PHY392 Midterm Solutions 2013

1 a) Climate sensitivity is a measure of how much global mean surface temperatures will change per unit change in radiative forcing. According to the IPCC, a doubling of CO_2, which is associated with a change in radiative forcing of 4 W/m^2, will produce a change in global mean surface temperatures of about 3°C. This is the most likely estimate of the change, but the likely range of the temperature change is between 2–4 °C.

b) Removing all O_3 above 30 km will increase the flux of UV radiation to the surface, which will result in a warming of the surface. The loss of UV absorption by stratospheric O_3 will result in a cooling of the stratosphere.

c) Ozone absorbes IR at 9.6 μm and is an effective greenhouse gas in the upper troposphere. It is in the upper troposphere where the optical path becomes small enough that IR photons emitted by O_3 can make it out to space. Removing O_3 at these altitudes will allow radiation from lower (warmer) regions of the atmosphere to escape to space, cooling the surface. Removing O_3 in the lower troposphere has little impact on surface temperatures since O_3 there is absorbing and emitting radiation at nearly the same temperature as the surface.

2) Equilibrium will reflect a balance between

a) Solar radiation and emission from the satellite

power absorbed $= L_0 \pi r^2$

power emitted $= 4\pi r^2 \sigma T^4$

the albedo $= 0$ since it absorbes all radiation

$$L_0 \pi r^2 = 4\pi r^2 \sigma T^4$$

$$\boxed{T = \left[\frac{L_0}{4\sigma}\right]^{1/4}}$$

When the satellite goes into Earth's shadow, it will lose energy at the rate $4\pi r^2 \sigma T^4$

For mass $m = 10^3$ kg and specific heat c

energy loss $= mc\frac{dT}{dt} = 4\pi r^2 \sigma T^4$

$$\Rightarrow \quad mc\frac{dT}{dt} = \pi r^2 L_0$$

$$\Rightarrow \quad \frac{dT}{dt} = \frac{\pi r^2 L_0}{mC} = \frac{\pi (1\,m)^2 (1370\,W m^{-2})}{10^3\,kg\,(10^3\,J\,kg^{-1}\,K^{-1})}$$

$$\boxed{\frac{dT}{dt} = 4 \times 10^{-3}\,K/s = 14\,K/hr}$$

3) a)

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/KT} - 1}$$

If $h\nu/KT \ll 1$ & $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$e^x \approx 1 + x$ for $x \ll 1$

$$\Rightarrow B_\nu(\nu, T) = \frac{2h\nu^3}{c^2}\left(\frac{h\nu}{KT}\right)^{-1} = \frac{2KT\nu^2}{c^2}$$

$$\Rightarrow F = \int_{\nu_1}^{\nu_2} B_\nu \, d\nu = \frac{2KT\nu^2}{c^2} \, d\nu$$

$$\Rightarrow \boxed{F = \frac{2KT}{3c^2}\left(\nu_2^3 - \nu_1^3\right)} \text{ in } W m^{-2} sr^{-1}$$

b) Integrating over all direction and over the total surface, gives the power

$$P = \pi F A$$

$$\Rightarrow P = \frac{2\pi KT(4\pi a_\nu^2)}{3c^2}\left(\nu_2^3 - \nu_1^3\right)$$

$$= \frac{8\pi^2 KT a_\nu^2}{3c^2}\left(\frac{c^3}{\lambda_1^3} - \frac{c^3}{\lambda_2^3}\right)$$

$$= \frac{8\pi^2 K T a_v^2 c}{3}\left[\frac{1}{\lambda_1^3} - \frac{1}{\lambda_2^3}\right]$$

$$\approx \frac{8\pi^2 K T a_v^2 c}{3\lambda_1^3} \quad \text{since} \quad \lambda_2 = 100\,\lambda_1$$

$$P \approx \frac{8\pi^2 (1.38\times10^{-23})(737)(6.0\times10^{6})^2(3.0\times10^{8})}{3(1.0\times10^{-3})^3}$$

$$\boxed{P = 2.9\times10^{12}\ W = 2.9\ TW}$$

c) at a distance $r = 41\times10^6$ km

$$F = \frac{P}{4\pi r^2} = \frac{2.9\times10^{12}\ W}{4\pi(41\times10^9)^2\ m^2}$$

$$\boxed{F = 1.4\times10^{-10}\ W/m^2}$$ not too different from flux from cell phone

with a $100\ m^2$ antenna $\Rightarrow$ $\boxed{P = 14\ nW}$