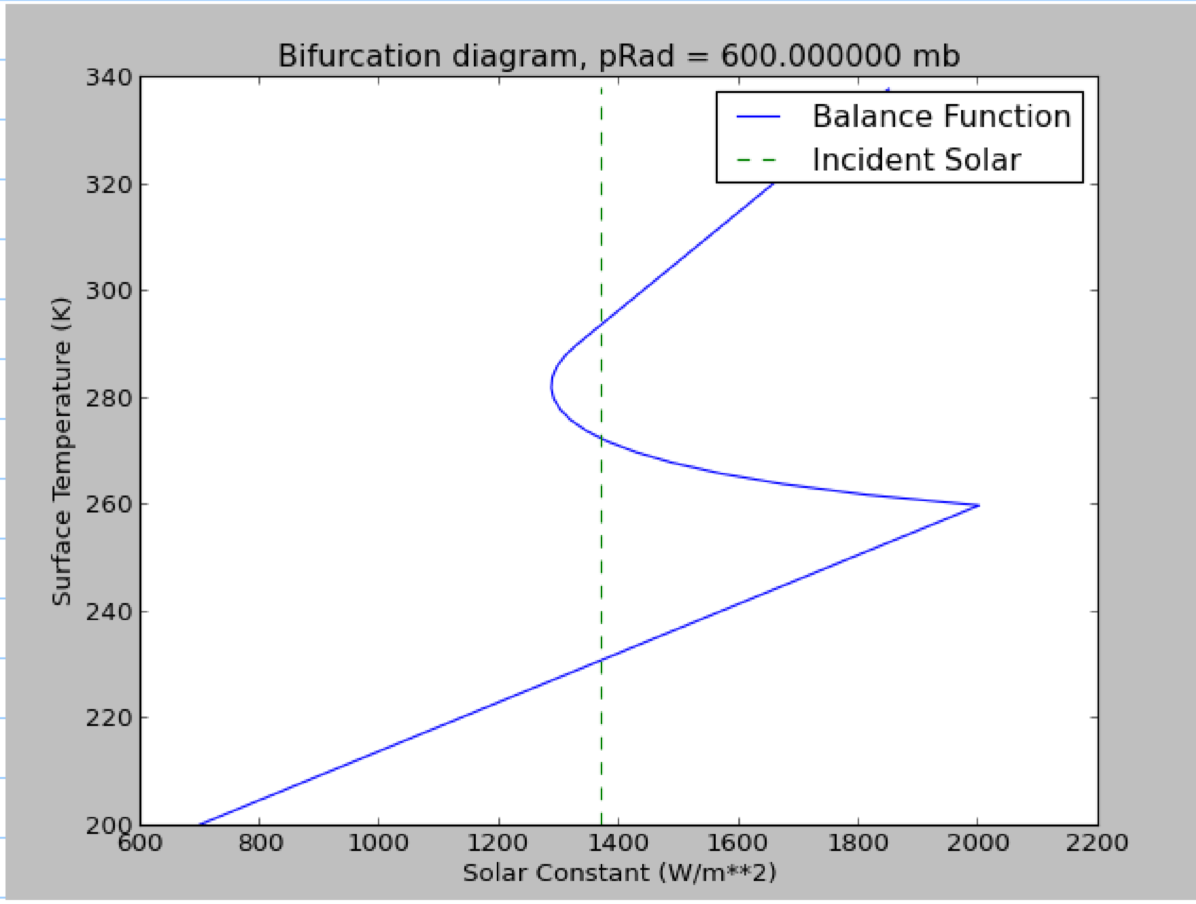
then $$T_e = \left[\frac{S_v}{4}(1 - A_p)\right]^{1/4}$$

$$= \left[\frac{2637}{4}\,\frac{(1-0.15)}{5.67\times10^{-8}}\right]^{1/4}$$

$$\Rightarrow \boxed{T_e = 315\,K}$$

d) As the atmospheric abundance of water vapour increased on both planets, the surface temperatures increased rapidly because of the greenhouse effect. Because Earth began with a lower emission temperature than Venus (since Earth is farther from the Sun), the partial pressure of water vapour eventually reach the saturation vapour pressure and water condensed out of the atmosphere. On Venus, on the other hand, because of the higher initial temperature ($T_s \gtrsim T_e$ without an absorbing atmosphere) the saturation vapour pressure was never reached, producing a runaway greenhouse effect.

3)



a) Based on the results shown in the plot, we would have to reduce L_0 to about 1280 Wm^{-2} to a transition the Snowball state. This a reduction in the solar constant of about 6.4%.

b) Power from the Sun: $P = 4\pi r_0^2 L_0 = 4\pi r^2 L$

where L is the solar constant at new radius r.

$$\Rightarrow \frac{r^2}{r_0^2} = \frac{L_0}{L} = \frac{1367\ Wm^{-2}}{1280\ Wm^{-2}}$$

$$\Rightarrow \boxed{\frac{r}{r_0} = 1.033}$$

Earth would have to be displaced by 3.3 % to force it into a Snowball state.

c) To prevent a Snowball state, we need L_0 greater than the "point" on the right of the graph, which is $\underline{L \approx 2000\ W m^{-2}}$

$$\frac{r^2}{r_0^2} = \frac{L_0}{L} = \frac{1367}{2000} \Rightarrow \boxed{\frac{r}{r_0} = 0.827}$$

For Venus, $r = 1.08 \times 10^{11} m$ and

for Earth, $r_0 = 1.5 \times 10^{11} m$

$$\Rightarrow \frac{r}{r_0} = \frac{1.08}{1.5} = 0.72$$

The required distance is greater than the Sun-Venus distance.