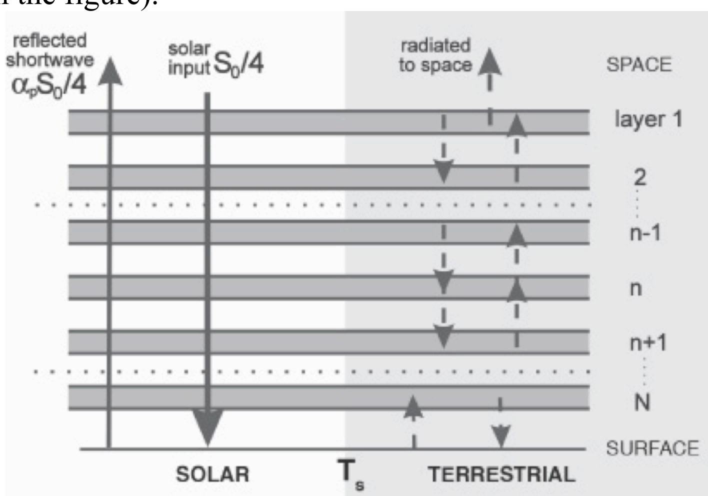
PHY392
Physics of Climate
Problem Set #2, 2014

Assigned: Friday, January 31, 2014
Due: Friday, February 14, 2014 (by 5 pm).
Late penalty = 5% per day, up to 7 days, after which material will not be accepted.

QUESTIONS:

1. Marshall and Plumb, Chapter 2, Problem 5.
Consider an atmosphere that is completely transparent to shortwave (solar) radiation, but very opaque to infrared radiation (IR). Specifically, assume that it can be represented by N slabs of atmosphere, each of which is completely absorbing of IR, as depicted in the figure below (not all layers are shown in the figure).



(a) By considering the radiative equilibrium of the surface, show that the surface must be warmer than the lowest atmospheric layer.

(b) By considering the radiative equilibrium of the n^{th} layer, show that, in equilibrium,

$$2T_n^4 = T_{n+1}^4 + T_{n-1}^4 \qquad (1)$$

Where T_n is the temperature of the n^{th} layer, for $n > 1$. Hence argue that the equilibrium surface temperature is

$$T_s = (n+1)^{1/4} T_e \qquad (2)$$

Where T_e is the planetary emission temperature. [Hint: Use your answer to part (a); determine T_1 and use Equation (1) to get a relationship for temperature differences between adjacent layers.]

2. Determine the emission temperature of the planet Venus. You may assume the following: the mean radius of Venus' orbit is 0.72 times that of Earth's orbit; the solar flux So decreases as the square of the distance from the sun and has a value of 1367 W m^{-2} at the mean Earth orbit; Venus' planetary albedo = 0.77.

The observed mean surface temperature of the planer Venus is about 750 K. How many layers of the N-layer model considered in Problem 1 would be required to achieve this degree of warming? Comment.

3. Pierrehumbert, Chapter 3, Problem 14.
Compute the total power radiated by a person with a normal body temperature of 37C. Why is this so much greater than the typical daily energy consumed by a person in form of food (equivalent to about 100 W)? Next, using the expression for the spectral flux density $\pi B_\lambda(T)$, compute the power radiated by the person in the visible wavelength band (0.5 to 1 μm). Approximately how many visible photons per second are radiated? How long would you have to wait for the person to emit a single ultraviolet photon at 0.1 μm? For the purposes of estimating the surface area needed in this problem, you may assume that the person is shaped approximately like a rectangular prism, with height 1.5 m, width 0.5 m, and depth 0.25 m.

4. Pierrehumbert, Chapter 3, Problem 19.
A cylindrical space station with length h and radius r is in orbit about the Sun at a distance where the solar constant is $L_\odot$. The space station has zero albedo in the shortwave and radiates as a perfect blackbody in the longwave (infrared) range. The flow of air inside keeps the entire station at the same temperature, and the skin is a good conductor of heat, so that its temperature is the same as that of the interior. The orientation of the station is such that the axis of the cylinder is always perpendicular to the line joining the center of the station to the center of the sun. Find an expression for the temperature of the station. Put in numbers corresponding to the mean solar constant at Earth's orbit, assuming $r = h$.

Now suppose that the equipment in the interior of the space station consumes 1 megawatt of solar-generated electrical power, which is dissipated as heat. How much warmer would this make the station once the equipment was turned on? To get rid of this excess heat, you are to design a radiator, which is a large, thin flat plate heated by pumped water from the space station so that its temperature is the same as the interior of the space station. The radiator is perfectively reflective in the shortwave, but acts as a perfect blackbody in the infrared region. How large should the radiator plate be in order to get rid of the excess heat? For this part of the problem you may assume $r = h =$ 50 m.

5. Pierrehumbert, Chapter 2, Problem 20.
In this computer problem you will compute the dry adiabat $T(p)$ for an ideal gas whose specific heat depends on temperature, in accord with the Shomate equation (Problem 2.13 in Pierrehumbert's book). In addition to basic skills such as defining functions and loops, you'll need to know how to write programs that find approximate solutions to an ordinary differential equation of the form $dY/dx = f(x,Y)$.

The Shomate equation is an empirical formula for the dependence of specific heat on temperature. It works well for a broad range of gases. The formula reads:

$$c_p = A + B(T/1000) + C(T/1000)^2 + D(T/1000)^3 + E(T/1000)^{-2}$$

where T is the temperature in Kelvin and $A,\ldots, E$ are gas-dependent constants. Some coefficients are given in the Table below.

Gas	A	B	C	D	E
N_2	931.857	293.529	−70.576	5.688	1.587
CO_2	568.122	1254.249	−765.713	180.645	−3.105
NH_3	1176.213	2927.717	−904.470	113.009	11.128

Table 2.2 Shomate coefficients for selected gases, valid from 300 K to 1300 K for CO_2 and NH_3, and 300 K to 6000 K for N_2.

First, use the First Law of Thermodynamics to derive a differential equation for $d\ln T / d\ln p$ assuming $dQ = 0$. This defines the dry adiabat. Note that since c_p is a function of T, you can no longer treat it as a constant in doing the integral.

Write a program that tabulates approximate solutions to the differential equation. Note that your dependent variable is $Y \equiv \ln T$ whereas the right hand side of the differential equation involves T. This is not a problem, since you can write $T = \exp(\ln T)$. In writing your program, assume that $c_p(T)$ is defined by the Shomate equation.

Apply your program to obtain an approximation to the dry adiabat in a pure CO_2 Venusian atmosphere. Start your computation at the ground (p_s = 92 bars) with the observed mean surface temperature of Venus (737 K). Integrate up to the 100 mb level, and compare the temperatures you get with those in the Magellan observations shown in Figure 2.2 of the text (slide 4 of the supplementary slides for lectures 3 and 4). Make a plot comparing your calculations with the dry adiabat obtained by keeping c_p constant at 820 J/kg.