PHY 499S
EARTH OBSERVATIONS FROM SPACE
Spring Term 2005
Problem Set #1

DUE: Thursday, February 10, 2005 (in class).
Late penalty = 5 marks/day, up to 1 week, after which problem sets will not be accepted.

NOTES: Marks, shown in brackets, will be given for workings, as well as for final answers.
Total marks will be scaled to 100. Show all workings and units (preferably SI units).
Mean radius of the Earth = 6371 km.

QUESTIONS:

1. **[12]** A remote sensing satellite is in an orbit with a radius of perigee of 7,000 km and a radius of apogee of 10,000 km.
   (a) What is the altitude of the satellite when the true anomaly is 90°?
   (b) What is the velocity of the satellite when the true anomaly is 90°?
   (c) Sketch the geometry of the satellite's orbit (in the orbital plane) for this situation.
   (d) What is the period of the satellite's orbit when the true anomaly is 90°?

2. **[20]** A satellite is in an elliptical polar orbit of eccentricity 0.1 and semi-major axis 7371 km.
   (a) Calculate the satellite position over one orbit, assuming $\theta = 0$ at $t_p = 0$. Specifically, determine: (i) the time, (ii) the true anomaly, (iii) the mean anomaly, and (iv) the satellite altitude (not radius) for eight points around the orbit corresponding to eccentric anomaly e = 0, $\pi/4$, $\pi/2$, $3\pi/4$, π, $5\pi/4$, $3\pi/2$, $7\pi/4$, and 2π. Provide the results in tabular form, and in graphical form with time on the "x-axis". Use of a spreadsheet or other computer program is recommended. Explicitly derive any intermediate variables needed for this calculation.
   (b) If the satellite is observed to have a true anomaly of 270° at a certain time, how much time will elapse before it reaches a true anomaly of 90°? Sketch the geometry (in 2-D).

3. **[14]** It is often desired that an orbit exactly retrace its path over the Earth's surface so that each site is revisited. For this to occur, a satellite must make an integral number of orbits in an integral number of days. A repeating orbit thus requires the relation

$$\Delta\mathrm{LON} = \tilde{T}\left(\frac{d\Omega_e}{dt} - \frac{d\Omega}{dt}\right) = 2\pi\frac{m}{mN + k}$$

where ΔLON is the difference in longitude between successive ascending nodes, $\tilde{T}$ is the nodal period, $\frac{d\Omega_e}{dt}$ is the Earth's rotation rate, and $\frac{d\Omega}{dt}$ is the rate of change of the right ascension of the ascending node. ERS-1 is a European remote sensing satellite currently in a sun-synchronous orbit with the following orbital parameters: $\varepsilon = 0.00117$, a = 7153.135 km, i = 98.5227°, and $\tilde{T}$ = 100.47 minutes.

(a) What are the values of $\frac{d\Omega_e}{dt}$ and $\frac{d\Omega}{dt}$?

(b) How often does ERS-1 revisit a given site, and after how many orbits? How many orbits does ERS-1 make in a day?

(c) What is the difference in longitude between successive ascending nodes? What is the spacing between any two adjacent sub-satellite tracks? Give both answers in degrees and in km at the equator.

4. **[12]** Given that the angular velocity of the rotating Earth is 2π radians per sidereal day, calculate the period, radius, and velocity of a satellite in geostationary orbit. Approximately what fraction of the Earth's surface will be visible from such a satellite?

5. **[15]** (a) A satellite has the following orbital elements: period = 480 minutes, $\varepsilon = 0.0$, and $i = 25°$. Sketch the ground track of this satellite on the attached map of the Earth. Explain your reasoning.

(b) Given the ground track on the attached map, what is the inclination, the difference in longitude between successive ascending nodes, and the period for a satellite in this orbit?

6. **[12]** A meteorological satellite circles the Earth at an altitude h above the surface.

(a) If the radius of the Earth is R_E, show that the solid angle under which the Earth is seen by the satellite sensor (i.e., the solid angle subtended by Earth as measured from the satellite) is given by $\Omega = 2\pi\left[1 - \frac{\sqrt{2hR_E + h^2}}{R_E + h}\right]$. Clearly sketch the geometry.

(b) Evaluate this expression for a satellite in a low Earth orbit of radius 7271 km, and for a satellite in a geostationary orbit.

7. **[15]** The instantaneous field-of-view (IFOV) of any detector is the angle through which it is sensitive to radiation, and can be written as L/r radians, where L is the length of the subtended arc and r is the distance from the detector to the object being viewed.

(a) An airborne cross-track scanner has an IFOV = 1.5 mrad, an angular FOV = 45°, and a scan mirror that rotates at 4000 revolutions/minute. The aircraft altitude is 10 km. Calculate the size of the ground resolution cell (in units of m by m, and area), the width of the ground swath, and the dwell time for a ground resolution cell.

(b) An along-track scanner has detectors with a 2 mrad IFOV. The scanner is carried in an aircraft at an altitude of 15 km and at a ground speed of 600 km/hr. Calculate the size of the ground resolution cell, and the dwell time for a ground resolution cell.

8. **[8]** Electromagnetic radiation from the Sun falls on the top of the Earth's atmosphere at the rate of 1368 W/m^2 (the solar constant). Assuming this to be plane wave radiation, calculate the magnitude of the electric and magnetic field amplitudes of the wave. Note: The units of the electric field are m kg s^{-2} Co^{-1} and the units of the magnetic field are kg s^{-1} Co^{-1} (Tesla).

9. Calculate the wavelength and the wavenumber at which the incoming solar radiation at the top of the Earth's atmosphere is equal to the outgoing terrestrial irradiance. Assume the Sun to be a blackbody at 5770 K and the Earth to be a blackbody at 255 K.
[6]

10. (a) Elements of surface ΔS_1 and ΔS_2 are a distance r apart, with normals inclined at angles θ_1 and θ_2 to the line joining them. Under what conditions is the net radiative power flow between them in the optical (frequency) passband $\Delta\nu$ given by
[20]

$$\Delta P_{1\to 2} = \frac{B_\nu(T_1)}{r^2}\Delta\nu\Delta S_1\Delta S_2\cos\theta_1\cos\theta_2 \ ?$$

(b) Integrate $\int_0^\infty B_\nu d\nu$ using the substitution $x = h\nu/(kT)$ and hence derive the Stefan-Boltzmann Law. Use the relations

$$\int_0^\infty \frac{x^3 dx}{e^x - 1} = \frac{\pi^4}{15} \quad \text{and} \quad \sigma = \frac{2\pi^5 k^4}{15h^3c^2}.$$

Hence use the results of part (a) to show that the solar constant F_s (i.e., the normal irradiance at the top of the atmosphere) is

$$F_s = \frac{\sigma T_s^4 r_s^2}{d^2}$$

where T_s is the effective black-body temperature of the Sun, r_s is the Sun's radius, and d is the Sun-Earth distance. Evaluate F_s, given that T_s = 5800 K.

(c) By carrying out the appropriate integration, find the power absorbed by a horizontal black disc of radius a from an infinite horizontal black plane at temperature T_1 over which it is suspended at height h.

(d) Find the power absorbed from the plane by a black sphere of radius a.