# PHY 499S
# EARTH OBSERVATIONS FROM SPACE
# Spring Term 2005
# Mid-Term Test
# 1:00 - 2:00 PM, Friday, March 4

**NOTES:** *Answer ALL five questions. Each question is worth a total of 20 marks.*
*Marks will be given for workings and units, as well as for final answers.*
*Non-programmable pocket calculators may be used.*
*A list of constants and equations is provided. No other aid sheets are allowed.*

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## QUESTIONS:

1. Indicate whether the following statements are true or false. If false, give a BRIEF (one sentence) correct version, but not simply the negative of the original statement. ***[Each part is worth 2 marks.]***

(a) $CO_2$ and $H_2O$ are two of the most important atmospheric absorbers at microwave wavelengths, where electronic transitions in the molecules give rise to discrete spectral lines.

(b) The inverse problem is complicated by non-uniqueness of the solution, discreteness of the measurements, and instability due to errors on the observations.

(c) The solar constant is the average radiant exitance leaving the surface (photosphere) of the Sun.

(d) The radiant exitance and the irradiance are both directionally defined terms for the radiant flux density.

(e) If the wavelength-dependent monochromatic intensity is directly proportional to wavelength, $I_\lambda \propto \lambda$, then the frequency-dependent monochromatic intensity is $I_\nu \propto \nu$.

(f) The Rayleigh-Jeans Approximation states that wavelength at which the Planck blackbody function peaks is inversely proportional to temperature.

(g) Cross-track or whiskbroom scanning is usually accomplished by using a scanning mirror, however, changing the pitch of a satellite will result in a similar scan pattern.

(h) The scattering behaviour of clouds depends on the wavelength of light being scattered. At ultraviolet wavelengths, clouds act almost like blackbodies, absorbing nearly all of the radiation incident on them.

(i) The terrestrial irradiance spectrum peaks in the ultraviolet near 0.5 μm, and is therefore often referred to as shortwave radiation.

(j) Newton's Laws of Motion and Gravitation can be used to show that the velocity of an Earth-orbiting satellite depends only on the mass of the satellite.

2. Sweden's third satellite, FREJA, was launched in 1992, and carried an Ultraviolet Auroral Imager from the University of Calgary to study the auroral regions. FREJA was launched into a highly elliptical polar orbit, having a perigee altitude of 597 km, an apogee altitude of 1763 km, an inclination of 63.0°, and a period of 108 minutes.

(a) What are the semi-major axis and eccentricity of FREJA's orbit? ***[4 marks]***

(b) If the time of perigee passage is 0, what are the eccentric anomaly, the mean anomaly, and the satellite altitude when the true anomaly is $\pi/2$ radians? Sketch the geometry. ***[6 marks]***

(c) How many orbits did FREJA make in a day? How often would FREJA revisit a given site, and after how many orbits? ***[4 marks]***

(d) What was the difference in longitude between successive ascending nodes (in degrees at the equator)? ***[4 marks]***

(e) Did FREJA have a prograde or retrograde orbit? Explain. ***[2 marks]***

3. The intrinsic sensitivity of a radiometric observation of thermal emission is determined by the Planck blackbody function, and is given by

$$S = \frac{1}{B_\lambda}\frac{\partial B_\lambda}{\partial T}$$

as this is the fractional change in the detected power for a unit change in the blackbody temperature of the source. Derive an explicit expression for this sensitivity in terms of $\lambda$, T, $c_1$, and $c_2$, and calculate this sensitivity at a temperature of 290 K for 4 μm and for 11 μm, wavelengths that lie in two of the Earth's IR atmospheric windows. Comment briefly on your results. ***[20 marks]***

4. Answer each of the following.

(a) Derive an expression relating the atmospheric pressure at which the Lorentz line width equals the Doppler line width ($\alpha_L = \alpha_D$) to the wavenumber at line centre ($\bar{\nu}_o$). ***[8 marks]***

(b) Calculate this pressure and the corresponding line width for the $H_2O$ absorption line at 6.25 μm, assuming $\alpha_L(T_o,p_o)$= 0.1 cm$^{-1}$ and a constant temperature of 300 K. In which regions of the atmosphere is Lorentz and Doppler line broadening each more important for this line? ***[12 marks]***

5. The Dobson spectrometer is a ground-based instrument used to determine the total vertical column abundance of ozone from measurements of the intensity of sunlight at a pair of wavelengths in the ultraviolet region, one at which absorption by ozone is strong and one at which absorption by ozone is weak.

(a) Sketch the viewing geometry for a Dobson instrument observing the sun at a solar zenith angle of 55°, assuming a plane parallel (i.e., not spherical) atmosphere. ***[5 marks]***

(b) Ignoring emission and scattering, write down Beer's Law for the monochromatic intensity, $I(\lambda)$, as a function of the intensity of the solar radiation incident on the top of the atmosphere ($I_o$), the absorption cross section ($k_a$), and the total optical mass (u) in the slant path. Explain any assumptions. ***[5 marks]***

(c) The terms $I_\lambda$, $I_o$, and $k_a$ are all wavelength dependent, so the equation from (b) is applied at both wavelengths, typically $\lambda_1$ = 325.4 nm and $\lambda_2$ = 305.5 nm. Derive an expression for optical mass u in terms of $I(\lambda_1)$, $I_o(\lambda_1)$, $k_a(\lambda_1)$, $I(\lambda_2)$, $I_o(\lambda_2)$, and $k_a(\lambda_2)$. ***[5 marks]***

(d) Given the known constant values $I_o(\lambda_1)/I_o(\lambda_2) = 1.57$, $k_a(\lambda_1) = 1.149 \times 10^{-20}$ cm$^2$/molecule, and $k_a(\lambda_2) = 1.62 \times 10^{-19}$ cm$^2$/molecule, and a measured value of $I(\lambda_1)/I(\lambda_2) = 6.20$, what are the slant path optical mass and the vertical path optical mass? Ozone optical masses, or column amounts, are usually expressed in Dobson units (DU), with 1 DU equivalent to $2.687 \times 10^{16}$ molecules/cm$^2$. What are the ozone slant column and vertical column in DU? ***[5 marks]***

## TABLE OF CONSTANTS

| CONSTANT | SYMBOL | VALUE |
|---|---|---|
| speed of light in a vacuum | C | $3.00 \times 10^{8}$ m s$^{-1}$ |
| gravitational constant | G | $6.673 \times 10^{-11}$ N m$^{2}$ kg$^{-2}$ |
| orbital constant | G $M_E$ | $3.986 \times 10^{14}$ m$^{3}$ s$^{-2}$ |
| standard gravitational acceleration | G | 9.81 m s$^{-1}$ |
| Planck's constant | H | $6.626 \times 10^{-34}$ J s |
| Boltzmann's constant | K | $1.381 \times 10^{-23}$ J K$^{-1}$ |
| first radiation constant | $c_1$ | $1.191 \times 10^{-16}$ W m$^{2}$ sr$^{-1}$ |
| second radiation constant | $c_2$ | $1.439 \times 10^{-2}$ m K |
| Stefan-Boltzmann constant | $\sigma$ | $5.670 \times 10^{-8}$ W m$^{-2}$ K$^{-4}$ |
| Wien's displacement law constant | None | $2.8979 \times 10^{-3}$ m K |
| Avogadro's number | $N_A$ | $6.022 \times 10^{23}$ molecules mole$^{-1}$ |
| Dobson unit | DU | 1 DU = $2.687 \times 10^{16}$ molecules/cm$^{2}$ |
| molar gas constant | R | 8.3143 J mole$^{-1}$ K$^{-1}$ |
| angular velocity of Earth | $d\Omega_E / dt$ | $7.292 \times 10^{-5}$ rad s$^{-1}$ |
| solar day | $T_{solar}$ | 86,400 s |
| sidereal day | $T_{sidereal}$ | 86,164.1 s |
| mean radius of Earth | $R_E$ | $6.371 \times 10^{6}$ m = 6371 km |
| mass of Earth | $M_E$ | $5.9737 \times 10^{24}$ kg |
| standard pressure | $p_o$ | 1013.25 mbar = $1.01325 \times 10^{5}$ N m$^{-2}$ |
| standard temperature | $T_o$ | 273.15 K |
| scale height of Earth's atmosphere | H = RT/Mg | ~ 7 km |
| radius of Sun (visible disk) | $R_{sun}$ | $6.96 \times 10^{8}$ m |
| mean Earth-Sun distance | $d_{sun}$ | $1.50 \times 10^{11}$ m |
| solar constant | $S_{sun}$ | 1368 W m$^{-2}$ |
| molecular mass of $CO_2$ | M | 44 g/mole=$7.3065\times10^{-26}$ kg/molecule |
| molecular mass of $H_2O$ | M | 18 g/mole=$2.9890\times10^{-26}$ kg/molecule |
| molecular mass of $O_2$ | M | 32 g/mole=$5.3138\times10^{-26}$ kg/molecule |

# LIST OF EQUATIONS

$$F = \frac{Gm_1 m_2}{r^2} \qquad T = 2\pi\sqrt{\frac{r^3}{Gm_E}} \qquad v = \sqrt{Gm_E\left(\frac{2}{r} - \frac{1}{a}\right)} \qquad r = \frac{a(1-\varepsilon^2)}{1+\varepsilon\cos\theta}$$

$$M = e - \varepsilon \sin e = n(t - t_p) \qquad \cos\theta = \frac{\cos e - \varepsilon}{1 - \varepsilon\cos e} \qquad \cos e = \frac{\cos\theta + \varepsilon}{1+\varepsilon\cos\theta} \qquad \tilde{T} = \frac{2\pi}{\bar{n} + \frac{d\omega}{dt}}$$

$$LT = UT + \frac{\psi}{15^\circ} \qquad ECT = UT + \frac{\psi_N}{15^\circ} = 12 + \frac{\Delta\psi}{15^\circ} \qquad \vec{\mathbf{S}} = c^2\varepsilon_0 \vec{\mathbf{E}} \times \vec{\mathbf{H}}$$

$$I = \int_0^\infty I_\lambda d\lambda = \int_0^\infty I_\nu d\nu = \int_0^\infty I_{\bar{\nu}} d\bar{\nu} \qquad B_\lambda = \frac{2hc^2\lambda^{-5}}{\exp\left(\frac{hc}{\lambda kT}\right) - 1} = \frac{c_1\lambda^{-5}}{\exp\left(\frac{c_2}{\lambda T}\right) - 1}$$

$$\lambda_{\max}(\mu m) = \frac{2897.9}{T(K)} \qquad M_{BB} = \sigma T^4 \qquad B_\lambda \cong \frac{c_1 T}{c_2 \lambda^4} \qquad \alpha_\lambda + R_\lambda + \tau_\lambda = 1 \qquad \alpha_\lambda = \varepsilon_\lambda$$

$$dI_\lambda = -\sigma_a(\lambda) I_\lambda ds = -\rho k_a(\lambda) I_\lambda ds = -k_a(\lambda) I_\lambda du$$

$$\tau_\lambda(z_1, z_2) = \exp\left[-\int_{z_1}^{z_2} \frac{k_a(\lambda)\rho}{\mu} dz\right] = \exp\left[-\int_{u_1}^{u_2} k_a(\lambda) du\right]$$

$$\frac{\mu}{\rho k_e(\lambda)} \frac{dI_\lambda}{dz} = -I_\lambda(\theta,\phi) + \tilde{\alpha}_\lambda B_\lambda(T) + \tilde{\omega}_\lambda < I_\lambda' > \qquad < I_\lambda' > = \frac{1}{4\pi} \int_0^{2\pi}\int_{-1}^{1} I_\lambda(\mu', \phi') p(\psi_S) d\mu' d\phi'$$

$$\tilde{\alpha}_\lambda = \frac{\sigma_a(\lambda)}{\sigma_e(\lambda)} \qquad \tilde{\omega}_\lambda = \frac{\sigma_s(\lambda)}{\sigma_e(\lambda)} \qquad \frac{\mu}{\rho k_e(\lambda)} \frac{dI_\lambda}{dz} = -I_\lambda(\mu,\phi) + B_\lambda(T)$$

$$k_a^L = \frac{S}{\pi} \frac{\alpha_L}{(\bar{\nu} - \bar{\nu}_o)^2 + \alpha_L{}^2} \qquad \alpha_L(T, p) = \alpha_L(T_o, p_o) \frac{p}{p_o} \sqrt{\frac{T_o}{T}}$$

$$k_a^D = \frac{S}{\alpha_D\sqrt{\pi}} \exp\left[-\frac{(\bar{\nu} - \bar{\nu}_o)^2}{\alpha_D{}^2}\right] \qquad \alpha_D(T, \bar{\nu}_o) = \sqrt{\frac{2kT}{M}} \frac{\bar{\nu}_o}{c}$$

$$\chi = \frac{2\pi r}{\lambda} \qquad Q_s = \frac{k_s}{\pi r^2} \qquad \sigma_s = \int_0^\infty \pi r^2 Q_s N(r) dr \qquad \sigma_s(\lambda) \cong \frac{8\pi^3}{3\lambda^4} \frac{\left[n_o(\lambda)^2 - 1\right]^2}{N_s{}^2} f(\delta)$$

$$m = n - i\,n' \qquad I_r(\theta_r, \varphi_r) = \int_0^{2\pi}\int_0^{\pi/2} I_i(\theta_i, \varphi_i)\gamma_r(\theta_r, \varphi_r; \theta_i, \varphi_i)\cos\theta_i \sin\theta_i d\theta_i d\varphi_i$$

$$\gamma_r(\theta_r, \phi_r; \theta_{sun}, \phi_{sun}) = \frac{I_r(\theta_r, \phi_r)}{I_{sun}\Omega_{sun}\cos\theta_{sun}} = \frac{I_r}{E_{sun}}$$

$$A = \frac{M}{E} = \int_0^{2\pi} \gamma_r(\theta_r, \varphi_r; \theta_{sun}, \varphi_{sun})\cos\theta_r d\Omega_r$$