PHY392
Physics of Climate
Problem Set #4, 2014

Assigned: Saturday, March 22, 2014
Due: Friday, April 4, 2014 (by 5 pm).
Late penalty = 5% per day, up to 7 days, after which material will not be accepted.

QUESTIONS:

1.
 a) What percentage of the incident monochromatic intensity with wavelength λ and zero zenith angle is absorbed in passing through the layer of the atmosphere extending from an optical depth $\tau_\lambda = 0.2$ to $\tau_\lambda = 4.0$?

 b) What percentage of the outgoing monochromatic intensity to space with wavelength λ and zero zenith angle is emitted from the layer of the atmosphere extending from an optical depth $\tau_\lambda = 0.2$ to $\tau_\lambda = 4.0$?

 c) In an isothermal atmosphere, through how many scale heights would the layer in (a) and (b) extend?

2. Given a nadir-viewing satellite designed to measure infrared emission in order to retrieve atmospheric temperature profiles using Schwarzchild's Equation. Assume that the absorption coefficient, $k_a(\bar{\nu})$, of the line used for sounding is constant for a given $\bar{\nu}$.

 a) Derive an expression for the atmospheric transmission from altitude z to the altitude of the satellite, $T_{\bar{\nu}}(z,\infty)$ with $z_{satellite} \equiv \infty$, in terms of the mass mixing ratio $Q = \rho_{gas} / \rho_{air}$ (assumed to be constant with altitude) and the atmospheric pressure $p(z)$.

 b) Derive an expression for the weighting function $W_{\bar{\nu}}(p) = dT_{\bar{\nu}}/dy$ for such a line, using $y = -\ln(p)$.

 c) At what pressure $p_{\max}$ does $W_{\bar{\nu}}(p)$ have a maximum?

 d) Express $W_{\bar{\nu}}(p)$ in terms of p and $p_{\max}$ and plot $W_{\bar{\nu}}(p)$ vs. $p / p_{\max}$. Explain the physical significance of $W_{\bar{\nu}}(p)$ and how weighting functions are relevant to the retrieval of vertical profiles of temperature.

3. Consider a CO_2 line at 15 μm (= wavenumber 667 cm^{-1}) with

$$\alpha_L(T,p) = \alpha_L^o(T_o,p_o)\frac{p}{p_o}\sqrt{\frac{T_o}{T}}$$

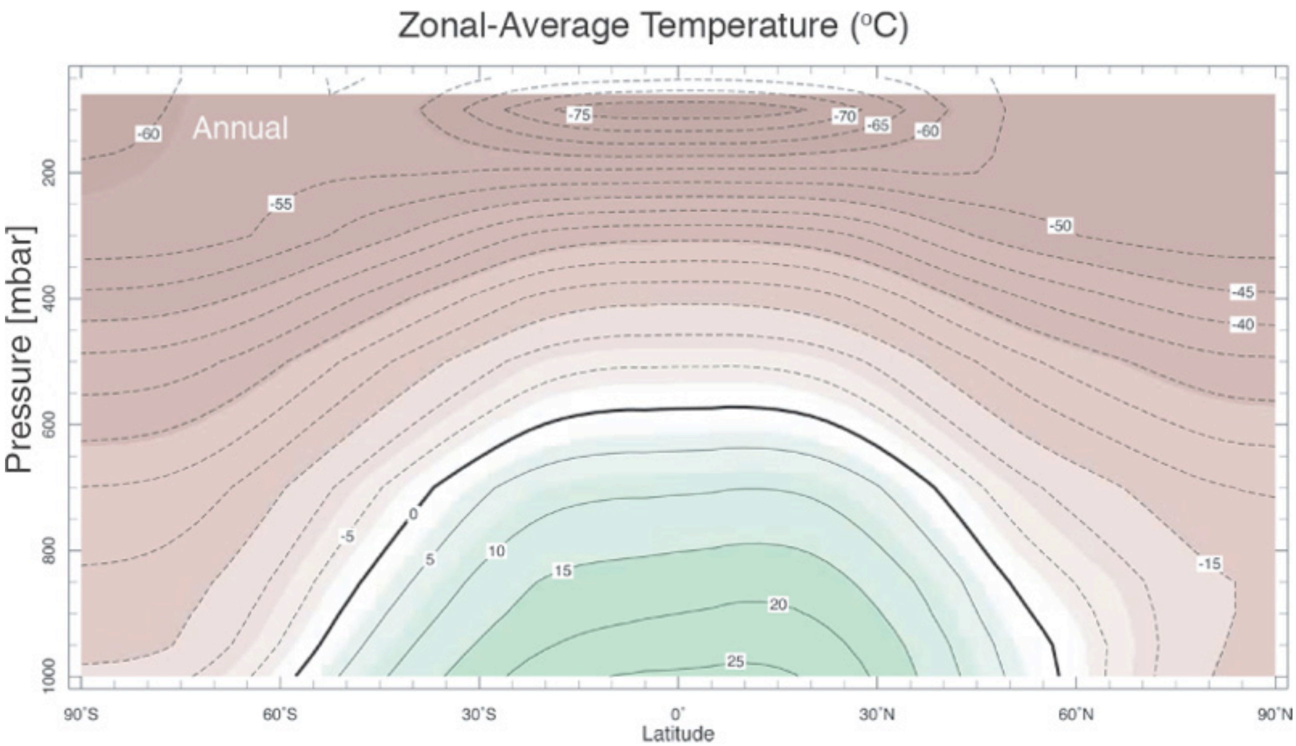
where p_o = 1000 hPa, T_o = 250 K and $\alpha_L^o(T_o,p_o) = 3\times10^9$ Hz (in units of frequency). Find the approximate pressure level in the atmosphere at which the transmittance of a horizontal path at the line centre under the assumption of pure Doppler broadening is the same as under the assumption of pure Lorentz broadening. Assume that the atmosphere is at a typical temperature of 250 K.

4. Suppose that Earth's rotation axis were normal to the Earth-Sun line. The solar flux, measured per unit area in a plane normal to the Earth-Sun line, is L_0. By considering the solar flux incident on a latitude belt bounded by latitudes (φ, $\varphi + d\varphi$), show that F, the 24-hr average solar flux *per unit area of the Earth's surface*, varies with latitude as

$$F = \frac{L_0}{\pi}\cos\varphi.$$

a) Using this result, suppose that the atmosphere is completely transparent to solar radiation, but opaque to infrared radiation such that, separately for each latitude belt, the radiation budget can be represented by the "one-layer" energy balance model. Determine how surface temperature varies with latitude.

b) Calculate the surface temperature at the equator, 30°, and 60° latitude if Earth's albedo is 0.30 and L_0 = 1367 W m^{-2}. Compare your estimated temperatures to those from the figure below.



5. The Earth's obliquity fluctuates between 22° and 24.5° on a time scale of roughly 40,000 years. Assuming that the albedo is 20%, compute the latitudinal profile of the difference in absorbed solar radiation between these two states. Convert these fluxes into temperature change using a climate of 0.5 K/(W m^{-2}). This gives you an estimate of the temperature change over extensice oceans, including the effect of water vapour feedback, but neglecting the additional sensitivity due to possible melting of sea ice or land glaciers. You should use the Chpater 7 Python script **solar.py** for this problem. Plot the temperature difference as a function of latitude for solicticial and annual mean conditions. The function **solar** takes as input the latitude, season, and obliquity and returns the flux. The function **AnnMeanFluxCirc** takes as input the latitude and obliquity and returns the flux.

6. In this problem set you will use a Python implementation (by Oliver Watt-Meyer) of the energy balance model based on Budyko [1969]. This model starkly demonstrates a possible non-linearity in the climate system due to the ice-albedo positive feedback. You should read Budyko [1969] before completing this problem set. Briefly, Budyko proposes a simple one-dimensional (depending only on latitude) model that assumes the total energies absorbed and emitted by the Earth are equal. The Earth is assumed to be covered only by ocean and sea ice, and the values for a number of parameters such as the albedo for the ocean versus for sea ice and the solar insolation as a function of latitude are estimated. The result is an implicit equation for temperature as a function of latitude.

a) Open and run the script **BudykoEnergyBalanceModel.py**. You should see a figure in which temperature as a function of latitude is plotted. This is the calculated temperature distribution based on the default reduction of solar insolation, -2%. Try adjusting the variable **delQp** to see what happens to the temperature distribution for different changes in the solar insolation. Does **delQp** = 0.0 give a realistic temperature distribution for the present day Earth? You should make plots for **delQp** = 0%, -2%, -4%, -6%, and -8% and comment on how the dependence of the temperature on **delQp**. Does the global mean temperature vary smoothly as you change **delQp**?

b) Set **delQp** equal to zero. Now, try adjusting the values of **albedoOcean** and **albedoIce**. How do the surface temperatures respond? You should Include these plots in your solutions.

c) Calculate the mean global temperature and ice edge boundary as a function of **delQp**, the change in solar insolation. Essentially, we want to recreate Figure 5 from Budyko [1969]. The script **BudykoEnergyBalanceModel.py** calculates the variables **Tp** and **iceEdge** which are the global mean temperature and latitude of sea ice edge respectively. You should adapt this script so that it calculates these variables for an array of **delQp** values (say, from 0% to -8%, with intervals of 0.1%), and then plots them.

   To do this, create the array of changes in insolation by calling something like

   delQp_list = arange$(0, -8.1, -0.1)$

   and add a **for** loop around the main part of the code that solves for the temperature distribution. You should make sure to save the values of **Tp** and **iceEdge** for each value of **delQp** so that you can make the appropriate plots.

d) Include plots of both **Tp** and **iceEdge** as a function of **delQp**. At what percentage decrease in solar insolation does the ice edge go to the equator, i.e. we get "snowball Earth" conditions? Does this agree with the value found in Budyko [1969]?

e) Now, try adjusting the albedos as in Part (b) and see what effect this has on the figures from the previous question. Comment on the differences.