PHY 499S
EARTH OBSERVATIONS FROM SPACE
Spring Term 2005
Problem Set #3

DUE: 5 PM, Thursday, April 7, 2005 (in MP314).
Late penalty = 5 marks/day, up to 1 week, after which problem sets will not be accepted.

NOTES: Marks, shown in brackets, will be given for workings, as well as for final answers.
Total marks will be scaled to 100. Show all workings and units.

QUESTIONS:

1. A grating spectrometer is a passive sounding instrument which uses a diffraction grating to disperse the incoming radiation (see attached Figures 1, 2, and 3). Light of wavelength λ incident on a grating at angle i will be diffracted at angle θ according to the grating equation

$$a(\sin i + \sin\theta) = m\lambda$$

where a is the distance between the grooves of a grating, and m is an integer and is the order of the diffraction. The angular dispersion of a grating is the change in diffraction angle with wavelength, $\frac{d\theta}{d\lambda}$, while the linear dispersion is $\frac{dl}{d\lambda} = f\,\frac{d\theta}{d\lambda}$, where f is the focal length of the system.

[2] (a) Show that the angular dispersion of a grating is given by $\frac{d\theta}{d\lambda} = \frac{m}{a\cos\theta}$.

[6] (b) GOME (Global Ozone Monitoring Experiment) is a nadir-viewing spectrometer, currently in orbit on the ERS-2 satellite, which uses four diffraction gratings in order to achieve good resolution over an extended spectral range. Each of these gratings disperses incoming backscattered solar radiation onto a separate detector having 1024 pixels each of 25 μm pixel width. Assuming first order diffraction (m=1), f=100 mm, and i=30° for all four channels, calculate

[i] the minimum diffraction angle θ_{min} corresponding to the minimum wavelength λ_{min},
[ii] the maximum diffraction angle θ_{max},
[iii] the maximum wavelength λ_{max} corresponding to θ_{max},
[iv] the total wavelength spread $\Delta\lambda = \lambda_{max} - \lambda_{min}$, and
[v] the pixel sampling (number of pixels per nm)

given the following data on each channel. You may just show the workings for channel 1, and then give the answers for channels 2, 3, and 4 in tabular form.

Channel	Grating Ruling (grooves/mm)	Minimum Wavelength (nm)
1	3600	240
2	2400	290
3	1200	400
4	1200	590

[2] (c) If only one detector (again 1024 pixels each of 25 μm pixel width) were available to cover the entire range from 240 to 800 nm, approximately what grating ruling would be required to fit this wavelength range onto the detector? What would be the pixel sampling in this case?

2. A spaceborne DIAL system is designed to measure the number density of ozone from orbit of 250 km. It transmits pulses at λ_1 = 308 nm and at λ_2 = 353 nm. The mass absorption coefficients of ozone at these wavelengths are $k_1 = 1.192 \times 10^{-19}$ cm^2/molecule and $k_2 = 8.41 \times 10^{-23}$ cm^2/molecule, respectively.

[2] (a) Write down the general lidar equation and briefly define all the terms used in the equation.

[4] (b) Take the ratio of the lidar equation at two wavelengths and derive an expression for the number density, $\bar{\rho}(R)$, averaged over the lidar volume that extends from R to R+ΔR, in terms of the normalized backscattered signals P_1 and P_2. Show your workings, define any new variables, and explain any assumptions.

[4] (c) The DIAL system is used to make observations at three altitudes, with a range interval of 2.0 km for each. It measures $P_2(R+\Delta R)/P_2(R) \cong 1$ for all three cases, while $P_1(R+\Delta R)/P_1(R) = 0.935$, 0.775, and 0.917 for observations from 10-12, 20-22, and 30-32 km, respectively. Calculate the average ozone number density for the three observations. Comment on the resulting ozone profile.

3. A scatterometer is an active non-imaging system, which can be used to derive surface wind speed

[10] and direction from a backscattered radar signal. A simplified model for the backscattering cross section of a sea surface is

$$\sigma^0 = A + B\cos 2\psi$$

where ψ is the azimuth angle between the wind direction and the radar look azimuth, $A = 0.75\,v - 28$, $B = 3.6 - 0.2\,v$, and v is the wind speed in m/s.

A scatterometer looking North and then East obtains values of σ^0, measured in decibels, as follows

$$\sigma^0_N = -23.2\text{dB} = 10\log(I_N / I_0)$$

$$\sigma^0_E = -21.7\text{dB} = 10\log(I_E / I_0)$$

where I_0 is the signal incident on the surface, and I_N, I_E are the backscattered signals. Calculate the wind speed and direction. Is there any ambiguity in your answer? If so, could it be removed by a third observation? Why or why not?

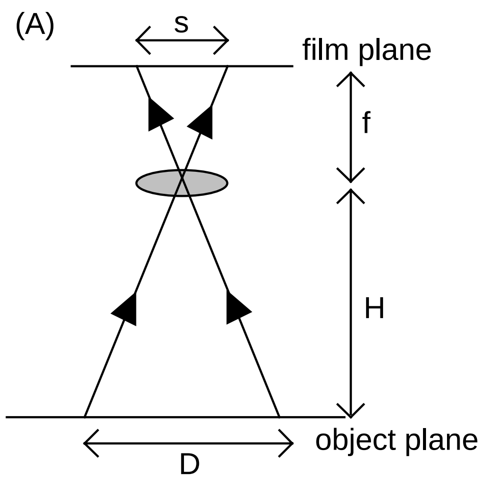
4. Aerial photography is a passive imaging technique. Assume that the photographic system can be treated as a simple lens of focal length f, imaging the surface from some height H. If the camera is looking vertically downwards, then the ground coverage will be distance $D = sH/f$, where s is the size (length) of the negative, as seen in the following figure (A).

[2] (a) Aerial photographs can also contain information, in the form of *relief displacement*, about the heights of surface features. Referring to distances defined in the figure (B) above, show that distance h', which is the projection of an object of height h onto the film, is given by

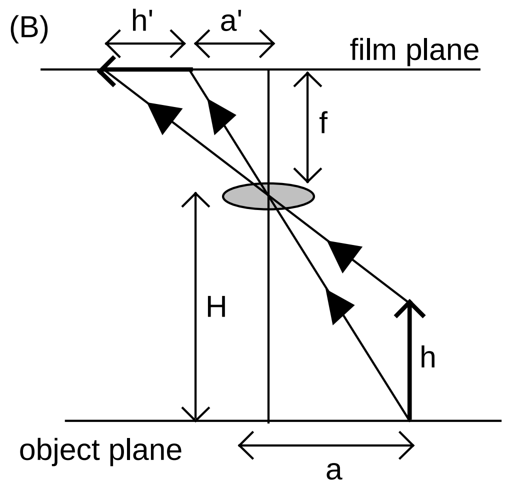
$$h' = \frac{ha'}{H - h}.$$

[8] (b) An aerial photograph taken with a camera of focal length 88 mm from height 212 m reveals a tall building. The base of one corner of the building has (x,y) coordinates (42.7, 88.2), both measured in mm from the lower left-hand corner of the negative, while the roof of the same corner is at (26.6, 82.6). The corresponding coordinates for an adjacent edge are (42.7, 103.6) and (26.6, 100.8) for the base and roof, respectively.

Find:
[i] the coordinates of the photograph's principal point (directly below the camera),
[ii] the height of the building, and
[iii] the width of the building.
Hint: It will help to draw the viewing geometry as seen looking down on the film and object planes. Note that there are two values of h' and two values of a'.



Geometric construction to find the coverage D of a vertical aerial photograph.



Relief displacement. The top and bottom of a vertical object are imaged at different places on the film.

5. Consider a two-layer atmosphere. The upper layer is 2 km thick and consists of a cloud with a mean temperature of 220 K. The lower surface behaves like a blackbody at a temperature of 285 K. The intervening layer is transparent to the radiation considered.

[3] (a) Sketch the viewing geometry for a satellite-based radiometer looking vertically downwards.

[6] (b) Calculate the 11 μm brightness temperature observed by the radiometer assuming that the cloud emissivity is [i] 0.2, [ii] 0.5, and [iii] 1.0, and that the cloud reflectance is 0 at this wavelength.

[6] (c) If the lapse rate (change in temperature with height) of the atmosphere is 6.5 K/km, what is the effective height of the emission for each case?

6. Consider a downward-looking, nadir-pointing radiometer observing the ocean surface from a
[10] satellite platform above a 3 km thick cloud that has a liquid water content W = 1.5 g/m^3 and a mean temperature of 275 K. The volume absorption coefficient of liquid water is given approximately by

$$\sigma(\nu) = 2.4 \times 10^{-4} \nu^{1.95} W \quad \text{km}^{-1}$$

where frequency ν is in GHz and liquid water content W is in g/m^3. Sketch the viewing geometry. For an ocean brightness temperature of 150 K, calculate the brightness temperature observed by the radiometer at frequencies of 22 and 37 GHz. Ignore the effects of water vapour absorption and explain any assumptions.

7. Measurements of outgoing radiation are made in three spectral intervals by a nadir-viewing
[10] radiometer in Earth orbit, with the following results:

(i) brightness temperature 310 K (atmospheric window at 11 μm),

(ii) brightness temperature 220 K (CO_2 band at 15 μm), and

(iii) brightness temperature 280 K (O_3 band at 9.6 μm).

Assuming that the surface is black, that the CO_2 absorption is so strong in interval (ii) that the stratosphere is black in this interval, and that all the ozone resides in the stratosphere, calculate the transmission of the ozone layer at 9.6 μm. Explain your reasoning. What could we learn about the ozone layer from this information?

Note: The Planck function, B(ν), at 9.6 μm, is approximately 5, 21, and 36, in units of 10^{-13} W m^{-2} $ster^{-1}$ Hz^{-1} at 220, 280, and 310 K, respectively.

8. The SBUV instrument measures backscattered UV-visible solar radiation for the retrieval of vertical column amounts of ozone. ***[Each part is worth 5 marks, for 15 total]***

(a) Write down the expression for the logarithmic attenuation $N(\lambda)$ and explain the significance of each term introduced. Derive an expression for the difference in logarithmic attenuation for two wavelengths, $\Delta N = N(\lambda_1) - N(\lambda_2)$ in terms of the total vertical column of ozone, U.

(b) Given that the SBUV instrument obtains the following values for the ratio of backscattered radiance to solar irradiance (I/E_{sun}): 0.0542 at 313 nm and 0.178 at 331 nm, with $k_a(313) = 5.687 \times 10^{-20}\,\mathrm{cm^2/molecule}$, $k_a(331) = 7.148 \times 10^{-21}\,\mathrm{cm^2/molecule}$, $\theta_{satellite} = 5°$, and $\theta_{sun} = 30°$. If the total reflectance term is assumed constant with wavelength, i.e. R(313 nm) = R(331 nm), then what is the total ozone column abundance?

(c) In practice, the total reflectance term must be taken into consideration. Assuming R exhibits Rayleigh λ^{-4} dependence only, i.e., $R(\lambda) = C\lambda^{-4}$, where C is a constant, what is the total ozone column abundance? Discuss the effect that ignoring this Rayleigh scattering term has on the retrieval of ozone using the UV-visible backscatter technique.