

PHY492F / PHY1498F
Fall Term 2012
List of Equations for the Mid-Term Test

$$p = \rho R T = n_o kT \quad p\alpha = R T \quad e = \rho_v R_v T \quad e \alpha_v = R_v T \quad VMR_i = \frac{N_i}{N_d} = \frac{(n_o)_i}{(n_o)_d} = \frac{p_i}{p_d}$$

$$MMR_i = \frac{m_i}{m_d} = \frac{\rho_i}{\rho_d} = \left(\frac{M_i}{M_d} \right) VMR_i \quad T_v \equiv \frac{T}{1 - \frac{e}{p}(1 - \varepsilon)} \quad p = \rho R_d T_v \quad \partial p = -\rho g \partial z$$

$$d\Phi \equiv g dz = -dp/\rho \quad Z \equiv \frac{\Phi(z)}{g_o} = \frac{1}{g_o} \int_0^z g dz \quad Z_2 - Z_1 = \frac{\Phi_2 - \Phi_1}{g_o} = \frac{1}{g_o} \int_{z_1}^{z_2} g dz = \frac{R_d}{g_o} \int_{p_2}^{p_1} T_v \frac{dp}{p}$$

$$p(z) = p(0) e^{-\frac{z}{H}} \quad H = \frac{RT}{g} \quad Z_2 - Z_1 = \bar{H} \ln \left(\frac{p_1}{p_2} \right) = \frac{R_d \bar{T}_v}{g_o} \ln \left(\frac{p_1}{p_2} \right) \quad du = dq - dw$$

$$c_p = c_v + R \quad \Gamma_d = -\frac{dT}{dz} = \frac{g}{c_p} \quad N = \sqrt{\frac{g}{T}(\Gamma_d - \Gamma)} \quad \frac{d^2 z'}{dt^2} = -\frac{g}{T}(\Gamma_d - \Gamma)z' \quad \text{or} \quad \frac{d^2 z'}{dt^2} + N^2 z' = 0$$

$$\theta = T \left(\frac{p_o}{p} \right)^{R/C_p} \quad dS \equiv \frac{dQ}{T} \quad s = c_p \ln \theta + \text{constant} \quad w \equiv \frac{m_v}{m_d} \quad q \equiv \frac{m_v}{m_v + m_d} \approx w$$

$$e = \frac{w}{w + \varepsilon} p \quad \varepsilon = R_d/R_v \quad \frac{de_s}{dT} = \frac{L_v}{T(\alpha_2 - \alpha_1)} \approx \frac{L_v}{T\alpha_2} \quad e_s(T) = Ae^{\beta T} \quad q_* = \frac{e_s/R_v T}{p/RT} = \left(\frac{R}{R_v} \right) \frac{e_s}{p}$$

$$w_s \equiv \frac{m_{vs}}{m_d} \approx 0.622 \frac{e_s}{p} \quad RH \equiv \frac{w}{w_s} \times 100\% \approx \frac{e_s}{p} \times 100\% = \frac{w_s (\text{at } T_d, p)}{w_s (\text{at } T, p)} \times 100\% \quad RH = \frac{q}{q_*} \times 100\%$$

$$\Gamma_s = \Gamma_d \left[\frac{1 + Lq_* / RT}{1 + L\beta q_* / c_p} \right] \quad d\Omega = d\left(\frac{A}{r^2} \right) = \sin \theta d\theta d\phi = d\mu d\phi \quad F = \pi I$$

$$B_\lambda(T) = \frac{2hc^2 \lambda^{-5}}{\exp\left(\frac{hc}{\lambda kT}\right) - 1} = \frac{c_1 \lambda^{-5}}{\exp\left(\frac{c_2}{\lambda T}\right) - 1} \quad B_{\bar{v}}(T) = \frac{2hc^2 \bar{v}^3}{\exp\left(\frac{hc\bar{v}}{kT}\right) - 1} \quad \Omega_{\text{Sun}} B_{\text{Sun}}(\lambda) = \pi B_{\text{Earth}}(\lambda)$$

$$\lambda_{\max} = \frac{2897.9}{T} \quad \bar{v}_{\max} = 1.962 T \quad M_{BB}(T) = \sigma T^4 \quad B_\lambda(T) \approx \frac{c_1}{c_2} \frac{T}{\lambda^4}$$

$$\alpha_\lambda = \varepsilon_\lambda = I - \tau_\lambda \quad \alpha_\lambda + r_\lambda + \tau_\lambda = 1 \quad F(1 - A) = 4\varepsilon_{\text{infrared}} \sigma T_e^4 \quad T_S = 2^{1/4} T_e$$

$$T_s=\left(\frac{2}{2-\varepsilon}\right)^{1/4}T_e \quad T_A=\left(\frac{1}{2-\varepsilon}\right)^{1/4}T_e \quad 2T_n^4=T_{n+1}^4+T_{n-1}^4 \quad T_S=(N+1)^{1/4}T_e$$

$$dI_{\bar{v}} = - I_{\bar{v}} k_{\bar{v}} \rho \, ds \quad I_{\lambda}(x) = I_{\lambda}(0) \exp \left[- \int_0^x k_{\lambda} \rho dx \right] = I_{\lambda}(0) \exp[-k_{\lambda} \rho x] = I_{\lambda}(0) \exp[-\chi_{\lambda}] = I_{\lambda}(0) \tau_{\lambda}$$

$$\tau_{\bar{v}}(z) = \exp \left(-k_{\bar{v}} \int_z^{\infty} \rho \, dz \right) = \exp \left[-k_{\bar{v}} p(z) / g \right] \quad I(\sec \theta) = I_0 e^{-k_a M \sec \theta} \quad k_s = \frac{8\pi^3}{3\lambda^4} \frac{(m^2 - 1)^2}{N^2} f(\delta)$$

$$(m_r - 1) \times 10^8 = 6432.8 + 2949810/(146 - \lambda^{-2}) + 25540/(41 - \lambda^{-2}) \quad (\lambda \text{ in } \mu\text{m}) \quad \chi \equiv 2\pi r / \lambda$$

$$Q_s = \frac{k_s}{\pi r^2} \quad \int_0^{\infty} f(\bar{v}) \, d\bar{v} = 1 \quad \text{and} \quad S = \int_0^{\infty} k_{\bar{v}} \, d\bar{v}$$

$$f_L(\bar{v} - \bar{v}_o) = \frac{1}{\pi} \frac{\alpha_L}{(\bar{v} - \bar{v}_o)^2 + \alpha_L^2} \quad \alpha_L(T, p) = \alpha_L^o(T_o, p_o) \frac{p}{p_o} \sqrt{\frac{T_o}{T}} \quad k_a(\bar{v}) = \frac{S}{\pi} \frac{\alpha_L}{(\bar{v} - \bar{v}_o)^2 + \alpha_L^2}$$

$$f_D(\bar{v} - \bar{v}_o) = \frac{1}{\sqrt{\pi} \alpha_D} \exp \left(- \frac{(\bar{v} - \bar{v}_o)^2}{\alpha_D^2} \right) \quad \alpha_D(T) = \sqrt{\frac{2k_B T}{M}} \frac{\bar{v}_o}{c} \quad k_a(\bar{v}) = \frac{S}{\sqrt{\pi} \alpha_D} \exp \left(- \frac{(\bar{v} - \bar{v}_o)^2}{\alpha_D^2} \right)$$

$$\frac{dI_{\bar{v}}}{k_{\bar{v}} \rho \, dx} = - I_{\bar{v}} + J_{\bar{v}} \quad I_{\bar{v}}(X) = I_{\bar{v}}(0) \tau_{\bar{v}}(X) + \int_{\tau_{\bar{v}}(X)}^1 B_{\bar{v}} \, d\tau \quad I_{\bar{v}}(X) = I_{\bar{v}}(0) e^{-k_{\bar{v}} \rho X} + B_{\bar{v}} \left(1 - e^{-k_{\bar{v}} \rho X} \right)$$

$$dF_{\bar{v}} = -F_{\bar{v}} k_{\bar{v}} \rho r \, dz \quad \frac{dF_{\bar{v}}}{k_{\bar{v}} \rho r \, dz} = -F_{\bar{v}} + \pi \bar{J}_{\bar{v}} \quad h_{\bar{v}} = \frac{dT_{\bar{v}}}{dt} = \frac{I_{\bar{v}}(\infty) \Omega_s k_{\bar{v}}}{c_p} \exp \left[\frac{-k_{\bar{v}} p(z)}{g} \right]$$

$$h_{\bar{v}} = \left(\frac{dT}{dt} \right)_{\bar{v}} = - \frac{\pi}{c_p} k_{\bar{v}} r B_{\bar{v}}(z) \frac{e^{-\tau_{\bar{v}}/\bar{\mu}}}{\bar{\mu}} \quad I_{\lambda}(z_1) = I_{\lambda}(0) \tau_{\lambda}(0, z_1) + \int_{\text{surface}}^{\text{satellite}} B_{\lambda}(T) K_{\lambda}(y) dy$$

$$K_{\lambda}(y) = \frac{d\tau_{\lambda}}{dy} \quad K_{\bar{v}}(p) = 2 \left(\frac{p}{p_{\max}} \right)^2 \exp \left[- \left(\frac{p}{p_{\max}} \right)^2 \right]$$

$$I_{\bar{v}}(z_{\text{tangent}}) = \int_{-\infty}^{\infty} B_{\bar{v}}[T(x)] \frac{\tau_{\bar{v}}(x, z_{\text{tangent}})}{dx} dx$$

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List of Constants for the Mid-Term Test

Universal Constants

Gravitational constant	G	$6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
Standard gravitational acceleration	g	9.81 m s^{-1}
Speed of light in vacuum	c	$3.00 \times 10^8 \text{ m s}^{-1}$
Planck's constant	h	$6.626 \times 10^{-34} \text{ J s}$
Boltzmann constant	k_B	$1.381 \times 10^{-23} \text{ J K}^{-1}$
First radiation constant	c_1	$1.191 \times 10^{-16} \text{ W m}^2 \text{ sr}^{-1}$
Second radiation constant	c_2	$1.439 \times 10^{-2} \text{ m K}$
Avogadro's number	N_A	$6.022 \times 10^{23} \text{ molecules mole}^{-1}$
Stefan-Boltzmann constant	σ	$5.670 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$

Earth and Sun

Global mean surface pressure	p_s	$1.013 \times 10^5 \text{ Pa} = 1013 \text{ mbar}$
Global mean surface temperature	T_s	288 K
Global mean effective radiating temperature	T_e	255 K
Global mean atmospheric surface density	ρ_s	1.235 kg m^{-3}
Earth's rotation rate	Ω	$7.27 \times 10^{-5} \text{ s}^{-1}$
Earth's mean radius	r_{Earth}	6370 km
Mean Earth-Sun distance	d_{Earth}	$1.496 \times 10^{11} \text{ m}$
Solar constant	F or S_0	1367 W m^{-2}
Mean Earth albedo	A	0.30
Radius of Sun (visible disk)	r_{sun}	$6.96 \times 10^8 \text{ m}$
Effective solar radiating temperature	T_{sun}	5780 K

Properties of Air

Standard pressure	p_0	$1013.25 \text{ mbar} = 1.01325 \times 10^5 \text{ N m}^{-2}$
Standard temperature	T_0	273.15 K
Specific heat at constant pressure	c_p	$1005 \text{ J kg}^{-1} \text{ K}^{-1}$
Specific heat at constant volume	c_v	$718 \text{ J kg}^{-1} \text{ K}^{-1}$
Dry air density at 273 K, 1013 hPa	ρ_0	1.293 kg m^{-3}
Universal gas constant	R^*	$8.3143 \text{ J mole}^{-1} \text{ K}^{-1}$
Gas constant for dry air	R or R_d	$287 \text{ J kg}^{-1} \text{ K}^{-1}$
Gas constant for water vapour	R_v	$461.39 \text{ J kg}^{-1} \text{ K}^{-1}$
Mean molecular weight of dry air	M_a	28.97 g/mole