Section 3. Radiative Transfer

References
- *Kidder and Vonder Haar: chapter 3*
- *Stephens: chapter 2, pp. 65-76; chapter 3, pp. 81-87, 99-121*
- *Liou: chapter 1; chapter 2, pp. 38-41; chapter 3, pp. 53-56, 60-63; chapter 4, pp. 87-93*
- *Lenoble: parts of chapters 1,3,5,6,17*

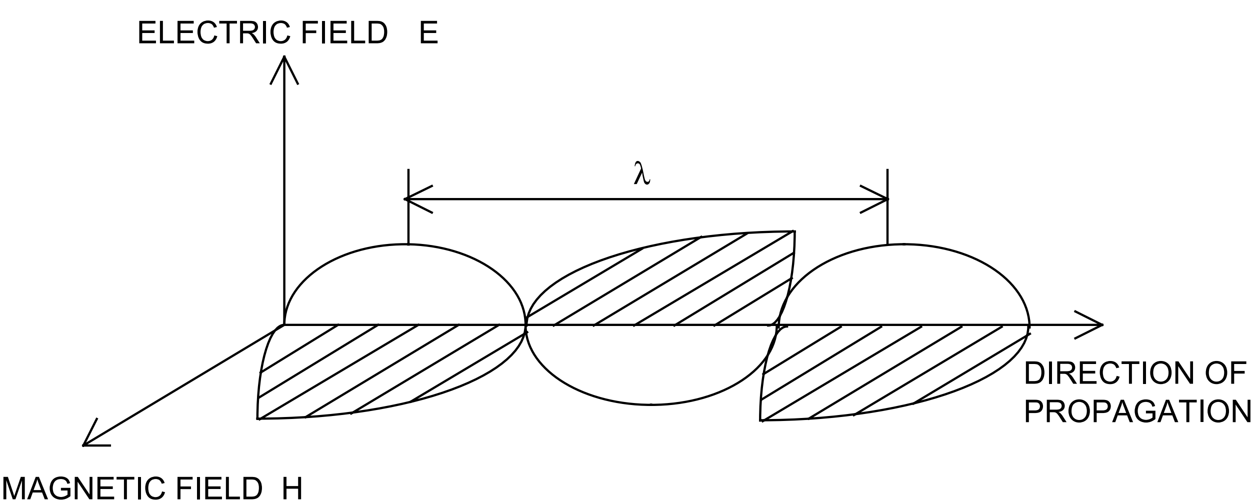
"All of the information received by a satellite about the Earth and its atmosphere comes in the form of electromagnetic radiation."

Therefore, we need to know
- how this radiation is generated
- how it interacts with the atmosphere

3.1 Electromagnetic Radiation

Electromagnetic (EM) waves are generated by oscillating electric charges which generate an oscillating electric field. This produces a magnetic field, which further produces an electric field. These fields thus propagate outwards from the charge, creating each other.

$\vec{E} \perp \vec{H} \perp$ direction of propagation.



EM radiation is usually specified by
- wavelength (λ) = distance between crests
- frequency (ν) = number of oscillations per second = c/λ
- wavenumber ($\bar{\nu}$ or κ) = number of crests per unit length (usually cm^{-1}) = $1/\lambda$

Can usually approximate the speed of light in the atmosphere by that in a vacuum. However, the change in density and humidity with height can cause significant refraction (bending of the EM rays) which must be taken into account in satellite pointing.

The EM spectrum can be divided into different spectral regions, which are useful in remote sounding:

- ultraviolet (UV) ~ 100 to 400 nm
- visible ~ 400 to 700 nm
- infrared ~ 700 nm to 1 mm
- microwave ~ 1 mm to 1 m
- radio ~ includes microwave to > 100 m

See figure (2.3) - the electromagnetic spectrum.

EM radiation carries energy that can be detected by sensors.

The energy per unit area per unit time flowing perpendicularly into a surface is given by the <u>Poynting vector</u> $\vec{S}$:

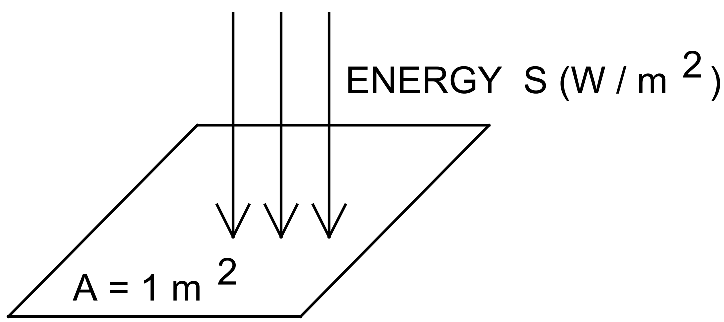
$$\vec{S} = c^2 \varepsilon_o \vec{E} \times \vec{H} \quad \text{units of W/m}^2$$

where

ε_o = vacuum permittivity

$\vec{E}$ = electric field

$\vec{H}$ = magnetic field

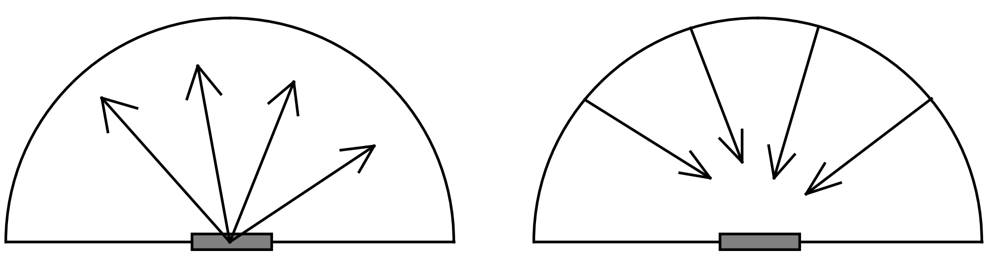


$\vec{E}$, $\vec{H}$, and $\vec{S}$ oscillate rapidly, so $\vec{S}$ is difficult to measure instantaneously. Usually measure the average magnitude over some time interval:

$$F = \langle S \rangle = \text{radiant flux density (W/m}^2\text{)}$$

The radiant flux density is redefined based on the direction of energy travel:

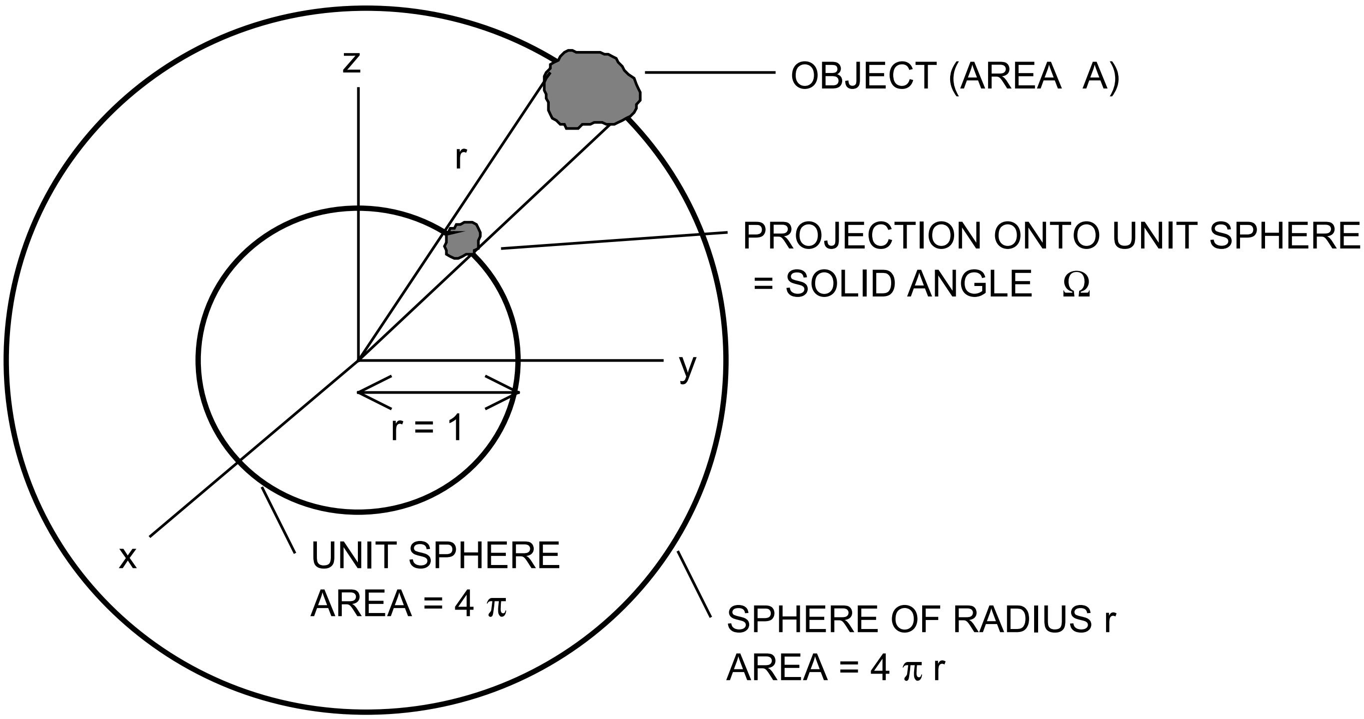
<u>radiant exitance</u> (M) = radiant flux density emerging from an area
<u>irradiance</u> (E) = radiant flux density incident on an area



Radiation is also a function of direction. This is accounted for by using the solid angle.

solid angle ($\Omega \neq$ right ascension!)

the area of the projection onto a unit sphere of an object, where lines are drawn from the centre of the sphere to every point on the surface of the object



Examples:

- solid angle of an object that completely surrounds a point: $\Omega = 4\pi$ steradians (sr)
- solid angle of an infinite plane: $\Omega = 2\pi$ sr
- solid angle of an object of cross sectional area A at distance r from a point: $\Omega = A / r^2$
  (using fractional areas $\frac{\Omega}{4\pi} = \frac{A}{4\pi r^2}$ or $d\Omega = \frac{dA}{r^2}$)

Differential element of solid angle is given by: $d\Omega = \sin\theta \, d\theta \, d\phi = d\mu \, d\varphi$

where

$\mu = \cos\theta$

θ = zenith angle = 90° – elevation angle

ϕ = azimuth angle

See figure (K&VH 3.4) - mathematical representation of solid angle.

We can now define

radiance or intensity (I or L) = radiant flux density per unit solid angle

$$I = \frac{\langle S \rangle}{d\Omega} \quad \text{W m}^{-2}\,\text{sr}^{-1}$$

This is the most important of the radiometric terms that we have defined.

Strictly, the radiance represents the EM radiation leaving or incident upon an area perpendicular to the beam. For other directions, it must be weighted by cos θ.

What is the radiant exitance, M, (i.e., the total amount of radiation leaving the surface) for a small surface area emitting radiance I?

$$M = \int_0^{2\pi} I(\theta,\varphi)\cos\theta\, d\Omega = \int_0^{2\pi}\int_0^{\pi/2} I(\theta,\varphi)\cos\theta\sin\theta\, d\theta\, d\varphi = \int_0^{2\pi}\int_0^{1} I(\mu,\varphi)\mu\, d\mu\, d\varphi$$

i.e., The radiant exitance M is obtained by integrating the radiance I over a hemisphere

For I independent of direction (isotropic): $M = I\int_0^{2\pi}\int_0^{\pi/2}\cos\theta\,\sin\theta\, d\theta\, d\varphi = \pi I.$

The radiation may be wavelength dependent.
→ prefix the energy-dependent terms by "monochromatic" or "spectral".

Symbols are subscripted accordingly, e.g., I_λ, I_ν, $I_{\bar{\nu}}$ refers to I per unit λ, ν, $\bar{\nu}$.

Units: e.g. monochromatic radiance: I_λ in W m^{-2} sr^{-1} nm^{-1}
I_ν in W m^{-2} sr^{-1} Hz^{-1}
$I_{\bar{\nu}}$ in W m^{-2} sr^{-1} $(cm^{-1})^{-1}$

The total radiance is then the integral over λ, ν, or $\bar{\nu}$ of the monochromatic radiance.

$$I = \int_0^\infty I_\lambda d\lambda = \int_0^\infty I_\nu d\nu = \int_0^\infty I_{\bar{\nu}} d\bar{\nu}$$

so

$$I_\lambda = -\frac{d\nu}{d\lambda} I_\nu = \frac{\nu^2}{c} I_\nu = \bar{\nu}^2 I_{\bar{\nu}}$$

$$\lambda I_\lambda = \nu I_\nu = \bar{\nu} I_{\bar{\nu}}$$

i.e., The radiation per unit λ interval is the same as radiation per unit ν or $\bar{\nu}$ interval, if the sizes of the intervals are accounted for.

Radiance is a useful quantity for satellite measurements because it is independent of distance from an object as long as the viewing angle and the amount of intervening matter do not change. (Both the irradiance and the solid angle decrease with r^2 so I remains constant.)

See table (K&VH 3.1) - radiation symbols and units.

3.2 Blackbody Radiation

All matter emits radiation if it is at a temperature > absolute zero.

A blackbody is a perfect emitter - it emits the maximum possible amount of radiation at each wavelength.

A blackbody is also a perfect absorber, absorbing at all wavelengths of radiation incident on it. Therefore, it looks black.

Planck's Blackbody Function

No real materials are perfect blackbodies. However, the radiation inside a cavity (whose walls are opaque to all radiation) is the radiation that would be emitted by a hypothetical blackbody at the same temperature. The cavity walls emit, absorb, and reflect radiation until equilibrium is reached.

Planck postulated that atoms oscillating in the walls of the cavity have discrete energies given by

$$E = n\,h\,\nu$$

where
n = integer (quantum number)
h = Planck's constant
ν = frequency

A quantum of energy emitted when an atom changes its energy state is then

$$\Delta E = h\,\nu \quad (\Delta n = 1).$$

Using these two assumptions, Planck derived the blackbody function, describing the radiance emitted by a blackbody

$$B_\lambda(T) = \frac{2hc^2\lambda^{-5}}{\exp\left(\frac{hc}{\lambda kT}\right) - 1}$$

where
B_λ = monochromatic radiance (W m^{-2} sr^{-1} μm^{-1})
k = Boltzmann's constant
T = absolute temperature

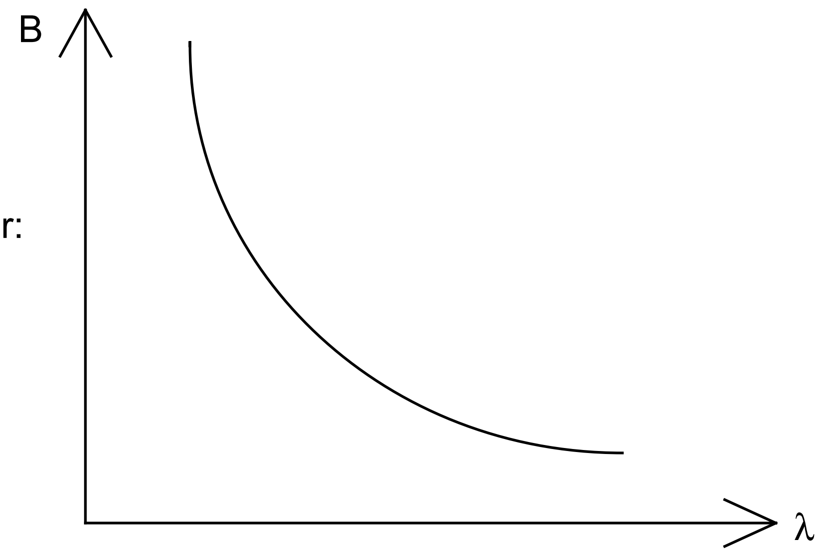
This can be written as: $B_\lambda(T) = \dfrac{c_1\lambda^{-5}}{\exp\left(\frac{c_2}{\lambda T}\right) - 1}$

where
c_1 = first radiation constant (1.191×10^{-16} W m^{-2} sr^{-1})
c_2 = second radiation constant (1.439×10^{-2} m K)

∴ The radiance emitted by a blackbody depends only on λ and T.

- $B_\lambda(T)$ increases with temperature
- the λ of maximum $B_\lambda(T)$ decreases with temperature

Sketch B vs. λ curves for:
$T_1 > T_2 > T_3 > T_4$



Several other relations can be derived from Planck's blackbody function.

(1) Wien's Displacement Law

The wavelength at which $B_\lambda(T)$ is a maximum is determined using $\frac{\partial B_\lambda(T)}{\partial \lambda} = 0$.

This gives $\lambda_m = \frac{2897.9}{T}$ μm, for T in K → this is Wien's displacement Law.

The hotter the object, the shorter the λ of its maximum intensity (e.g., element on a stove). This law can be used to determine the T of a blackbody from the position of the maximum monochromatic radiance.

(2) Stefan Boltzmann Law

The monochromatic radiant exitance is simply $M_\lambda(T) = \pi B_\lambda(T)$
because the blackbody radiance is isotropic (independent of direction).
The total radiant exitance from a blackbody is then

$$M(T) = \int_0^\infty M_\lambda(T)d\lambda = \int_0^\infty \pi B_\lambda(T)d\lambda = ... = \frac{\pi^5}{15}\frac{c_1}{c_2^4}T^4$$

∴ $M_{BB}(T) = \sigma T^4$ or $B(T) = \frac{\sigma T^4}{\pi}$ → this is the Stefan Boltzmann Law.

where
σ = Stefan Boltzmann constant = 5.67×10^{-8} W m^{-2} K^{-4}

This states that the total amount of radiation emitted from a given surface area is proportional to T^4.

(3) Rayleigh Jeans Approximation

At longer wavelengths, the Planck blackbody function can be simplified.

For λ in the microwave region, $c_2 / \lambda T << 1$ for T relevant to Earth.

$$\therefore \qquad \exp\left(\frac{c_2}{\lambda T}\right) \approx 1 + \frac{c_2}{\lambda T} \text{ , giving } B_\lambda(T) \approx \frac{c_1}{c_2}\frac{T}{\lambda^4}$$

$\rightarrow$ this is the Rayleigh Jeans Approximation.

It states that in the microwave region, the radiance is simply linearly proportional to T.

Often the radiance will be scaled by $(c_1/c_2)\lambda^{-4}$ to get the brightness temperature, T_b, which is radiance expressed in units of temperature.

T_b is the temperature required to match the measured intensity to the Planck blackbody function at a given λ.

T_b is also used in the infrared (referred to as the equivalent brightness temperature), but it must be derived from the blackbody function.

(4) Kirchoff's Radiation Law

Thus far, we have been discussing only blackbody radiance. In order to quantify how closely a material approximates a blackbody, the emittance or emissivity of the material is defined.

$$\text{emittance } \varepsilon_\lambda \equiv \frac{\text{emitted radiance at } \lambda}{\text{blackbody radiance at } \lambda} = \frac{I_\lambda}{B_\lambda(T)}$$

For a blackbody, $\varepsilon_\lambda = 1$.
For other materials, called grey bodies, $0 \le \varepsilon_\lambda < 1$.

Can also define:
absorptance α_λ $\equiv$ absorbed radiance at λ / incident radiance at λ
reflectance R_λ, ρ_λ $\equiv$ reflected radiance at λ / incident radiance at λ
transmittance τ_λ $\equiv$ transmitted radiance at λ / incident radiance at λ

These three quantities describe the three possibilities for incident radiation. All have values between 0 and 1.

By the conservation of energy, $\alpha_\lambda + R_\lambda + \tau_\lambda = 1$.

Kirchoff's radiation Law states that for a body in local thermodynamic equilibrium (having a single temperature $\rightarrow$ true in atmosphere below ~40 km (Liou; 100 km K&VH)

$$\alpha_\lambda = \varepsilon_\lambda$$

Simple Application to Atmospheric Remote Sounding

See table (Stephens 2.2) - properties of blackbody emission.

Consider three wavelengths suitable for remote sounding of temperature:
4.3 μm in CO_2 absorption band
15 μm in CO_2 absorption band
5 mm in O_2 absorption band

Relative emission of energy:
→ 15 μm appears to be the best λ for remote sounding at both 200 K and 300 K

Sensitivity of the emission to a small change in temperature (related to dB_λ/dT):
→ 4.3 μm is better for measuring warmer T, but 15 μm is still best for lower T

So far, the 5 mm region emits the least energy and is least sensitive to temperature.

However, the optimum λ for remote sounding of temperature is also determined by factors other than the available energy and the instrument sensitivity, such as the atmospheric transmission.

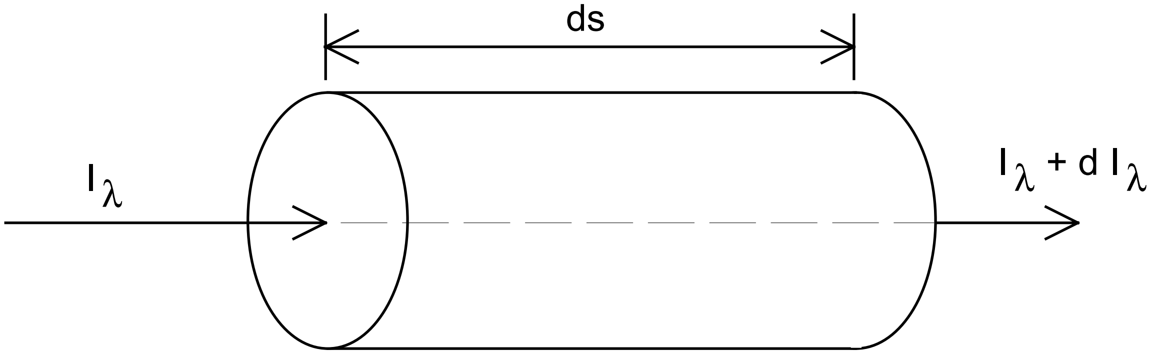
Typical transmission properties of clouds:
→ clouds strongly absorb radiation at 4.3 and 15 μm, but are almost transparent at 5 mm at both 200 K and 300 K

(See Smith, *Bull. Am. Met. Soc.*, **53**, 1074, 1972.)

3.3 Radiative Transfer Equation

Now we can discuss the transfer of EM radiation through the atmosphere.

Consider the radiation incident on a differential volume of material (atmosphere).



Four processes will change the intensity of the EM radiation as it passes through the volume:
A – absorption from the beam (depletion term)
B – emission by the material (source term)
C – scattering out of the beam (depletion term)
D – scattering into the beam (source term)

The change of intensity with distance is: $\frac{dI_\lambda}{ds} = A + B + C + D$.

Let's consider each of these four terms.

(A) The absorption term is given by the Beer-Bouguer-Lambert Law which states that the change in intensity is:

$$dI_\lambda = -\sigma_a(\lambda) I_\lambda ds = -\rho k_a(\lambda) I_\lambda ds \quad \text{[i.e., term A} = -\sigma_a(\lambda) I_\lambda = -\rho k_a(\lambda) I_\lambda \text{]}$$

where
$\sigma_a(\lambda)$ = volume absorption coefficient (m^{-1})
$k_a(\lambda)$ = mass absorption coefficient or cross section ($m^2\ kg^{-1}$ or cm^2 molecule^{-1})
ρ = density of the absorbing material (kg m^{-3} or molecules cm^{-3})

Integrating over a finite distance a to b:

$$I_\lambda(b) = I_\lambda(a) \exp\left[-\int_a^b \sigma_a(\lambda) ds\right] = I_\lambda(a) \tau_\lambda(a, b)$$

where
$\tau_\lambda(a, b)$ = transmittance between a and b
= $1 - \alpha_\lambda$, with no scattering.

(B) The emission term is given by Kirchoff's Law, because a material is as good an emitter as it is an absorber.

$$\text{term B} = \sigma_a(\lambda)B_\lambda(T) = \rho k_a(\lambda)B_\lambda(T)$$

(C) The scattering sink term also follows the form of Beer's Law.

$$\text{term C} = -\sigma_s(\lambda)I_\lambda = \rho k_s(\lambda)I_\lambda$$

where

$\sigma_s(\lambda)$ = volume scattering coefficient (m^{-1})

$k_s(\lambda)$ = mass scattering coefficient or cross section ($m^2\ kg^{-1}$ or cm^2 molecule^{-1})

(D) The scattering source term is more complex because radiation is scattered into the beam from all directions.

$$\text{term D} = +\frac{\sigma_s(\lambda)}{4\pi}\int_0^{2\pi}\int_0^{\pi} I_\lambda(\theta',\phi')p(\psi_s)\sin\theta'\,d\theta'\,d\phi'$$

where

(θ',ϕ') represents the direction of the incoming radiation

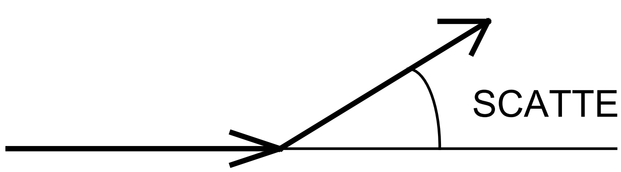
(θ,ϕ) represents the direction of the original beam

ψ_s = <u>scattering angle</u> = angle between (θ,ϕ) and (θ',ϕ')

$p(\psi_s)$ = <u>scattering phase function</u>, which specifies what portion of the radiation from direction (θ',ϕ') is scattered into direction (θ,ϕ)

Note: $\psi_s = 0^\circ$ for scattering in the forward direction

$\psi_s = 180^\circ$ for scattering in the backward direction



$p(\psi_s)$ = 1 for an isotropic scatterer and $\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} p(\psi_s)\sin\theta'\,d\theta'\,d\phi' = 1$

Can regard term D as the product of $\sigma_s(\lambda)$ and a directionally weighted average of I_λ, i.e., $\text{term D} = \sigma_s(\lambda)\langle I'_\lambda\rangle$.

Now these four terms can be combined to get the <u>radiative transfer equation</u> (RTE) for unpolarized radiation:

$$\begin{aligned}\frac{dI_\lambda}{ds} &= -\sigma_a(\lambda)I_\lambda(\theta,\phi) + \sigma_a(\lambda)B_\lambda(T) - \sigma_s(\lambda)I_\lambda(\theta,\phi) \\ &\quad + \frac{\sigma_s(\lambda)}{4\pi}\int_0^{2\pi}\int_0^{\pi} I_\lambda(\theta',\phi')p(\psi_s)\sin\theta'\,d\theta'\,d\phi' \qquad \text{[RTE1]} \\ &= -\sigma_a(\lambda)I_\lambda(\theta,\phi) + \sigma_a(\lambda)B_\lambda(T) - \sigma_s(\lambda)I_\lambda(\theta,\phi) + \sigma_s(\lambda)\langle I'_\lambda\rangle\end{aligned}$$

Rearranging

$$\frac{dI_\lambda}{ds} = \sigma_a(\lambda)[B_\lambda(T) - I_\lambda(\theta,\phi)] + \sigma_s(\lambda)[\langle I'_\lambda \rangle - I_\lambda(\theta,\phi)] \qquad \text{[RTE2]}$$

Let's apply this expression to a beam of radiation propagating upward through a thin layer of atmosphere towards a satellite instrument.

The first term $\sigma_a(\lambda)[B_\lambda(T) - I_\lambda(\theta,\phi)]$ represents the effects of absorption and emission.

- if $\sigma_a(\lambda) = 0$, then the layer is transparent with respect to radiation
- if $\sigma_a(\lambda) \neq 0$, then the temperature of the layer and the incident intensity determine the change of intensity across the layer

The second term $\sigma_s(\lambda)[\langle I'_\lambda \rangle - I_\lambda(\theta,\phi)]$ represents the effects of scattering.

- if $\sigma_s(\lambda) = 0$, then no scattering particles are present so no scattering occurs
- if $\sigma_s(\lambda) \neq 0$, then the intensity of the beam increases for $\langle I'_\lambda \rangle > I_\lambda(\theta,\phi)$
  decreases for $\langle I'_\lambda \rangle < I_\lambda(\theta,\phi)$

The full RTE can be formally solved, but we will simplify it for use in remote sounding applications.

Rearrange RTE2:

$$\begin{aligned}\frac{dI_\lambda}{ds} &= -[\sigma_a(\lambda) + \sigma_s(\lambda)]I_\lambda(\theta,\phi) + \sigma_a(\lambda)B_\lambda(T) + \sigma_s(\lambda)\langle I'_\lambda \rangle \\ &= -\sigma_e(\lambda)I_\lambda(\theta,\phi) + \sigma_a(\lambda)B_\lambda(T) + \sigma_s(\lambda)\langle I'_\lambda \rangle\end{aligned} \qquad \text{[RTE3]}$$

where

$\sigma_e(\lambda) = \sigma_a(\lambda) + \sigma_s(\lambda)$ = <u>volume extinction coefficient</u>

Similarly, $k_e(\lambda) = k_a(\lambda) + k_s(\lambda)$.

Further introducing

$\tilde{\alpha}_\lambda = \dfrac{\sigma_a(\lambda)}{\sigma_e(\lambda)}$ = <u>absorption number</u> $\qquad$ $\tilde{\omega}_\lambda = \dfrac{\sigma_s(\lambda)}{\sigma_e(\lambda)}$ = <u>single scatter albedo</u>

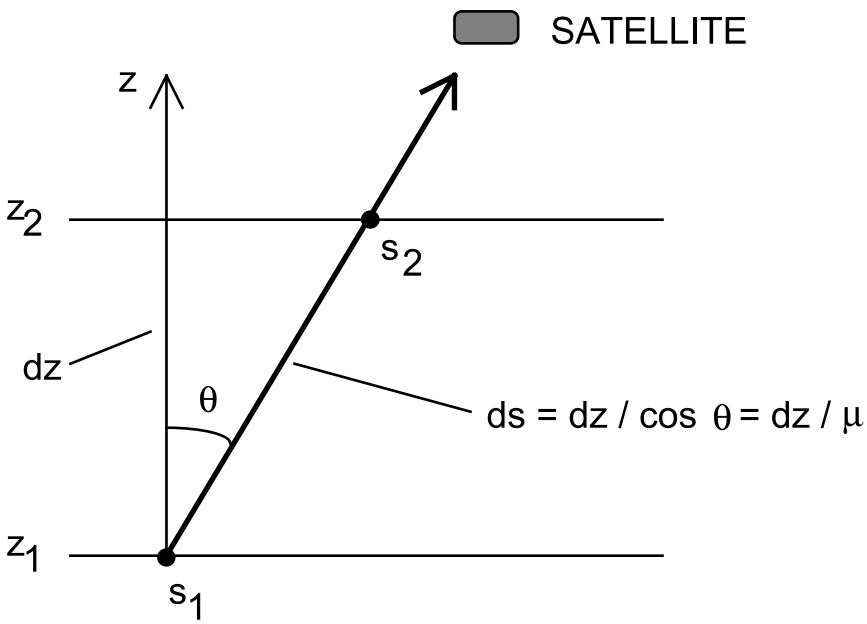
$\sigma_e(\lambda) = \rho k_e(\lambda)$ and $ds = dz/\cos\theta = dz/\mu$

yields

$$\begin{aligned}\frac{1}{\sigma_e(\lambda)}\frac{dI_\lambda}{ds} &= -I_\lambda(\theta,\phi) + \tilde{\alpha}_\lambda B_\lambda(T) + \tilde{\omega}_\lambda\langle I'_\lambda \rangle \\ \frac{\mu}{\rho k_e(\lambda)}\frac{dI_\lambda}{dz} &= -I_\lambda(\theta,\phi) + \tilde{\alpha}_\lambda B_\lambda(T) + \tilde{\omega}_\lambda\langle I'_\lambda \rangle\end{aligned} \qquad \text{[RTE4]}$$

where

$$\langle I'_\lambda \rangle = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} I_\lambda(\theta', \phi')p(\psi_s)\sin\theta' \, d\theta' \, d\phi' = \frac{1}{4\pi}\int_0^{2\pi}\int_{-1}^{1} I_\lambda(\mu', \phi')p(\psi_s)d\mu' \, d\phi'$$



Some textbooks express the RTE in terms of the optical depth δ rather than the vertical distance z, where

$$\delta_{slant}(s_1, s_2) = \int_{s_1}^{s_2} \sigma_e(\lambda, s)ds$$

$$\delta_{vertical}(z_1, z_2) = \int_{z_1}^{z_2} \sigma_e(\lambda, z)dz = \mu \times \delta_{slant}(s_1, s_2)$$

Let $\delta_\lambda = \delta_{vertical}(0, z)$ i.e., = optical depth from the surface to height z.

Then $d\delta_\lambda = \sigma_e(\lambda)dz = \mu\sigma_e(\lambda)ds = \rho k_e(\lambda)dz = \mu\rho k_e(\lambda)ds$.

We can now rewrite the RTE as

$$\mu \frac{dI_\lambda}{d\delta_\lambda} = -I_\lambda(\mu, \phi) + \tilde{\alpha}_\lambda B_\lambda(T) + \tilde{\omega}_\lambda \langle I'_\lambda \rangle \qquad \text{[RTE5]}$$

where (again)

$$\langle I'_\lambda \rangle = \frac{1}{4\pi}\int_0^{2\pi}\int_{-1}^{1} I_\lambda(\mu', \phi')p(\psi_s)d\mu' \, d\phi'$$

This is a general form of the radiative transfer equation. It can be simplified in several ways to give versions that are very useful in remote sounding.

Simplified Case #1: The RTE with No Scattering

The assumption of negligible scattering in the atmosphere is valid at infrared wavelengths when no clouds are available (assuming local thermodynamic equilibrium).

No scattering: $\tilde{\omega}_\lambda = 0 \qquad \tilde{\alpha}_\lambda = 1$

The RTE becomes

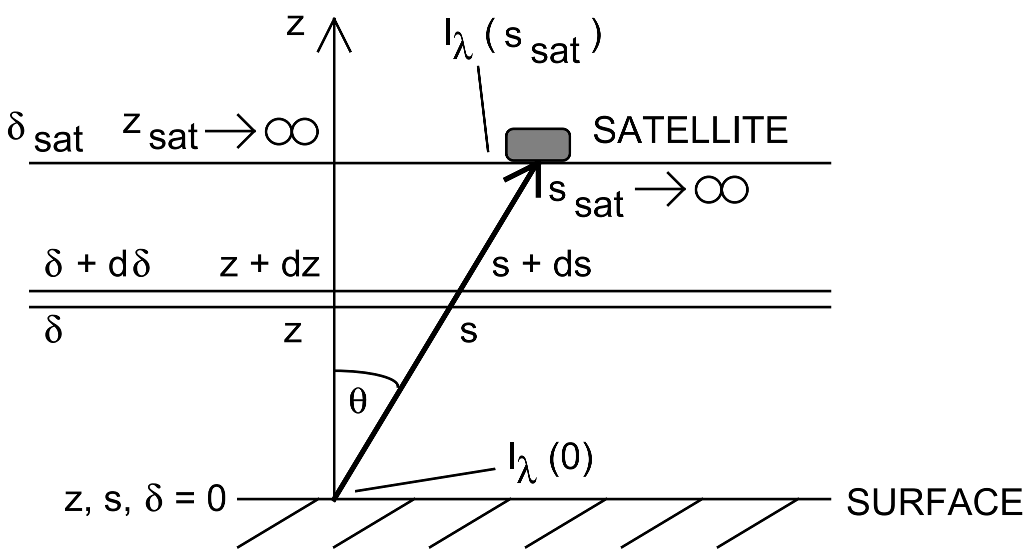
$$\mu \frac{dI_\lambda}{d\delta_\lambda} = -I_\lambda(\mu, \phi) + B_\lambda(T) \qquad \text{[RTE6]}$$

This can also be written using $\rho k_e(\lambda)dz$ or $\sigma_e(\lambda)dz$ instead of $d\delta_\lambda$.

This is **Schwarzchild's Equation**, a fundamental relation for radiative transfer in the atmosphere. It is a first order, linear, ordinary differential equation which can be integrated to calculate the infrared radiance seen by a satellite instrument.

Schwarzchild's Equation can also be derived from a consideration of the transfer of energy between two states in a molecule. This results in a more general form of the equation, where the blackbody term B is replaced by a more general emission term J. The substitution of B for J is valid in the lower atmosphere, where Local Thermodynamic Equilibrium holds.

The first term on the RHS describes the decrease in radiance due to absorption, and the second term on the RHS describes the increase in radiance due to blackbody emission.



Let's solve Schwarzchild's Equation. Each textbook has its own approach, as the path from the ground to the satellite can be represented by s, z/μ, δ_{slant}, or $\delta_{vertical}/\mu$. The following is based on Kidder & Vonder Haar.

First multiply both sides of the equation by $\exp[\delta_\lambda / \mu] / \mu$, and rearrange. Keep in mind that $\delta_\lambda = \delta_{vertical}(0, z)$ = the vertical optical depth from the surface to height z, with δ_λ decreasing as z increases.

$$\exp\left(\frac{\delta_\lambda}{\mu}\right)\frac{dI_\lambda}{d\delta_\lambda} + \frac{1}{\mu}\exp\left(\frac{\delta_\lambda}{\mu}\right) I_\lambda(\mu, \phi) = \frac{1}{\mu}\exp\left(\frac{\delta_\lambda}{\mu}\right) B_\lambda(T)$$

This can be written as:

$$\frac{d}{d\delta_\lambda}\left[\exp\left(\frac{\delta_\lambda}{\mu}\right) I_\lambda(\mu, \phi)\right] = \frac{1}{\mu}\exp\left(\frac{\delta_\lambda}{\mu}\right) B_\lambda(T)$$

Integrate this from the surface ($\delta_\lambda = \delta_\lambda(0,0) = 0$) to the satellite ($\delta_z = \delta_z(0, z_{sat}) = \delta_{sat}$), using δ_λ as the vertical coordinate:

$$\int_0^{\delta_{sat}} \frac{d}{d\delta_\lambda}\left[\exp\left(\frac{\delta_\lambda}{\mu}\right) I_\lambda(\mu, \phi)\right] d\delta_\lambda = \int_0^{\delta_{sat}} \exp\left(\frac{\delta_\lambda}{\mu}\right) B_\lambda(T)\frac{d\delta_\lambda}{\mu}$$

This gives:

$$\exp\left(\frac{\delta_\lambda}{\mu}\right) I_\lambda(\mu, \phi)\Big|_0^{\delta_{sat}} = \int_0^{\delta_{sat}} \exp\left(\frac{\delta_\lambda}{\mu}\right) B_\lambda(T)\frac{d\delta_\lambda}{\mu}$$

$$\exp\left(\frac{\delta_{sat}}{\mu}\right) I_{sat} - I_o = \int_0^{\delta_{sat}} \exp\left(\frac{\delta_\lambda}{\mu}\right) B_\lambda(T)\frac{d\delta_\lambda}{\mu}$$

where
I_o = radiance leaving the surface
I_{sat} = radiance reaching the satellite

Rearranging to solve for I_{sat}:

$$\boxed{I_{sat} = I_o \exp\left(-\frac{\delta_{sat}}{\mu}\right) + \int_0^{\delta_{sat}} B_\lambda(T)\exp\left(-\frac{\delta_{sat} - \delta_\lambda}{\mu}\right)\frac{d\delta_\lambda}{\mu}} \qquad \text{[RTE7a]}$$

Alternative solutions in terms of z and s are:

$$I_\lambda(s_{sat}) = I_\lambda(0)\exp\left(-\frac{\rho k_e(\lambda) z_{sat}}{\mu}\right) + \int_0^{z_{sat}} \frac{\rho k_e(\lambda)}{\mu} B_\lambda(T)\exp\left(-\frac{\rho k_e(\lambda) z}{\mu}\right) dz$$

$$= I_\lambda(0)\exp(-\rho k_e(\lambda) s_{sat}) + \int_0^{s_{sat}} \rho k_e(\lambda) B_\lambda(T)\exp[-\rho k_e(\lambda) s] ds \qquad \text{[RTE7b]}$$

Introduce the vertical transmission between two optical depths:

$$\tau_\lambda(\delta_1, \delta_2) = \exp(-|\delta_2 - \delta_1|)$$

Then define

$$\tau_\lambda = \tau_\lambda(\delta_\lambda, \delta_{sat})$$

i.e., τ_λ is the vertical transmission of the atmosphere from level δ_λ to the satellite, with $\tau_{sat} = \tau_\lambda(0, \delta_{sat})$ as the vertical transmission from the surface to the satellite. (Note: this can be confusing because sometimes τ_λ is defined as $\tau_\lambda = e^{-\delta_\lambda}$.)

Substituting this into RTE7 gives the following solution to Schwarzchild's Equation:

$$I_{sat} = I_o(\tau_{sat})^{\frac{1}{\mu}} + \int_{\tau_{sat}}^{1} B_\lambda(T)\tau_\lambda^{\left(\frac{1}{\mu}-1\right)} \frac{d\tau_\lambda}{\mu} \qquad \text{[RTE8]}$$

For overhead viewing ($\theta = 0$, $\mu = 1$), this simplifies to:

$$I_{sat} = I_o\tau_{sat} + \int_{\tau_{sat}}^{1} B_\lambda(T) \frac{d\tau_\lambda}{\mu} \qquad \text{[RTE9]}$$

Now the physical interpretation becomes obvious.

- first term = surface radiance × atmospheric transmittance to the satellite
- second term = radiance emitted by each layer × transmittance from layer to satellite

This equation is fundamental to the interpretation of satellite measurements of radiance in terms of atmospheric temperature and composition.

Simplified Case #2: The RTE with Scattering but No Emission

The interpretation and use of the RTE for the scattering case is more complex.

However, in the ultraviolet, visible and near infrared, neither the Earth nor its atmosphere emit significant radiation, allowing the RTE to be simplified by dropping the blackbody emission term.

Thus:

$$\mu \frac{dI_\lambda}{d\delta_\lambda} = -I_\lambda(\mu, \phi) + \tilde{\omega}_\lambda \langle I'_\lambda \rangle = \frac{\tilde{\omega}_\lambda}{4\pi} \int_0^{2\pi} \int_{-1}^{1} I_\lambda(\mu', \phi')p(\psi_s)d\mu'\, d\phi'$$

In order to solve this equation, need to formulate an expression for $\tilde{\omega}_\lambda$ based on the relevant scattering processes.

For more details of this derivation, see Kidder & Vonder Haar, Section 3.3.2.

Now want to look at the parameters that are needed in order to apply the RTE:

- absorption and scattering properties of the atmosphere
- reflection properties of the Earth's surface
- characteristics of the Sun's radiation

3.4 Gaseous Absorption and Spectroscopy

Absorption and emission spectra provide a means of identifying and measuring the composition of the atmosphere.

Interaction of Radiation with Gases

There are four primary ways in which radiation can interact with atmospheric gases; each results in a characteristic absorption spectrum.

(1) Ionization - dissociation
- radiation strips an electron from an atom or molecule, or breaks apart a molecule
- produces relatively smooth spectra
- UV and visible
- spectra are characterized by the absorption cross section:
  $k_a = \sigma_a / \rho$ (cm^2/molecule)
- important bands for the detection of ozone (Hartley and Chappuis bands)

(2) Electronic transitions
- valence electrons jump between energy levels within an atom or molecule
- also characterized by the absorption cross section, which has some structure
- UV and visible
- also important for the detection of ozone (Huggins band)

(3) Vibrational transitions
- a molecule changes vibrational energy levels, usually from the ground state to the first excited state
- produces discrete spectral lines
- infrared

(4) Rotational transitions
- a molecule changes rotational energy levels
- produces discrete spectral lines
- far infrared and microwave (pure rotational lines)

In practice, the IR spectrum of many molecules is due to a combination of vibrational and rotational transitions.
→ CO_2 and H_2O are the most important absorbers in this region

See figure - UV-visible absorption cross sections.

Line-Broadening Processes

These vibrational-rotational transitions occur at discrete wavelengths and are broadened by three processes:

(1) Natural line broadening due to uncertainties in the energy levels.
$\rightarrow$ only important in the upper stratosphere and mesosphere

(2) Pressure (or Lorentz) broadening due to collisions between molecules which distort them and cause absorption at slightly different frequencies.
$\rightarrow$ most relevant to the lower atmosphere below 40 km

The mass absorption coefficient of a Lorentz-broadened line is

$$k_a(\bar{\nu}) = \frac{S}{\pi} \frac{\alpha_L}{(\bar{\nu} - \bar{\nu}_o)^2 + \alpha_L{}^2}$$

where
S = line strength, a function of temperature and lower state energy E"
$\bar{\nu}_o$ = central wavenumber
α_L = Lorentz half-width (HW at HM)

$$\alpha_L(T, p) = \alpha_L^o(T_o, p_o) \frac{p}{p_o} \sqrt{\frac{T_o}{T}}$$

where
T_o and p_o = reference temperature and pressure (273.15 K, 1013.25 mbar)

Every Lorentz-broadened line can be specified by four parameters: $\bar{\nu}_o$, S, α_L^o, E".

See figure (K&VH 3.16) - Lorentz lines at surface and tropopause.

(3) Doppler broadening due to the motions of molecules.
$\rightarrow$ most relevant to the atmosphere above 40 km, becoming comparable to Lorentz broadening at 40 km

The mass absorption coefficient of a Doppler-broadened line is

$$k_a(\bar{\nu}) = \frac{S}{\sqrt{\pi}\alpha_D} \exp\left(-\frac{(\bar{\nu} - \bar{\nu}_o)^2}{\alpha_D{}^2}\right)$$

where
α_D = Doppler line-width = HWHM / $\sqrt{\ln 2}$ (or HWHM = $\alpha_D \sqrt{\ln 2}$)

$$\alpha_D(T) = \sqrt{\frac{2kT}{M}} \frac{\bar{\nu}_o}{c}$$

where
k = Boltzmann's constant
M = molecular mass

Note: Doppler lines are more intense at the centre and weaker in the wings than Lorentz lines.

See figure - Lorentz and Doppler lines.

The influence of Lorentz and Doppler broadening can be combined in a convolution function called the Voigt lineshape.

Lorentz and Doppler-broadened vibrational-rotational IR lines are the basis for temperature sounding and measurements of the concentrations of many trace gases in the atmosphere.

e.g., 4.3 μm and 15 μm CO_2 → temperature sounding
6.3 μm H_2O → water vapour measurements
9.6 μm ozone → total ozone measurements
also NO, N_2O, NO_2, HNO_3, HCl, HF, CH_4, CO, and CO_2

Application of Beer's Law

Practical application of UV-visible absorption cross sections and IR mass absorption coefficients can be demonstrated using Beer's Law.

$$dI_\lambda = -\rho k_a(\lambda) I_\lambda ds$$

$$\int_{I_1}^{I_2} \frac{dI_\lambda}{I_\lambda} = \int_{s_1}^{s_2} -\rho k_a(\lambda) ds$$

$$I_\lambda(s_2) = I_\lambda(s_1) \exp\left(\int_{s_1}^{s_2} -\rho k_a(\lambda) ds \right) = I_\lambda(s_1) \tau(s_1, s_2)$$

Measurement of the radiances $I_\lambda(s_1)$ incident and $I_\lambda(s_2)$ outgoing, allows the monochromatic transmission function $\tau_\lambda(s_1, s_2)$ to be determined.

Typically, in the UV-visible region:

- introduce optical mass $u(s_1, s_2) = \int_{s_1}^{s_2} \rho ds$ (units – molecules cm^{-2})

so $\tau_\lambda(s_1, s_2) = \exp\left(-\int_{u(s_1)}^{u(s_2)} k_a(\lambda) du \right) = \exp(-k_a(\lambda) u(s_1, s_2))$ for $k_a(\lambda)$ constant from s_1 to s_2

- measure $k_a(\lambda)$ for a gas in the laboratory - typically only weakly dependent on T and p

- thus the total optical mass (or slant column abundance) of the gas can be determined

Similar calculations can be done in the IR region, except that k_a has a stronger dependence on T and p through Lorentz and Doppler broadening.

- have $\tau_\lambda(s_1, s_2) = \exp\left(- \int_{u(s_1)}^{u(s_2)} k_a(\lambda, T, p) du \right)$

- there are various approximations for the transmission along such an inhomogeneous path

e.g., $\tau_\lambda(s_1, s_2) \approx \exp\left(- k_a(\lambda, T_o, p_o)\tilde{u}\right)$

where

$\tilde{u} = u\left(\frac{p}{p_o}\right)^m \sqrt{\frac{T_o}{T}}$ = a scaled optical mass

m = constant dependent on the gas

T, p = temperature and pressure representative of the path

The accuracy of abundances and temperatures derived in this way clearly depends on the accuracy of the UV-visible absorption cross sections and the IR line parameters, which are typically only accurate to 5-10%.

Instrumental Considerations

Strictly, the RTE and transmission functions apply to monochromatic cases.

However, typical instruments measure over some spectral region, which may include several IR lines, for example. In this case, the measured radiance becomes

$$I_{measured} = \int_{\lambda_1}^{\lambda_2} f(I_\lambda) d\lambda$$

where

f includes the instrument resolution function and the spectral response function of the instrument over its bandpass from λ_1 to λ_2.

Such calculations of radiance are most accurately done "line-by-line", calculating I_λ at many closely spaced wavelengths and then integrating over the instrument bandpass.

These calculations are very intensive, and are therefore often approximated by band models which are used to calculate the transmission over an entire band using some suitable parameterization.

See figures (K&VH 3.11, 3.12, 3.13, 3.14) - transmittance of gases in Earth's atmosphere.

3.5 Atmospheric Scattering

This is a complex topic - we will just touch on the basics.

Radiation scattered from a particle depends on:
- particle shape
- particle size
- particle index of refraction
- wavelength of the incident radiation
- viewing geometry

Mie (1908) applied Maxwell's EM equations to the case of a plane EM wave incident on a sphere, and showed that the scattered radiation for a sphere depends only on:
- viewing angle
- index of refraction
- size parameter $\chi \equiv 2\pi r / \lambda$, where r = radius of the sphere

Even Mie scattering is an approximation for most particles, since they are nonspherical. χ provides a means for defining three scattering regimes:

(1) Mie scattering: $0.1 < \chi < 50$

- λ is comparable to the circumference of the particle
- radiation interacts strongly with the particle
- full Mie equations must be used
- useful in the detection of raindrops using radar, cloud drops in the IR, and aerosols (smoke, dust, haze) in the visible

See figure (K&VH 3.18) - scattering regimes.

Several terms can be defined in order to characterize the scattering:

(i) Scattering efficiency

Q_s = total scattered radiation / incident radiation
= volume absorption coefficient / (number density × cross-sectional area of particle)

$$= \frac{\sigma_s}{\rho \pi r^2} = \frac{k_s}{\pi r^2}$$

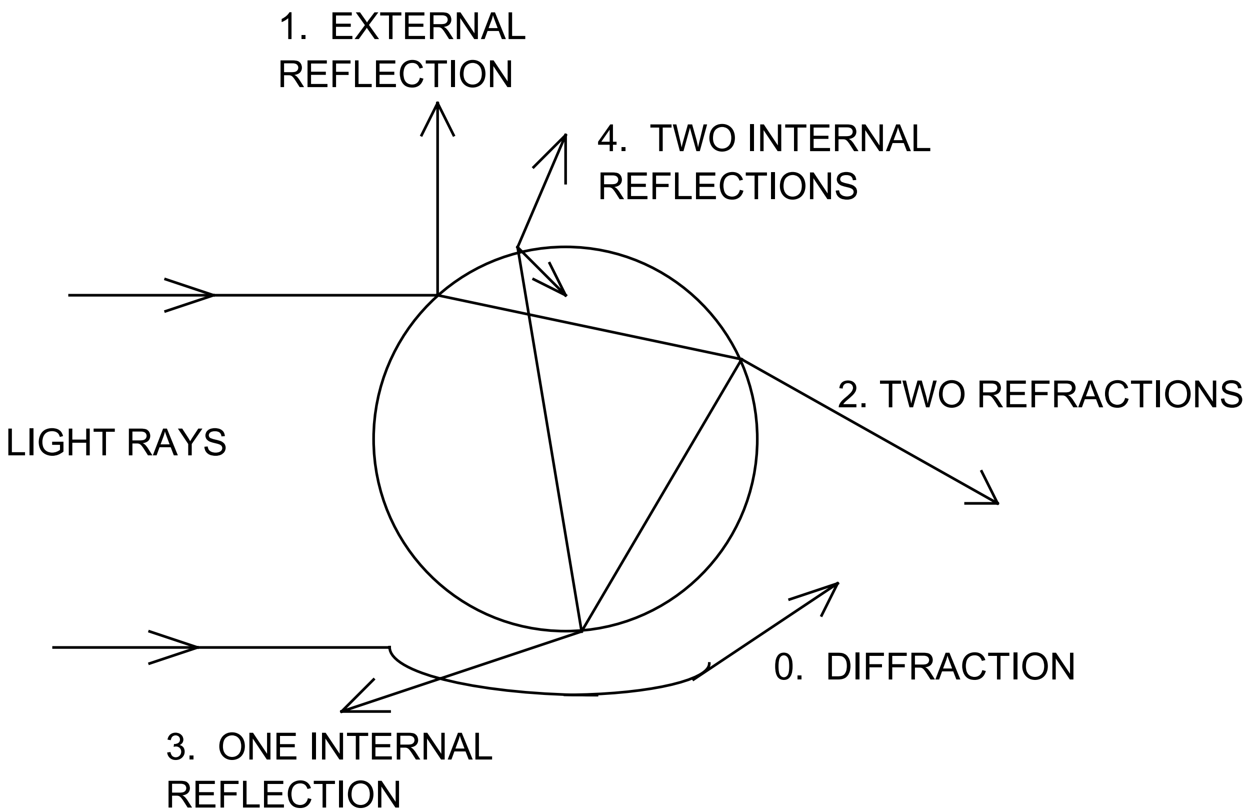
See figure (K&VH 3.20) - scattering efficiency of water spheres.

This plot of the scattering efficiency shows a series of maxima and minima, with smaller ripples superimposed. The maxima and minima are caused by interference between the light diffracted and transmitted by the sphere, and the ripple is due to rays that graze the sphere and deflect energy in all directions.

Note also that the scattering efficiency can be greater than one, and that it tends to oscillate about a value of 2 as $\chi \rightarrow \infty$. This is called the extinction paradox.

Q_s = 2 means that a large particle removes twice as much light from the incident beam as it intercepts. A purely geometrical consideration of extinction would suggest a limiting value of 1 determined by the amount of radiation blocked by the cross-sectional area of the particle.

However, the edge of the particle causes diffraction which is concentrated in a narrow lobe about the forward direction and contains an equal amount of energy to that incident on the cross section of the particle. The light removed from the forward direction of the incident beam thus consists of a diffracted component that passes by the particle, and a scattered (or blocked) component that undergoes reflection and refraction inside the particle.



LIGHT RAYS SCATTERED BY A SPHERE

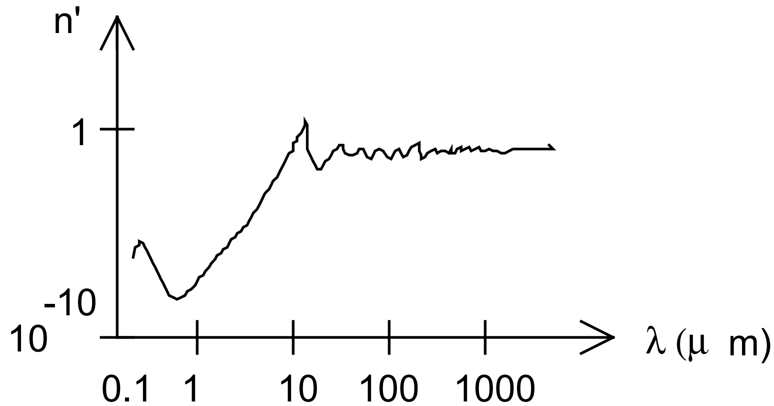
Note: for N(r) dr = the number of drops of radius r to r + dr per unit volume, the Mie scattering cross section is

$$\sigma_s(\lambda) = \int_0^\infty \pi r^2 Q_s N(r) dr$$

(ii) Complex index of refraction
→ accounts for both scattering and absorption

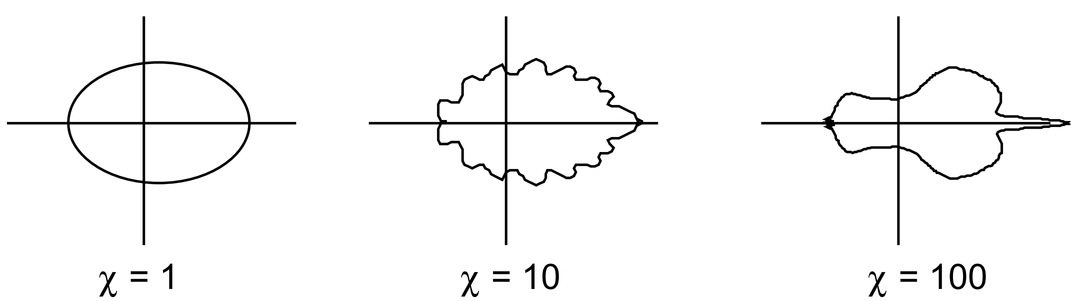
$$m = n - i n'$$

where
n = real part = speed of light in a vacuum / speed of light in the medium
n' = imaginary part which accounts for absorption inside the scattering particles (important in the IR)



See figure (K&VH 3.19) - imaginary part of the index of refraction of water and ice.

(iii) Scattering phase function
→ determines the direction in which the radiation is scattered
→ peaks in the forward direction as χ increases



See figure (K&VH 3.21) - polar plots of the scattering phase function of water drops.

(2) Geometric (optics) scattering: $\chi > 50$

- λ is much smaller than the particle
- radiation can be treated using geometric optics, tracing the reflection and refraction of rays at the surface of the sphere
- for nonabsorbing spheres, Q_S approaches 2 as χ increases because radiation striking the sphere is scattered and radiation approaching the sphere is diffracted around it
- used to detect drizzle and raindrops, to detect cloud drops in the infrared, and to explain rainbows and halos

(3) <u>Rayleigh scattering</u>: $\chi < 0.1$

- λ is much larger than the particle
- this allows the Mie equations to be simplified
- the scattering becomes independent of particle shape
- air molecules (not particles) scatter UV and visible radiation, so Rayleigh scattering must be taken into account when observing at these wavelengths (e.g. retrieving ozone in the UV)

The Rayleigh scattering cross section, in cm^2/molecule, can be calculated as

$$\sigma_s(\lambda) \cong \frac{8\pi^3}{3\lambda^4} \frac{\left[n_o(\lambda)^2 - 1\right]^2}{N_s^{\,2}} f(\delta)$$

where

$n_o(\lambda)$ is the real part of the refractive index of standard air,

N_s is the number density in molecules cm^{-3}

$f(\delta) \cong 1.061$ is a correction for anisotropy

Thus
$$\sigma_s \propto 1/\lambda^4$$
$$\sigma_s \propto \rho$$

This is important because it means that radiation of shorter λ is scattered more.

Aside:

The sky is blue because the λ of blue light (~425 nm) is shorter than that of red light (~650 nm). Thus blue light is scattered 5.5 times more than red light.

The sky is red at sunset because more blue light is scattered <u>out</u> of the long atmospheric path, leaving red light in the direct path to the viewer.

See figure (K&VH 3.11) - vertical transmission through the atmosphere due to Rayleigh scattering.

See figure ("1") - molecular vs. aerosol scattering.

In more detail, following the approach of Witt *et al.* …

The Rayleigh molecular scattering cross section in cm^2/molecule is given by Penndorf

$$\sigma_R(\lambda) = \frac{32\pi^3}{3\lambda^4} \frac{[n_o(\lambda) - 1]^2}{N_o{}^2} \frac{6 + 3\rho_n(\lambda)}{6 - 7\rho_n(\lambda)}$$

where

$n_o(\lambda)$ is the refractive index of standard air, based on Equation (2) of Edlen, and

N_o is Loschmidt's number, 2.54743×10^{19} cm^{-3} (for 15°C and ρ_n = 0.035 [Penndorf]).

Note that: $$\frac{8}{3}\left[n_o(\lambda)^2 - 1\right]^2 = \frac{32}{3}\left[n_o(\lambda) - 1\right]^2.$$

The depolarization factor is: $$\rho_n(\lambda) = 6 \times \frac{F_K(\lambda) - 1}{3 + 7F_K(\lambda)}.$$

The King correction factor for air given by (*e.g.*, Bates)

$$F_K(\lambda) = 1.0367 + (5.381 \times 10^{-12})\,\bar{\nu}_{vac}^2 + (0.304 \times 10^{-20})\bar{\nu}_{vac}^4$$

where

$\bar{\nu}_{vac}$ is wavenumber in vacuum (cm^{-1}).

The Rayleigh scattering phase function is calculated using (Goody and Yung, p. 298)

$$P_R(\theta) = 2\,\frac{0.75}{2 + \rho_n}\left[1 + \rho_n + (1 - \rho_n)\cos^2\theta\right].$$

References

G. Witt, J.E. Dye, and N. Wilhelm, Rocket-Borne Measurements of Scattered Sunlight in the Mesosphere, *J. Atmos. Terr. Phys.*, **38**, 223 (1976).

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B. Edlen, The Dispersion of Standard Air, *J. Opt. Soc. Am.*, **43 (5)**, 339 (1953).

D.R. Bates, Rayleigh Scattering by Air, *Planet. Space Sci.*, **32 (6)**, 785 (1984).

R.M. Goody and Y.L. Yung, *Atmospheric Radiation, Second Edition*, Oxford University Press, Oxford (1989).

Clouds and Scattering

Clouds consist of water drops and ice crystals of radius ≈ 10 μm.

	cloud drops	<	drizzle	<	rain drops
r =	10 μm	<	100 μm	<	1000 μm (1 mm)

The scattering behaviour of clouds depends on wavelength. For a 2 km thick (stratus cloud) with the sun overhead:

visible	~0.2%	radiation is absorbed *
(0.55 μm)	~79.8%	scattered out the top of the cloud (geometric optics)
	~20%	scattered out the bottom of the cloud

* Liquid water does not absorb much in the visible.

Because all visible λs are scattered equally well, due to the drop size distribution and large χ, clouds are white.

near infrared	~10%	absorbed (liquid water absorbs more in the near IR)
(averaged over IR	~73.8%	scattered out the top (geometric optics)
solar spectrum)	16.2%	scattered out the bottom

infrared (8.5-12.5 μm)
→ clouds drops are Mie scatterers but absorb very strongly
→ act almost as blackbodies

microwave
→ scattering by cloud drops is negligible, absorption is small ($\tau \approx 90\%$) for non-raining clouds
→ but raining clouds interact more with radiation, providing a means of detecting precipitation

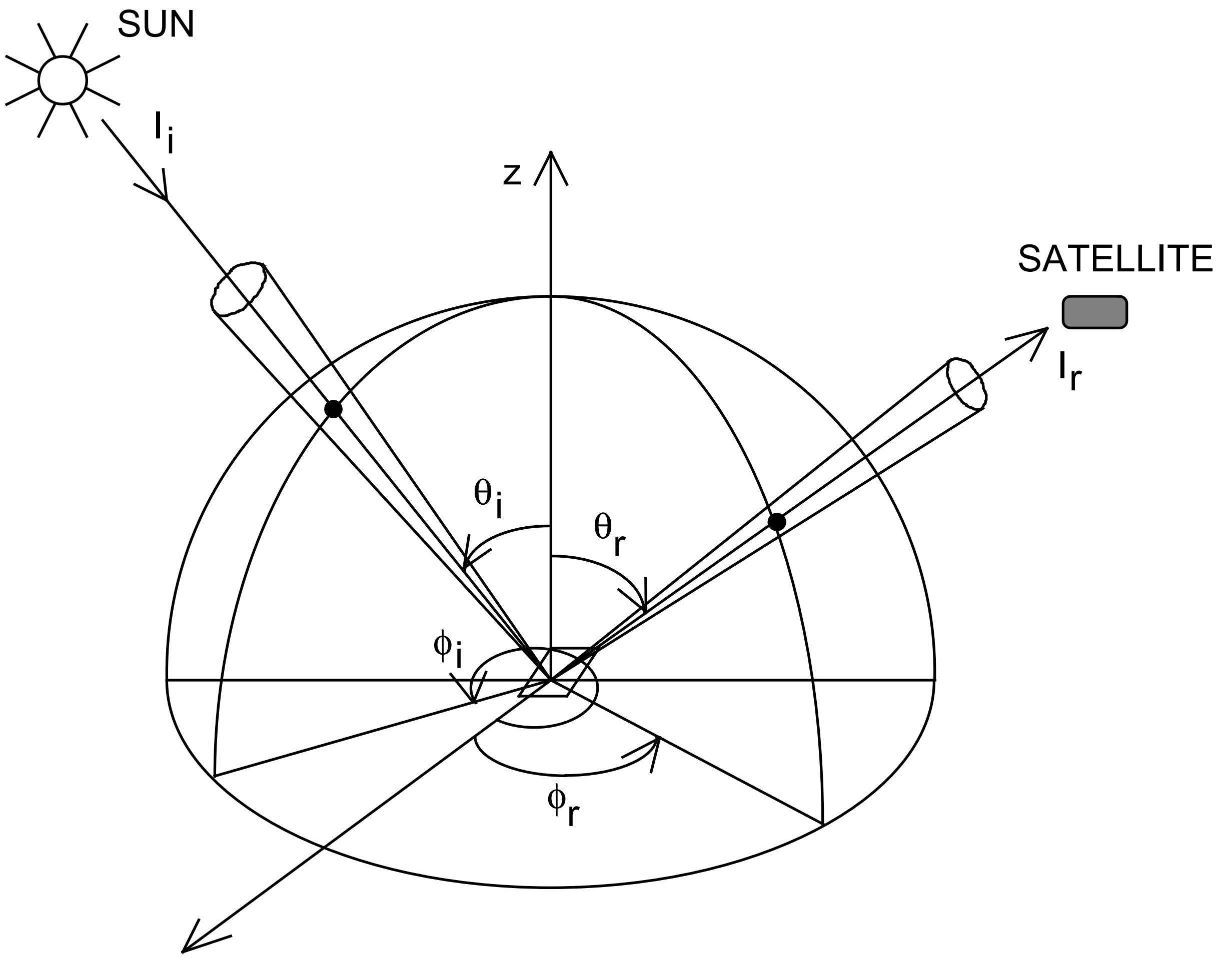
3.6 Surface Reflection

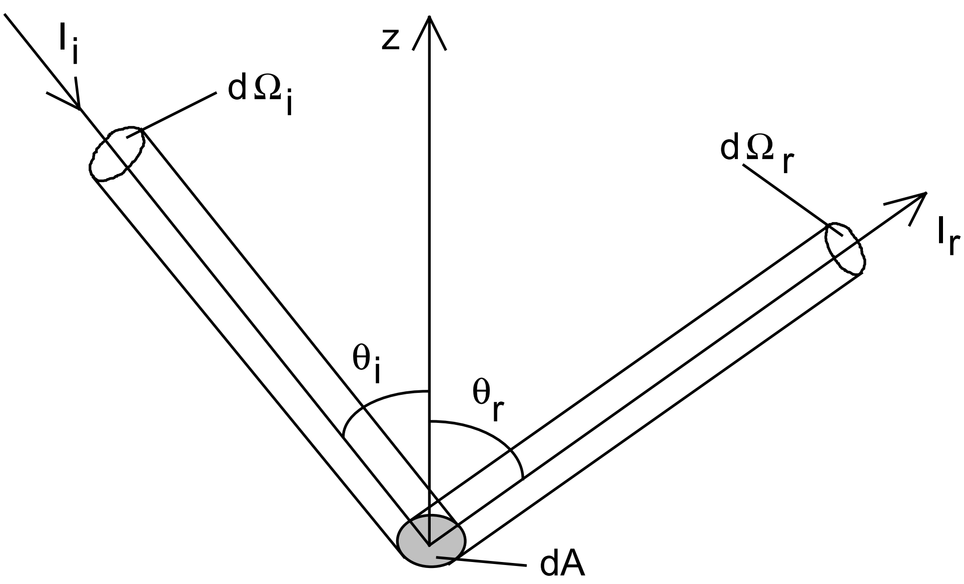
Remote sounding of the atmosphere and surface often relies on measurements of reflected radiation.

Analysis of this radiation requires knowledge of the reflection properties of the surface.

Several terms are used to describe these reflection properties.
All the following terms are λ dependent (monochromatic).

Bidirectional reflectance (γ_r or R) relates the radiation reflected in direction (θ_r, ϕ_r) to the radiation incident in direction (θ_i, ϕ_i).


SUN
I_i
z
SATELLITE
I_r
θ_i
θ_r
ϕ_i
ϕ_r


I_i
z
$d\Omega_i$
$d\Omega_r$
I_r
θ_i
θ_r
dA

The radiance reflected from a small element of surface is

$$I_r(\theta_r,\phi_r) = \int_0^{2\pi}\int_0^{\pi/2} I_i(\theta_i,\phi_i)\gamma_r(\theta_r,\phi_r;\theta_i,\phi_i)\cos\theta_i\ \sin\theta_i d\theta_i d\phi_i$$

→ radiation $I_i \cos\theta_i$ from direction (θ_i,ϕ_i) is incident on the surface area dA

→ $I_i \cos\theta_i$ is available to be reflected

→ fraction γ_r is reflected into direction (θ_r,ϕ_r)

→ integrating over all incident solid angles ($d\Omega_i = \sin\theta_i d\theta_i d\phi_i$) gives the total reflected radiance $I_r(\theta_r,\phi_r)$

Note that γ_r is unchanged if the directions of incoming and outgoing radiance are switched:

$$\gamma_r(\theta_r,\phi_r;\theta_i,\phi_i) = \gamma_r(\theta_i,\phi_i;\theta_r,\phi_r)$$

This is the Helmholtz reciprocity principle.

One of the most commonly used reflection properties is the albedo A.

$$A = \frac{M}{E} = \frac{\text{radiant exitance due to reflection}}{\text{irradiance}} \qquad \text{(no units)}$$

where

$0 \le A \le 1$.

$$A = \frac{\int_0^{2\pi}\int_0^{\pi/2} I_r(\theta_r,\phi_r)\cos\theta_r\ \sin\theta_r d\theta_r d\phi_r}{\int_0^{2\pi}\int_0^{\pi/2} I_i(\theta_i,\phi_i)\cos\theta_i\ \sin\theta_i d\theta_i d\phi_i}$$

Consider the simplified case of direct-beam solar radiation (comes directly from the Sun with no scattering) which comes from a narrow range of angles:

$$E = I_{sun}\Omega_{sun}\cos\theta_{sun}\left(= \int I_{sun}\cos\theta_{sun} d\Omega_{sun}\right) = \text{incident irradiance}$$

where

Ω_{sun} = solid angle of the Sun subtended at Earth

$$I_r(\theta_r,\phi_r) = (I_{sun}\Omega_{sun}\cos\theta_{sun})\gamma_r(\theta_r,\phi_r;\theta_{sun},\phi_{sun}) = \text{reflected intensity}$$

Thus:

$$\gamma_r(\theta_r,\phi_r;\theta_{sun},\phi_{sun}) = \frac{I_r(\theta_r,\phi_r)}{I_{sun}\Omega_{sun}\cos\theta_{sun}}$$

$$\gamma_r = \frac{I_r}{E_{sun}}$$

In addition,

$$M = I_{sun}\Omega_{sun}\cos\theta_{sun}\int_0^{2\pi}\int_0^{\pi/2}\gamma_r(\theta_r,\phi_r;\theta_{sun},\phi_{sun})\cos\theta_r\,\sin\theta_r d\theta_r d\phi_r$$

$$\therefore \qquad A = \frac{M}{E} = \int_0^{2\pi}\int_0^{\pi/2}\gamma_r(\theta_r,\phi_r;\theta_{sun},\phi_{sun})\cos\theta_r\,\sin\theta_r d\theta_r d\phi_r = \int_0^{2\pi}\gamma_r\cos\theta_r d\Omega_r$$

so A is a function of solar direction (through $\cos\theta_{sun}$).

Two limiting cases for reflecting surfaces:

(1) a Lambertian or isotropic reflector

- reflects radiation uniformly in all directions
- is "perfectly rough"
- $\gamma_r = \frac{A}{\pi} = \frac{1}{\pi}$ = constant for all directions
- $A \approx 1$ as all of the incident radiation is scattered
- e.g., snow, flat white paint

(2) a specular reflector

- reflects radiation in one direction only: $\theta_r = \theta_i$ and $\phi_r = \phi_i + \pi$
- is "perfectly smooth" like a mirror
- e.g., the surface of water which is pseudo-specular because it is rough (referred to as sun glint or sun glitter)

Note: All terms above are monochromatic (should have subscript λ). They can be integrated over the λ bandpass of a satellite instrument, but E_λ and M_λ must be integrated separately before calculating A_λ.

Typical values of A integrated over the solar spectrum:

bare soil	0.10 - 0.25
desert sand	0.25 - 0.40
grass	0.15 - 0.25
forest	0.10 - 0.20
clean snow	0.75 - 0.95 approaches Lambertian
sea surface (sun > 25° above horizon)	< 0.10 approaches specular

One additional reflection term: the <u>anisotropic reflectance factor</u> (ξ_r):

- often used to specify the behaviour of real scattering surfaces
- can make surfaces that have ξ = 1 for many wavelengths and angles

$$\xi_r(\theta_r, \phi_r; \theta_i, \phi_i) \equiv \frac{\pi}{A}\gamma_r(\theta_r, \phi_r; \theta_i, \phi_i)$$

For a Lambertian surface, $\xi = (\pi / 1)(\pi / 1) = 1$.

For a non-Lambertian surface, ξ is the ratio of the surface radiance to that of a Lambertian surface having the same albedo A.

ξ > 1 if the surface reflects more than a Lambertian surface
ξ < 1 if the surface reflects less than a Lambertian surface

For any incident direction $(\theta_{sun}, \phi_{sun})$,

$$\int_0^{2\pi}\int_0^{\pi/2}\xi_r(\theta_r, \phi_r; \theta_{sun}, \phi_{sun}) \cos\theta_r \sin\theta_r d\theta_r d\phi_r = \pi.$$

3.7 Solar Radiation

The source of most radiation used in remote sounding is the Sun.

Most of the solar EM radiation reaching Earth is emitted by the <u>photosphere</u> which is a gaseous about 500 km thick. It is sometimes called the surface of the Sun and coincides with its visible disk.

The temperature in the photosphere ranges from 4000 to 8000 K, and the emitted radiation approximates a Planck blackbody curve at ~6000 K.

This radiation traverses the <u>chromosphere</u> (several 1000 km thick, 4000 to 1,000,000 K) and the <u>corona</u>, where gases absorb and emit radiation.

The <u>solar spectrum</u> is the solar EM radiation as a function of λ which is incident on the top of the Earth's atmosphere.

The solar spectrum is usually specified as irradiance (W/m^2) instead of radiance (W/m^2/sr) because the solid angle subtended by the Sun (6.8×10^{-5} sr) at Earth is so small that the solar radiation effectively comes from the same direction.

$$E_{Earth} = I_{Sun}\Omega_{Sun}\left[= I_{Sun}\frac{\pi r^2}{R^2} = M_{Sun}\frac{r^2}{R^2}\right]$$

Once the solar irradiance has reached the top of the atmosphere, it is further absorbed and scattered before it reaches the surface.

See figure (K&VH 3.23) – solar spectral irradiance.
→ shows dominant absorbers H_2O, CO_2, O_3, O_2

We can compare the solar spectrum with the radiation emitted by Earth.

Solar irradiance
- peaks at visible λ, near 0.48 μm
- falls off rapidly at IR λ
- hence known as <u>shortwave</u> radiation

Earth's irradiance
- similar to that of a blackbody at 245-250 K
- peaks at IR λ, near 10 μm
- emits no visible radiance
- known as <u>longwave</u>

Solar and terrestrial radiation are equal at ~5.7 μm.

In order to maintain the Earth in thermal equilibrium, the total amounts of incoming solar and emitted terrestrial energy must be equal.

See figure (Harries 2.3) – the transmission spectrum of the atmosphere
→ shows λB_λ so that areas are equal
→ the solar spectrum has higher irradiance but over a smaller λ range and solid angle than the terrestrial spectrum

The <u>solar constant</u>, S_{sun}, is defined as the annual average total irradiance incident on the top of the Earth's atmosphere.

Currently accepted value, as measured by satellite radiometers: S_{sun} = 1368 W/m^2.

This is equivalent to a blackbody at 5774 K.

It varies with time,
e.g., by ± 0.6 W/m^2 due to the sunspot cycle
by ± 3.4% due to the eccentricity of Earth's orbit.