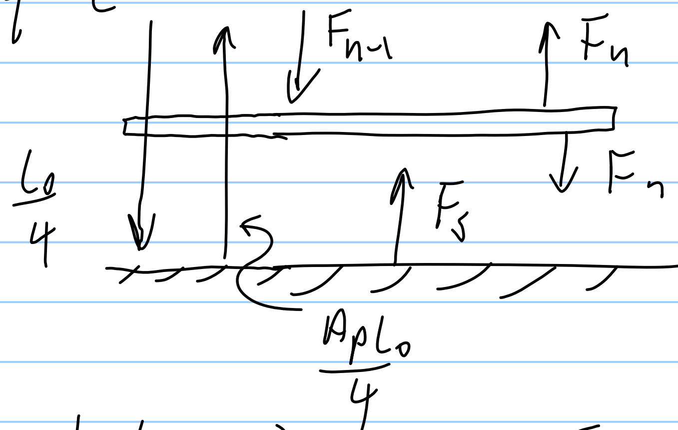
PHY392 Solution Set 2 2014

1)

(a) For the bottom layer (n) and the surface



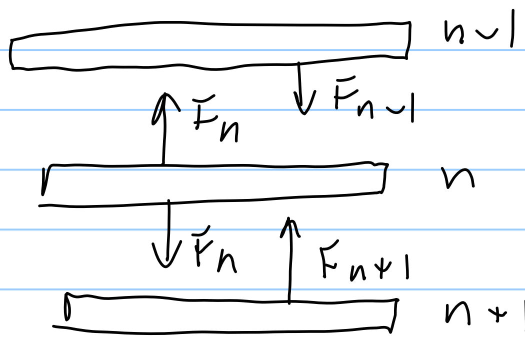
$$\frac{L_0}{4}\left(1 - A_p\right) + F_n = F_s$$

$$\sigma T_e^4 + \sigma T_n^4 = \sigma T_s^4$$

For the lowest atmospheric layer N

$$\boxed{T_s = \left[T_e^4 + T_N^4 \right]^{1/4}} \qquad \underline{T_s > T_N}$$

b) For the nth layer



$$F_{n-1} + F_{n+1} = 2F_n$$

$$\boxed{T_{n-1}^4 + T_{n+1}^4 = 2T_n^4}$$

$$T_{n+1}^4 - T_n^4 = T_n^4 - T_{n-1}^4$$

For the top layer : $T_1^4 = T_e^4$

$\Rightarrow$ $$T_2^4 = 2T_1^4 = 2T_e^4$$

$\Rightarrow$ $$T_2^4 - T_1^4 = T_e^4$$

$\Rightarrow$ $$T_n^4 - T_{n-1}^4 = T_e^4$$

$\Rightarrow$ $$T_n^4 = T_e^4 + T_{n-1}^4 .$$

$\Rightarrow$ $$T_n^4 = nT_e^4$$

For the lowest layer N, $T_N^4 = NT_e^4$

Since $T_s^4 = T_e^4 + T_N^4$

$\Rightarrow$ $$\boxed{T_s = (N+1)^{1/4} T_e}$$

2)

$$\frac{r_v}{r_e} = 0.72 \qquad L_0 = 1367\ W\,m^{-2}$$

Solar flux at venus $L_V = L_0 \left(\frac{r_e}{r_v}\right)^2$

$$\Rightarrow \boxed{L_V = 2637\ W\,m^{-2}}$$

$$T_e = \left[\frac{L_V}{4\sigma}(1 - A_p)\right]^{1/4}$$

$$= \left[\frac{2637\,W\,m^{-2}}{4}\ \frac{(1-0.77)}{5.67\times10^{-8}\,W\,m^{-2}K^{-4}}\right]^{1/4}$$

$$\boxed{T_e = 227\ K}$$

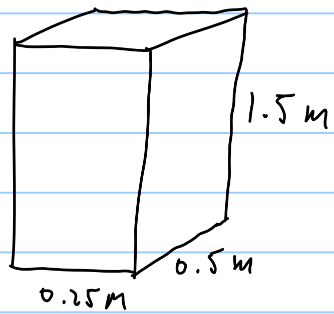
$$T_s^4 = (n+1)\,T_e^4$$

$$\left(\frac{T_s}{T_e}\right)^4 - 1 = n \qquad T_s = 750K$$

$$\Rightarrow \boxed{n \simeq 118 \text{ layers}}$$

not surprising given the high CO_2 and dense atmosphere on venus.

3)

a) 

$F = \sigma T^4$ $\quad T = 310\,K$

the total power radiated $P = FA$

Area of prism $= 2.5\,m^2$ $\Rightarrow P = (2.5)(5.67 \times 10^{-8})(310)^4$

$$\boxed{P = 1309\ W}$$

The energy lost through radiation is much less than the energy consumed ($100\,W \approx 2000$ calories/day) because the person absorbs radiation from the environment.

As Pierrehumbert notes, with the environment at freezing ($T = 273\,K$), the flux from the environment

$$F = \sigma T^4 = 315\ W\,m^{-2}$$

So the person would absorb $FA = \underline{788\ W}$ from the environment.

b) $F_\lambda = \pi B_\lambda$

Power radiated between λ_1 and λ_2 is

$$P = A \int_{\lambda_1}^{\lambda_2} \pi B_\lambda \, d\lambda$$

$$P = A \int_{\lambda_1}^{\lambda_2} \pi B_\lambda \, d\lambda \approx A \pi B_\lambda(\lambda_0, T) \, \Delta\lambda$$

where $\lambda_0 = 0.75 \ \mu m$

$\Delta\lambda = 0.5 \ \mu m$

$$B_\lambda = \frac{2\pi h c^2}{\lambda^5} \left[\exp(hc/\lambda k T) - 1 \right]^{-1}$$

$$B_\lambda(0.75 \ \mu m, 210 K) = 6.235 \times 10^{-13}$$

$$\Rightarrow \quad P = (2.5) \pi (6.235 \times 10^{-13}) (0.5 \ \mu m)$$

$$\boxed{P = 2.4 \times 10^{-18} \ W}$$

at $\lambda_0 = 0.75 \ \mu m$ $\quad E = h\nu_0 = 2.65 \times 10^{-19} \ J/\text{photon}$

rate of photon emission $= \dfrac{P}{E} = \dfrac{2.4 \times 10^{-18} \ W}{2.65 \times 10^{-19} \ J}$

$$= \boxed{9.1 \text{ photons/second}}$$

note $\dfrac{hc}{\lambda k T} = 61.95$ for $\lambda = 0.75 \ \mu m$, $T = 310 K$

$$\Rightarrow \quad e^{hc/\lambda k T} \gg 1 \quad \Rightarrow \quad B_\lambda \approx \frac{2hc^2}{\lambda^5} e^{-hc/\lambda k T}$$

$\nu = \dfrac{c}{\lambda}$ & $B_\lambda \, d\lambda = -B_\nu \, d\nu$

$$\Rightarrow \quad B_\nu \, d\nu = \frac{2hc}{\lambda^3} e^{-h\nu/kT} d\nu = \frac{2h\nu^3}{c^2} e^{-h\nu/kT} d\nu$$

$$\text{rate of emission of photons} = A \int_{\nu_1}^{\nu_1} \frac{\pi B_\nu \, d\nu}{h\nu}$$

$$= A \int_{\nu_1}^{\nu_1} \frac{2\nu^2}{c^2} e^{-h\nu/kT} d\nu$$

which can be solve analytically!

c) At $\lambda_0 = 0.1\ \mu m$ (assume $\Delta\lambda = 0.05\ \mu m$)

$$P = A \pi B_\lambda(\lambda_0, T) \Delta\lambda$$

$$= 7.5 \times 10^{-190}\ W$$

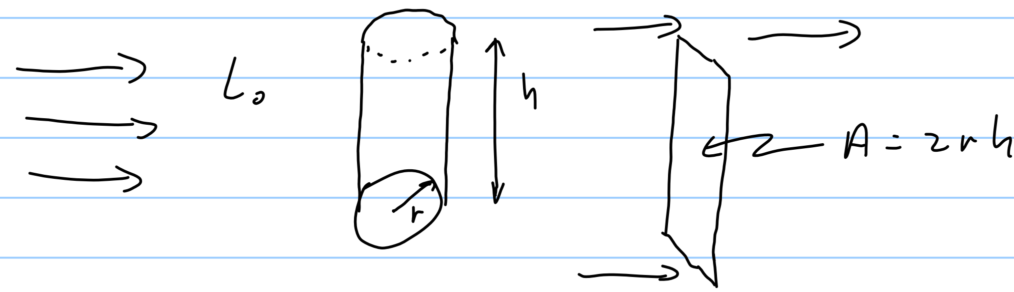
$$E = h\nu = \frac{(6.626 \times 10^{-34})(3. \times 10^8)}{0.1\ \mu m} = 2.0 \times 10^{-18}\ J/\text{photon}$$

$$\text{time per photon} = \frac{E}{P} = \frac{2.0 \times 10^{-18}}{7.5 \times 10^{-190}}$$

$$\boxed{= 2.7 \times 10^{171}\ \text{seconds}}$$

effectivey forever!

4)



Power absorbed $= 2rhL_o$

Power emitted $= \sigma T^4 (2\pi rh + 2\pi r^2)$ total surface area

$$\Rightarrow \quad 2rhL_o = \sigma T^4 (2\pi rh + 2\pi r^2)$$

$$L_o = \sigma T^4 (\pi + \pi r/h)$$

$$\Rightarrow \quad \boxed{T = \left[\frac{L_o}{\sigma \pi (1 + r/h)}\right]^{1/4}}$$

for $r = h = 50\,m$ $\quad T = \left[\frac{1367}{\pi (5.67 \times 10^{-8}) 2}\right]^{1/4}$

$$\boxed{T = 249\,K}$$

With an additional $1 \times 10^6\,W$ of heat, the energy balance is

$$2rhL_o + 1 \times 10^6\,W = \sigma T^4 (2\pi rh + 2\pi r^2)$$

assuming $r = h = 50\,m$

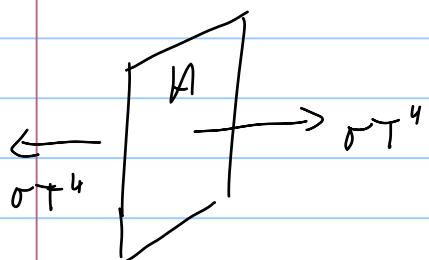
$$\Rightarrow \quad 2r^2 L_o + 1 \times 10^6\,W = \sigma T^4 4\pi r^2$$

$$T = \left[\frac{2r^2 L_o + 1 \times 10^6}{\sigma 4 \pi r^2}\right]^{1/4}$$

$$T = \left[\frac{2(50)^2(1367) + 1\times10^6}{(5.67\times10^{-8})\,4\pi(50^2)} \right]^{1/4}$$

$$\boxed{T = 258\,K}$$

To radiate the additional 1 MW of heat, we need a radiator of area A that radiates 1 MW of power with the station at the original T = 249K.



$$1\times10^6\,W = 2\,\sigma T^4 A$$

$$= 2A\,(5.67\times10^{-8}\,W m^{-2} K^{-4})\,(249K)^4$$

$$\Rightarrow \boxed{A = 2.3\times10^3\,m^2}$$