

PHY392 USEFUL CONSTANTS AND FORMULAS

Constants

Gravitational constant	G	$6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
Universal gas constant	R^*	$8.3143 \text{ J K}^{-1} \text{ mole}^{-1}$
Speed of light in vacuum	c	$2.998 \times 10^8 \text{ m s}^{-1}$
Planck's constant	h	$6.626 \times 10^{-34} \text{ J s}$
Boltzmann's constant	k	$1.381 \times 10^{-23} \text{ J K}^{-1}$
Avogadro's number	N_A	$6.02 \times 10^{23} \text{ particles/mole}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
First radiation constant	c_1	$1.191 \times 10^{-16} \text{ W m}^2 \text{ sr}^{-1}$
Second radiation constant	c_2	$1.439 \times 10^{-2} \text{ m K}$

Planetary and Sun

Mean surface pressure on Earth	p_s	$1.013 \times 10^5 \text{ Pa} = 1013 \text{ mbar}$
Mean surface temperature on Earth	T_s	288 K
Earth's emission temperature	T_e	255 K
Mean surface density on Earth	ρ_s	1.235 kg m^{-3}
Earth's rotation rate	Ω	$7.27 \times 10^{-5} \text{ s}^{-1}$
Earth's mean radius	a	$6.38 \times 10^6 \text{ m}$
Mean Earth-Sun distance	d_{Earth}	$1.5 \times 10^{11} \text{ m}$
Solar constant	L_0	1367 W m^{-2}
Mean Earth albedo	A_p	0.30

Properties of Dry Air

Standard pressure	p_o	1013.25 hPa
Standard temperature	T_o	273.15 K
Specific heat at constant pressure	c_p	$1005 \text{ J kg}^{-1} \text{ K}^{-1}$
Specific heat at constant volume	c_v	$718 \text{ J kg}^{-1} \text{ K}^{-1}$
Air density at 273 K, 1013 hPa	ρ_0	1.293 kg m^{-3}
Gas constant for dry air	R or R_d	$287 \text{ J kg}^{-1} \text{ K}^{-1}$
Gas constant for water vapour	R_v	$461.39 \text{ J kg}^{-1} \text{ K}^{-1}$
Mean molecular weight of dry air	M_a	28.97 g/mole

PHY392
POSSIBLY USEFUL EQUATIONS

$$p = \rho RT = n_0 kT$$

$$pV = NkT$$

$$R = \frac{R^*}{M_a}$$

$$\text{volumetric mixing ratio (VMR}_i) = \frac{N_i}{\sum_i N_i} = \frac{n_i}{\sum_i n_i} = \frac{p_i}{\sum_i p_i}$$

$$\text{mass mixing ratio (MMR}_i) = \frac{m_i}{\sum_i m_i} = \frac{\rho_i}{\sum_i \rho_i} = \left(\frac{M_i}{\bar{M}} \right) \text{VMR}_i$$

$$\text{Relative humidity for gas A} = \frac{p_A}{p_{\text{sat},A}} \times 100\%$$

$$H = \frac{R^* T}{M_a g} = \frac{RT}{g}$$

$$\text{For water } p_{\text{sat}}(T) = A e^{\beta T} \text{ where } A = 6.11 \text{ hPa and } \beta = 0.067^\circ\text{C}^{-1}$$

$$\frac{dP}{dz} = -\rho g$$

$$p(z) = p_0 e^{-\frac{z}{H}}$$

$$\rho(z) = \rho_0 e^{-\frac{z}{H}}$$

$$\theta = T \left(\frac{p_0}{p} \right)^{R/c_p}$$

$$c_p = c_v + R$$

$$\Gamma_d = \frac{g}{c_p}$$

$$\frac{d \ln T}{d \ln P} = \frac{R}{c_p}$$

$$dS = c_p d \ln \theta$$

$$T(p) = \frac{T_0}{1 - \frac{RT_0}{L} \ln \left(\frac{p}{p_{\text{sat}}(T_0)} \right)}$$

$$d\Omega = d \left(\frac{A}{r^2} \right) = \sin \theta \, d\theta \, d\varphi$$

$$\lambda_{\text{max}} = \frac{2897.9}{T}$$

$$F_\lambda = \pi B_\lambda$$

$$B_\lambda(T) = \frac{2hc^2 \lambda^{-5}}{\exp \left(\frac{hc}{\lambda kT} \right) - 1} = \frac{c_1 \lambda^{-5}}{\exp \left(\frac{c_2}{\lambda T} \right) - 1}$$

$$B_\nu(T) = \frac{2h\nu^3 c^{-2}}{\exp \left(\frac{h\nu}{kT} \right) - 1}$$

$$T_s = (N+1)^{1/4} T_e$$

$$T_s = \left(\frac{2}{2-\epsilon} \right)^{1/4} T_e$$

$$T_a = \left(\frac{1}{2-\epsilon} \right)^{1/4} T_e$$

$$T_e = \left[\frac{L_0(1-A_p)}{4\sigma} \right]^{1/4}$$

$$\Delta T_s = \lambda \Delta F$$

$$F = \sigma T^4$$

$$\frac{1}{\alpha_v} \frac{dI_v}{ds} = -I_v + B_v(T)$$

$$\alpha_v = \rho k_v$$

$$\tau_v(s_2, s_1) = \int_{s_1}^{s_2} \rho k_v ds$$

$$T_v(s_2, s_1) = \exp \left(- \int_{s_1}^{s_2} \rho k_v ds \right)$$

$$ds = \frac{dz}{\cos \theta}$$

$$W_v(z) = \frac{dT_v(z)}{dz}$$

$$I_v(s) = I_v(0) e^{-\tau_v(s,0)} + \int_0^s B(T) e^{-\tau(s,s')} ds'$$

$$\tau_v(z) = \int_z^\infty \rho k_v dz'$$

$$I_v(s) = I_v(0) e^{-\tau_v(s,0)} = I_v(0) \exp \left(- \int_0^s \rho k_v ds' \right) = I_v(0) T_v(s,0)$$