PHY 499S
EARTH OBSERVATIONS FROM SPACE
Spring Term 2005
Problem Set #2

DUE: Thursday, March 24, 2005 (in class).
Late penalty = 5 marks/day, up to 1 week, after which problem sets will not be accepted.

NOTES: Marks, shown in brackets, will be given for workings, as well as for final answers. Total marks will be scaled to 100. Show all workings and units. You may find useful reference data in the Appendices of The Physics of Atmospheres by Houghton (1986).

QUESTIONS:

1. Absorption in the atmospheric window between 8 and 13 μm is mostly due to the water vapour dimer. This has an absorption coefficient of the form $k_2 e$ where e is the water vapour pressure (in kPa), $k_2 \approx 0.1$ (g cm^{-2})$^{-1}$ kPa^{-1}. Given that the water vapour pressure near the surface is 1 kPa.

[5] (a) Calculate the transmission of a horizontal path 1 km long near the surface.

[5] (b) Calculate the transmission of a vertical column of water vapour, assuming that the distribution of water vapour pressure is proportional to pressure (in units of atmospheres) raised to the fourth power.

Note: The mass mixing ratio of water vapour is $\rho(H_2O)/\rho(\text{air}) = 0.622e/p$. The density of dry air at 273 K and 101.325 kPa is 1.293 kg/m^3.

2. (a) Assume that the atmosphere follows the Ideal Gas Law, with the density decreasing as

[5] $\rho(z) = \rho(0)\exp(-z/H)$, where H = RT/Mg = scale height ≈ 8 km. If the Lorentz line width is 100 times larger than the Doppler line width at ground level, at what altitude are both line widths equal?

[5] (b) Show that the relative importance of Doppler broadening compared to Lorentz broadening can be expressed as

$$\frac{\alpha_D}{\alpha_L} \approx 10^{-12}\frac{\nu_o}{p}$$

where ν_o is in Hz and p is in mbar. Explain any assumptions. At approximately what pressures are the Doppler and Lorentz line widths equal for a CO_2 at 15 μm and for an O_2 at 2.5 mm?

3. Using the following expression for the Rayleigh scattering coefficient and data in Appendix 1 of

[10] Houghton, calculate the extinction due to Rayleigh scattering for a vertical column of atmosphere at wavelengths of 0.3, 0.6, and 1 μm. Comment.

$$k_R(\lambda) = \frac{32\pi^3}{3\lambda^4}\frac{[n_o(\lambda)-1]^2}{N_o^{\;2}}$$

4. The Angstrom turbidity factor for an atmosphere that contains aerosol is defined as the optical depth of a vertical column due to aerosol scattering.

(a) Assuming that the scattering cross section of the aerosol particles is twice their geometric cross

[6] section, what number density of particles of diameter 1 μm would be needed in the lowest kilometre of the atmosphere to produce a turbidity factor of 1?

(b) It is commonly assumed that the Angstrom turbidity factor varies with wavelength as $\lambda^{-1.3}$. For **[6]** an atmosphere with an optical depth due to aerosol of 0.3 at 0.6 μm, what is the value of the turbidity factor at 0.3 μm and at 1 μm?

(c) Given that the extinction due to Rayleigh scattering for a vertical column of atmosphere is **[8]** 0.637 at 0.3 μm, what is the total optical depth and what is the total extinction due Rayleigh and aerosol scattering for such an atmosphere at this wavelength?

5. (a) The hydrostatic equation, $dp = -g\rho dz$, describes the change in atmospheric pressure with **[3]** altitude. Combine this with the ideal gas law for a gas of molecular weight M at temperature T, $\rho = Mp / RT$, where R = 8.3143 J mol^{-1} K^{-1}, to show that the density can be written as

$$\rho(z) = \frac{p(z_0)}{gH} \exp\left[-\frac{(z - z_0)}{H} \right]$$

where $H = RT / Mg$ is the scale height, the increase in altitude required to reduce the pressure by a factor of e.

[7] (b) A satellite-mounted radiometer is observing the limb of the atmosphere at tangent height h. Show that the path of atmosphere traversed through the limb (i.e., $\int \rho(x)dx$, where x is along the line-of-sight) is approximately 75 times that in a vertical path ($\int \rho(z)dz$) above the tangent point of the path. Assume that H = 7 km and is approximately constant with altitude. *Hint:* You will need to use the equation derived in (a) and $\int_{-\infty}^{\infty} e^{-u^2} du = \sqrt{\pi}$.

6. In class, we derived an expression for the weighting function $K_{\bar{\nu}}(p)$ in terms of p/p_{max} for a nadir- **[10]** viewing radiometer designed to measure infrared emission in the far wings of a Lorentz-broadened line. We also sketched $K_{\bar{\nu}}(p)$ vs. p/p_{max} and claimed that although p_{max} depends on $\bar{\nu}$, the full width at half maximum (FWHM) of the weighting function is independent of $\bar{\nu}$. Prove this by determining $K_{\bar{\nu}}(p_{max})$, p/p_{max} at the half-maximum points, and hence the FWHM. Assuming that the atmosphere is isothermal, then what is the FWHM in terms of altitude? Comment.

7. Given a nadir-viewing radiometer designed to measure infrared emission in order to retrieve atmospheric temperature profiles using Schwarzchild's Equation. Assume that the absorption coefficient, $k_a(\bar{\nu})$, of the line used for sounding is <u>constant</u> for a given $\bar{\nu}$.

[4] (a) Derive an expression for the atmospheric transmission from altitude z to the altitude of the satellite, $\tau_{\bar{\nu}}(z, \infty)$ with $z_{satellite} \equiv \infty$, in terms of the mass mixing ratio $Q = \rho_{gas} / \rho_{air}$ (assumed to be constant with altitude) and the atmospheric pressure p(z).

[3] (b) Derive an expression for the weighting function $K_{\bar{\nu}}(p)$ for such a line, using $y = -\ln(p)$.

[3] (c) At what pressure p_{max} does $K_{\bar{\nu}}(p)$ have a maximum?

[5] (d) Express $K_{\bar{\nu}}(p)$ in terms of p and p_{max} and plot $K_{\bar{\nu}}(p)$ vs. p/p_{max}. Explain the physical significance of $K_{\bar{\nu}}(p)$ and how weighting functions are relevant to the retrieval of vertical profiles of temperature.

8. Figure 1 below shows emission spectra from Mars.

[3] (a) What do the fine features between 50 and 25 μm, especially in the South Polar spectra, indicate? Explain your reasoning.

[3] (b) What is responsible for the large feature centred on 15 μm? Explain your reasoning.

[6] (c) What can be said about the surface and atmospheric temperatures for each of the three geographical regions? Explain your reasoning.

[3] (d) What can be learned from a comparison of the terrestrial mid-latitude spectrum (see Figure 2 below) and the Martian mid-latitude spectrum? Explain your reasoning.