The Atmospheric Kinetic Energy Spectrum and Nonlinear Spectral Fluxes as seen in ECMWF Operational Analyses

Andre R. Erler    A. B. Helen Burgess    Theodore G. Shepherd

University of Toronto

July 10\textsuperscript{th}, 2013
Outline

The Large-scale Kinetic Energy Spectrum
  Introduction: Synoptic to Planetary Scales
  Synoptic to Planetary Scales in ECMWF Analysis

Meso-scale Shallowing of the Kinetic Energy Spectrum
  Introduction: The Mesoscale KE Spectrum
  Meso-scale Shallowing and Change-point Analysis
  The Divergent Component and Gravity Waves

Nonlinear Spectral Fluxes
  Turbulence at the Tropopause
  Eddy / Mean–flow Interaction in the Stratosphere
Outline

The Large-scale Kinetic Energy Spectrum
  Introduction: Synoptic to Planetary Scales
  Synoptic to Planetary Scales in ECMWF Analysis

Meso-scale Shallowing of the Kinetic Energy Spectrum
  Introduction: The Mesoscale KE Spectrum
  Meso-scale Shallowing and Change-point Analysis
  The Divergent Component and Gravity Waves

Nonlinear Spectral Fluxes
  Turbulence at the Tropopause
  Eddy / Mean–flow Interaction in the Stratosphere
Atmospheric Turbulence at Large Scales

Eddy Kinetic Energy spectrum from ERA-40 reanalysis. Large scales are forced at the scale of the Rossby radius; and any inverse cascade would be halted at the Rhines scale.

In the atmosphere macro-turbulence is forced by synoptic-scale baroclinic instability. A direct enstrophy cascade exists, but no inverse energy cascade.

No Inverse Cascade

Baroclinic eddies always adjust the stratification such that the Rhines scale coincides with the scale of baroclinic instability.
The Kinetic Energy Spectrum at Large Scales

Boer & Shepherd (1983) first computed KE spectra for meteorological analysis data (T31), showing resemblance to 2D geostrophic turbulence. However, a strong stationary zonal flow also exists. 2D turbulence–like flow is limited to transient component.

Kinetic energy components for:
- a) stationary/transient,
- b) zonal/meridional, and
- c) zonal/meridional transient.

[BS83]
ECMWF Analysis Data

Model

- IFS (Integrated Forecast System) with data assimilation (4DVar)
- Global spectral model, run at T799 resolution
- Hydrostatic with hybrid vertical coordinate

Data

- Gridded data: $0.25^\circ \times 0.25^\circ$ ($\sim 25$ km), 91 vertical levels
- Provided in gridded form by ECMWF for IPY (2008/09)
- We use one month of data (January 2008)

Note: All spectra are in total spherical harmonic wavenumber
ECMWF Analysis Data

Model

- IFS (Integrated Forecast System) with data assimilation (4DVar)
- Global spectral model, run at T799 resolution
- Hydrostatic with hybrid vertical coordinate

Data

- Gridded data: 0.25° × 0.25° (∼ 25 km), 91 vertical levels
- Provided in gridded form by ECMWF for IPY (2008/09)
- We use one month of data (January 2008)

Note: All spectra are in total spherical harmonic wavenumber
The Kinetic Energy Spectrum in ECMWF Analysis

Total Kinetic Energy

- Steep power law in synoptic scales
- Meso-scale shallowing in stratosphere
- Planetary scales quite “noisy”
- Strong dissipation beyond $n \approx 200$

Total Kinetic Energy spectra; $-3.1$ and $-2.5$ slopes have been added.
The Stationary and Transient Components

Stationary and Transient KE spectra at 250 hPa (a) and 10 hPa (b), and transient KE at several levels (c).
Zonal-mean Spectrum and Anisotropy

Zonal and meridional components of the transient flow at 250 hPa (a) and 10 hPa (b), and anisotropy at the synoptic and meso-scale.
Summary: Large-scale Spectra

- Synoptic scale slope $\approx -3$, steepening with altitude
- Meso-scale shallowing emerging in the tropopause region
- Strong dissipation beyond $n \approx 200$
- Planetary scales dominated by stationary flow
- Transients dominate synoptic and smaller scales
- Anisotropy of is relatively large in synoptic scale (jets)
- Much stronger anisotropy in the stratosphere
Outline

The Large-scale Kinetic Energy Spectrum
  Introduction: Synoptic to Planetary Scales
  Synoptic to Planetary Scales in ECMWF Analysis

Meso-scale Shallowing of the Kinetic Energy Spectrum
  Introduction: The Mesoscale KE Spectrum
  Meso-scale Shallowing and Change-point Analysis
  The Divergent Component and Gravity Waves

Nonlinear Spectral Fluxes
  Turbulence at the Tropopause
  Eddy / Mean–flow Interaction in the Stratosphere
Observations: the Nastrom & Gage–Spectrum

Wind and Temperature measurements from commercial aircraft.
Flight levels in tropopause region.
Horizontal resolution: 2.5 km

- $k^{-5/3}$-spectrum in mesoscale up to $\sim 400$ km
- $k^{-3}$-spectrum at large scales
- relatively isotropic

N.B.: these data are much higher resolution than any GCM, even now!

Zonal and meridional wind and temperature (offset by one decade each).
Observations: the Nastrom & Gage–Spectrum

Wind and Temperature measurements from commercial aircraft.
Flight levels in tropopause region.
Horizontal resolution: 2.5 km

- $k^{-5/3}$-spectrum in mesoscale up to $\sim 400$ km
- $k^{-3}$-spectrum at large scales
- relatively isotropic

N.B.: these data are much higher resolution than any GCM, even now!

Zonal and meridional wind and temperature (offset by one decade each).
High Resolution GCM Simulations

From *Takahashi et Al. 2006*, using the Earth Simulator at T639.

- The first mesoscale decade is well resolved in the model.
- The mesoscale spectral shallowing is more pronounced than in observations.

Wind spectra from the model and aircraft measurements [THO06]
Controversy over the Meso–scale KE Spectrum I

The nature and forcing of the mesoscale cascade are still controversial, but most recent evidence points towards a direct cascade.

**Direct Cascade**
- VanZandt 1982
- Lindborg 1999
- Koshyk & Hamilton 2001

Unbalanced motion (e.g. gravity waves) excited by, e.g., baroclinic waves. Cascading down from larger to smaller scales.

**Inverse Cascade**
- Lilly 1983
- Vallis, Shutts, & Gray 1997

Balanced motion forced from small scales, e.g. by convection. Energy cascades from small to large scales.
Meso–scale Shallowing of the Kinetic Energy Spectrum

In the tropopause region the spectrum develops two distinct wavenumber regimes, which remain in the stratosphere.

- Spectral shallowing between 250 hPa and 100 hPa
- Shallowing starts at larger scales at higher altitudes

Total Kinetic Energy spectra; -3.1 and -2.5 slopes have been added.
Meso–scale Shallowing of the Kinetic Energy Spectrum

In the tropopause region, the spectrum develops two distinct wavenumber regimes, which remain in the stratosphere.

- Spectral shallowing between 250 hPa and 100 hPa
- Shallowing starts at larger scales at higher altitudes

Total KE spectra between 250 hPa and 100 hPa; -3.1 and -2.5 slopes added.
Change–point Analysis

Piece–wise linear fit with free parameter \( H \):

\[
\begin{align*}
\text{for } x < H &;& y(x) = y_0 + \gamma_1(x - x_0) \\
\text{for } x > H &;& y(x) = y_H + \gamma_2(x - H)
\end{align*}
\]

with \( y_H = y_0 + \gamma_1(H - x_0) \)

\( H \) and \( \gamma_{1,2} \) are adjusted to minimize Root–Mean–Square Error of the fit against the profile.
Change–point Analysis

Piece–wise linear fit with free parameter $H$:

- $y(x) = y_0 + \gamma_1 (x - x_0)$ for $x < H$
- $y(x) = y_H + \gamma_2 (x - H)$ for $x > H$

with $y_H = y_0 + \gamma_1 (H - x_0)$

$H$ and $\gamma_{1,2}$ are adjusted to minimize Root–Mean–Square Error of the fit against the profile.

Sample function (blue), fitted function (green)
Meso-scale Shallowing

Use change–point analysis to fit a two–segment linear function to the KE spectrum.

- Estimate transition wavenumber $n_{cp}$,
- and synoptic and meso–scale slopes

The fit is robust; but if there is no shallowing, $n_{cp}$ is not meaningful.
Meso-scale Shallowing

Use change–point analysis to fit a two–segment linear function to the KE spectrum.

- Estimate transition wavenumber $n_{cp}$,
- and synoptic and meso–scale slopes

The fit is robust; but if there is no shallowing, $n_{cp}$ is not meaningful.

Profile of change-point wavenumber and slopes.

Andre R. Erler, A. B. Helen Burgess, Theodore G. Shepherd
The Kinetic Energy Spectrum in ECMWF Operational Analysis
Controversy over the Meso–scale KE Spectrum II

Currently the inverse cascade hypothesis is out of favor, but forcing and dynamics are still unclear.

Gravity Waves

among others:
- VanZandt 1982
- Koshyk & Hamilton 2001

Unbalanced motion excited by, e.g., baroclinic waves or convection. Associated with divergent component.

Stratified Turbulence

among others:
- Lindborg 2007
- Hakim et Al. 2002

Balanced motion forced, e.g. by synoptic scales or convection. Associated with rotational component.
The Rotational and Divergent Components

Divergent and Rotational components at 250 hPa.

- Rotational component represents balanced flow
- Divergent component represents unbalanced flow

Divergent KE

Meso-scale shallowing is driven by divergent KE.
The Rotational and Divergent Components

Divergent and Rotational components at 10 hPa.

- Rotational component represents balanced flow
- Divergent component represents unbalanced flow

Divergent KE

Meso-scale shallowing is driven by divergent KE.
The Divergent Component and Meso-scale Shallowing

Blow-up of divergent and rotational components at different levels, and their intersection points (open circles).

Intersection Point

A clear progression of the intersection point to larger scales with altitude is evident.
The Divergent Component and Meso-scale Shallowing

Wavenumber at which rotational and divergent components intersect as a function of height. Change-point wavenumber is also plotted.

- Intersection and change-point follow the same pattern
- Note: divergent KE becomes significant before intersection

Andre R. Erler, A. B. Helen Burgess, Theodore G. Shepherd
Rotational and Divergent KE

- (a, b) Rotational and Divergent components
- (c) Intersection of components
- (d) Wavenumber where $div = ro$ and $n_{cp}$
Meso-scale Shallowing and Divergent KE

Slope of the divergent KE spectrum (purple).

Hypothesis

In the absence of dissipation the meso-scale slope may converge towards the observed slope of divergent KE.

R: Profiles of change-point wavenumber and slopes.
Density-weighted Spectra

Divergent motion is associated with unbalanced flow; the rotational component with balanced motion.

- “Normal” KE spectra are actually velocity variance spectra
- Scaling by density yields actual kinetic energy

Divergent (top) and rotational (bottom) KE components.
Left: Normal KE spectrum
Right: Density-scaled spectrum
Gravity Wave Propagation in the Stratosphere

- Divergent KE is almost constant in the troposphere
- Drops most between 100 hPa and 50 hPa
- Rotational KE drops off rapidly

Gravity Waves

Consistent with upwards propagating (inertia) gravity waves.

Closeup of density–scaled divergent (left) and rotational (right) components
Gravity Wave Propagation in the Stratosphere

- Divergent KE is almost constant in the troposphere
- Drops most between 100 hPa and 50 hPa
- Rotational KE drops off rapidly

Gravity Waves

Consistent with upwards propagating (inertia) gravity waves.
Gravity Wave Breaking in the Stratosphere

Relative gradient of (density weighted) kinetic energy components:

- Rotational (top): decrease in upper troposphere (Charney-Drazin Filtering)
- Divergent (bottom): decrease in stratosphere (gravity wave breaking)
Summary: Meso-scale Shallowing

- A shallow meso-scale range emerges at tropopause and persists into the stratosphere
- With increasing altitude the change-point wavenumber moves to larger scales
- Shallowing is mainly associated with the divergent component
- The slope of the divergent component is close to $-\frac{5}{3}$
- The rotational component is preferentially damped with altitude, allowing the divergent component to emerge.
- Hypothesis: the shallow meso–scale spectrum may be caused by gravity waves propagating into the stratosphere
Outline

The Large-scale Kinetic Energy Spectrum
   Introduction: Synoptic to Planetary Scales
   Synoptic to Planetary Scales in ECMWF Analysis

Meso-scale Shallowing of the Kinetic Energy Spectrum
   Introduction: The Mesoscale KE Spectrum
   Meso-scale Shallowing and Change-point Analysis
   The Divergent Component and Gravity Waves

Nonlinear Spectral Fluxes
   Turbulence at the Tropopause
   Eddy / Mean–flow Interaction in the Stratosphere
Nonlinear Triad Interactions

- Consider tendencies due to advection term in wavenumber space

\[
\frac{\partial u}{\partial t} = u \cdot \nabla u , \quad u = \sum_k u_k e^{2\pi i k \cdot x}
\]

\[
\sum_k e^{2\pi i k \cdot x} \frac{\partial u_k}{\partial t} = \sum_{m,l} u_m u_l e^{2\pi i (m+l) \cdot x}
\]

- By orthogonality of the basis functions, the wavenumbers have to add up:

\[
k = m + l
\]
Rotational Kinetic Energy Fluxes

KE fluxes are the integral of the nonlinear interaction terms (rotational component only).

- Nonlinear fluxes are much stronger in the troposphere
- Synoptic source region

![Graphs showing KE tendencies and fluxes in different pressure levels in the troposphere and stratosphere.](image)

Nonlinear interaction terms (top) and fluxes (bottom) in the troposphere (left) and stratosphere (right).
Rotational Kinetic Energy Fluxes

Troposphere (left):
- Upscale KE flux in synoptic scales
- Downscale KE flux in meso-scale

Stratosphere (right):
- Upscale KE flux to planetary scale
- Meso-scale KE flux is very small

Large-scale (top) and meso-scale (bottom) nonlinear fluxes in the troposphere (left) and stratosphere (right).
Rotational Kinetic Energy Fluxes

**Two centers** of inverse KE flux emerge in the tropopause region:

- at large synoptic scales \( (n = 10) \)
- and small planetary scales \( (n = 5) \)

**2D Turbulence**

The upscale KE flux is limited to large scales at the tropopause.

Kinetic energy fluxes resulting from nonlinear interactions of the rotational component only (quasi–2D)
Two centers of inverse KE flux emerge in the tropopause region:

- at large synoptic scales \( n = 10 \)
- and small planetary scales \( n = 5 \)

2D Turbulence

The upscale KE flux is limited to large scales at the tropopause

Kinetic energy fluxes resulting from nonlinear interactions of the rotational component only (quasi–2D)
Eddy / Mean–flow Interaction

Troposphere (250 hPa)

- Eddy / Eddy interactions dominate in synoptic scales
- Eddy / Mean–flow interactions dominate planetary scale

Upscale KE Transfer

The inverse KE cascade is a hand–off from Eddy / Eddy to Eddy / Mean–flow interaction.
Total, Eddy / Eddy, and Eddy / Mean–flow interactions (of the rotational component) at 250 hPa

**Upscale KE Transfer**

The inverse KE cascade is a hand–off from Eddy / Eddy to Eddy / Mean–flow interaction.

**Troposphere (250 hPa)**

- Eddy / Eddy interactions dominate in synoptic scales
- Eddy / Mean–flow interactions dominate planetary scale
Eddy / Mean–flow Interaction

Stratosphere (10 hPa)

- Nonlinear interactions are weaker ($\sim \frac{1}{3}$)
- Eddy / Mean–flow Interactions dominate even more

Total, Eddy / Eddy, and Eddy / Mean–flow interactions (of the rotational component) at 10 hPa

Wave / Mean–flow

Eddy / Mean–flow interaction probably mostly due to waves, consistent with E-P theory.
Eddy / Mean–flow Interaction

Stratosphere (10 hPa)

- Nonlinear interactions are weaker ($\sim \frac{1}{3}$)
- Eddy / Mean–flow Interactions dominate even more

Wave / Mean–flow

Eddy / Mean–flow interaction probably mostly due to waves, consistent with E-P theory.

Total, Eddy / Eddy, and Eddy / Mean–flow interactions (of the rotational component) at 10 hPa
Eddy / Mean–flow Interaction and 2D Turbulence

The troposphere has a turbulent enstrophy cascade.

In the stratosphere the enstrophy cascade is weak.

Right: KE (top) and enstrophy (bottom) flux at 250 hPa (left) and 10 hPa (right).
Summary: Nonlinear Fluxes

- Strong 2D-turbulence–like fluxes in the tropopause region: upscale KE flux and downscale enstrophy flux from synoptic source
- Meso-scale KE flux is downscale, but relatively weak
- Nonlinear Eddy / Eddy interactions in the stratosphere are weak
- Energy transfer to large planetary scales is achieved by Eddy / Mean-flow interactions, in both stratosphere and troposphere
Comparison with a Higher Resolution Forecast Model

Total Kinetic Energy spectra for the N721 interpolated (IPY) analysis, the T799 analysis, and a T1279 forecast at 250 hPa (a) and 100 hPa (b); slopes of −3 and −\( \frac{5}{3} \) have been added.
Summary

- The large-scale KE spectrum is reproduced in the ECMWF analysis data, but the meso-scale spectrum is too steep.
- Meso-scale Shallowing occurs very suddenly in the tropopause region and persists throughout the stratosphere.
- The shallowing is driven by divergent KE; there is some evidence for upward propagating (inertia) gravity waves.
- Strong turbulence occurs predominantly in the tropopause region (direct and indirect cascade).
- Upscale KE flux is taken over by Eddy / Mean–flow Interaction at the planetary scale (arrest of the inverse cascade).
Summary

- The large-scale KE spectrum is reproduced in the ECMWF analysis data, but the meso-scale spectrum is too steep.
- Meso-scale Shallowing occurs very suddenly in the tropopause region and persists throughout the stratosphere.
- The shallowing is driven by divergent KE; there is some evidence for upward propagating (inertia) gravity waves.
- Strong turbulence occurs predominantly in the tropopause region (direct and indirect cascade).
- Upscale KE flux is taken over by Eddy / Mean–flow Interaction at the planetary scale (arrest of the inverse cascade).
Summary

▶ The large-scale KE spectrum is reproduced in the ECMWF analysis data, but the meso-scale spectrum is too steep

▶ Meso-scale Shallowing occurs very suddenly in the tropopause region and persists throughout the stratosphere

▶ The shallowing is driven by divergent KE; there is some evidence for upward propagating (inertia) gravity waves

▶ Strong turbulence occurs predominantly in the tropopause region (direct and indirect cascade)

▶ Upscale KE flux is taken over by Eddy / Mean–flow Interaction at the planetary scale (arrest of the inverse cascade)
Caveats

- All spectra and fluxes are global, hence averaged over tropical troposphere and extra-tropical stratosphere

- Comparison with aircraft measurements is difficult because these are hemispherically biased and averaged over the troposphere

- Strong dissipation beyond $n \approx 300$, downscale fluxes not fully converged
References I


References II


Nonlinear Triad Interactions

- Consider tendencies due to advection term in wavenumber space

\[
\frac{\partial u}{\partial t} = u \cdot \nabla u , \\
\sum_k e^{2\pi i k \cdot x} \frac{\partial u_k}{\partial t} = \sum_{m,l} u_m u_l e^{2\pi i (m+l) \cdot x}
\]

- By orthogonality of the basis functions, the wavenumbers have to add up:

\[
k = m + l
\]
Rotational and Divergent KE

- (a, b) Rotational and Divergent components
- (c) Intersection of components
- (d) Wavenumber where $\text{div} = \text{rot}$ and $n_{cp}$