PHY392
Physics of Climate
Problem Set #1, 2014

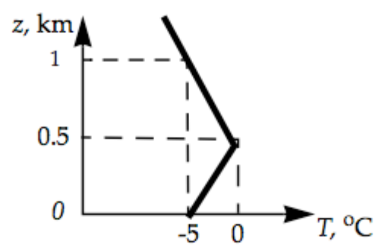
Assigned: Wednesday, January 15, 2014
Due: Wednesday, January 29, 2014 (by 5 pm).
Late penalty = 5% per day, up to 7 days, after which material will not be accepted.

QUESTIONS:

1. Marshall and Plumb, Chapter 3, Problem 2.
Using the hydrostatic equation, derive an expression for the pressure at the center of a planet in terms of its surface gravity, radius a, and density ρ, assuming that the latter does not vary with depth. Insert values appropriate for Earth and evaluate the central pressure. [Recall that $g(r) = Gm(r)/r^2$, where $m(r)$ is the mass inside radius r and $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}. You may assume the density of rock is 2000 kg m^{-3}.]

2. Pierrehumbert, Chapter 2, Problem 24.
Venus has a surface pressure of 92 bar, and a surface gravity of 8.87 m s^{-2}. 3.5% of the atmosphere (by mole fraction) consists of N_2. Compute the mass of N_2 per unit surface area of Venus, and compare with the corresponding number for Earth's atmosphere.

3. Jacob, Chapter 4, Problem 5
A town suffers from severe nighttime smoke pollution during thr winter months because of domestic wood burning and strong temperature inversion. Consider the temperature profile measured at dawn shown in the figure below. We determine in this problem the amount of solar heating necessary to break the inversion and ventilate the town.



(a) Based on the figure, what is the minimum temperature rise between 0 – 0.5 km required to ventilate the town?

(b) Show that the corresponding heat input per unit area of surface is $Q = 2.5 \times 10^6$ J m^{-2}. Use $\rho = 1$ kg m^{-3} for the density of air and $c_p = 1 \times 10^3$ J kg^{-1} K^{-1} for the specific heat at constant pressure.

(c) Solar radiation heats the surface after sunrise, and the resulting heat flux F from the surface to the atmosphere is approximated by

$$F = F_{\max} \cos\left(\frac{2\pi(t - t_{\text{noon}})}{\Delta t}\right), \qquad 6 \text{ a.m.} < t < 6 \text{ p.m.},$$

where $F_{\max} = 300$ W m^{-2} is the maximum flux at $t_{\text{noon}} = 12$ p.m., and $\Delta t = 24$ hours. At what time of day will the town finally be ventilated?

4. Pierrehumbert, Chapter 2, Problem 46.
Something is about to happen. Something wonderful. To promote life on Jupiter's moon Europa, which currently is composed of a liquid water ocean covered by a very thick water ice crust, the alien race which built Tycho Magnetic Anomaly 1 ignites thermonuclear fusion on Jupiter, heating Europa to the point that its icy crust melts, leaving it with a globally ocean covered surface having a temperature of 280 K. Water vapor is the only source of atmosphere for this planet. Describe what the atmosphere would be like, and calculate $T(p)$ for this atmosphere. Here you will use Pierrehumbert's unconventional definition of the dry adiabatic lapse rate to make a plot of temperature versus pressure, similar to Figure 2.6. Give a rough estimate of the depth (in km) of the layer containing most of the mass of the atmosphere. (The gravitational acceleration at Europa's surface is 1.3 m s^{-2}.)

5. Marshall and Plumb, Chapter 4, Problem 12.
Observations show that, over the Sahara, air continuously subsides (hence the Saharan climate). Consider an air parcel subsiding in this region, where the environmental temperature T_e decreases with altitude at the constant rate of 7 K/km.

(a) Suppose the air parcel leaves height z with the environmental temperature. Assuming the displacement to be adiabatic, show that, after time δt, the parcel is warmer then its environment by an amount $w\Lambda_e\delta t$, where w is the subsidence velocity and

$$\Lambda_e = \frac{dT_e}{dz} + \frac{g}{c_p},$$

where c_p is the specific heat at constant pressure.

(b) Suppose now that the displacement is not adiabatic, but that the parcel cools radiatively at such a rate that its temperature is *always the same as* its environment (so the circulation is in equilibrium). Show that the radiative rate of energy loss per unit volume must be $\rho c_p w\Lambda_e$, and hence that the net radiative loss to space per unit horizontal area must be

$$\int_0^\infty \rho c_p w\Lambda_e dz = \frac{c_p}{g}\int_0^{P_s} w\Lambda_e \, dP$$

where P_s is the surface pressure and ρ is the air density.

(c) Radiative measurements show that, over the Sahara, energy is being lost to space at a net, annually-averaged rate of 20 W m^{-2}. Estimate the vertically-averaged (and annually-averaged) subsidence velocity.