

PHY492F / PHY1498F

Fall Term 2012

List of Equations for the Final Exam

$$p = \rho R T = n_o k T \quad p\alpha = R T \quad e = \rho_v R_v T \quad e\alpha_v = R_v T \quad \text{VMR}_i = \frac{N_i}{N_d} = \frac{(n_o)_i}{(n_o)_d} = \frac{p_i}{p_d}$$

$$\text{MMR}_i = \frac{m_i}{m_d} = \frac{\rho_i}{\rho_d} = \left(\frac{M_i}{M_d} \right) \text{VMR}_i \quad T_v \equiv \frac{T}{1 - \frac{e}{g}(1 - \varepsilon)} \quad p = \rho R_d T_v \quad \partial p = -\rho g \partial z$$

$$d\Phi \equiv g dz = -dp/\rho \quad Z \equiv \frac{\Phi(z)}{g_o} = \frac{1}{g_o} \int_0^z g dz \quad Z_2 - Z_1 = \frac{\Phi_2 - \Phi_1}{g_o} = \frac{1}{g_o} \int_{z_1}^{z_2} g dz = \frac{R_d}{g_o} \int_{p_2}^{p_1} T_v \frac{dp}{p}$$

$$p(z) = p(0) e^{-\frac{z}{H}} \quad H = \frac{RT}{g} \quad Z_2 - Z_1 = \bar{H} \ln \left(\frac{p_1}{p_2} \right) = \frac{R_d \bar{T}_v}{g_o} \ln \left(\frac{p_1}{p_2} \right) \quad du = dq - dw$$

$$c_p = c_v + R \quad \Gamma_d = -\frac{dT}{dz} = \frac{g}{c_p} \quad N = \sqrt{\frac{g}{T} (\Gamma_d - \Gamma)} \quad \frac{d^2 z'}{dt^2} = -\frac{g}{T} (\Gamma_d - \Gamma) z' \quad \text{or} \quad \frac{d^2 z'}{dt^2} + N^2 z' = 0$$

$$\theta = T \left(\frac{p_o}{p} \right)^{R/C_p} \quad dS \equiv \frac{dQ}{T} \quad s = c_p \ln \theta + \text{constant} \quad w \equiv \frac{m_v}{m_d} \quad q \equiv \frac{m_v}{m_v + m_d} \approx w$$

$$e = \frac{w}{w + \varepsilon} p \quad \varepsilon = R_d/R_v \quad \frac{de_s}{dT} = \frac{L_v}{T(\alpha_2 - \alpha_1)} \approx \frac{L_v}{T\alpha_2} \quad e_s(T) = A e^{\beta T} \quad q_* = \frac{e_s / R_v T}{p / RT} = \left(\frac{R}{R_v} \right) \frac{e_s}{p}$$

$$w_s \equiv \frac{m_{vs}}{m_d} \approx 0.622 \frac{e_s}{p} \quad RH \equiv \frac{w}{w_s} \times 100\% \approx \frac{e_s}{p} \times 100\% = \frac{w_s \text{ (at } T_d, p)}{w_s \text{ (at } T, p)} \times 100\% \quad RH = \frac{q}{q_*} \times 100\%$$

$$\Gamma_s = \Gamma_d \left[\frac{1 + Lq_* / RT}{1 + L\beta q_* / c_p} \right] \quad d\Omega = d\left(\frac{A}{r^2} \right) = \sin \theta d\theta d\phi = d\mu d\phi \quad F = \pi I$$

$$B_\lambda(T) = \frac{2hc^2 \lambda^{-5}}{\exp\left(\frac{hc}{\lambda kT}\right) - 1} = \frac{c_1 \lambda^{-5}}{\exp\left(\frac{c_2}{\lambda T}\right) - 1} \quad B_{\bar{v}}(T) = \frac{2hc^2 \bar{v}^3}{\exp\left(\frac{hc\bar{v}}{kT}\right) - 1} \quad \Omega_{\text{Sun}} B_{\text{Sun}}(\lambda) = \pi B_{\text{Earth}}(\lambda)$$

$$\lambda_{\max} = \frac{2897.9}{T} \quad \bar{v}_{\max} = 1.962 \text{ T} \quad M_{\text{BB}}(T) = \sigma T^4 \quad B_\lambda(T) \approx \frac{c_1}{c_2} \frac{T}{\lambda^4}$$

$$\alpha_\lambda = \varepsilon_\lambda = 1 - \tau_\lambda \quad \alpha_\lambda + r_\lambda + \tau_\lambda = 1 \quad F(1 - A) = 4\varepsilon_{\text{infrared}} \sigma T_e^4 \quad T_S = 2^{1/4} T_e$$

$$T_s = \left(\frac{2}{2 - \varepsilon} \right)^{1/4} T_e \quad T_A = \left(\frac{1}{2 - \varepsilon} \right)^{1/4} T_e \quad 2T_n^4 = T_{n+1}^4 + T_{n-1}^4 \quad T_S = (N + 1)^{1/4} T_e$$

$$dI_{\bar{v}} = -I_{\bar{v}} k_{\bar{v}} \rho ds \quad I_\lambda(x) = I_\lambda(0) \exp \left[- \int_0^x k_\lambda \rho dx \right] = I_\lambda(0) \exp[-k_\lambda \rho x] = I_\lambda(0) \exp[-\chi_\lambda] = I_\lambda(0) \tau_\lambda$$

$$\tau_{\bar{v}}(z) = \exp \left(-k_{\bar{v}} \int_z^\infty \rho dz \right) = \exp[-k_{\bar{v}} p(z)/g] \quad I(\sec \theta) = I_0 e^{-k_a M \sec \theta} \quad k_s = \frac{8\pi^3}{3\lambda^4} \frac{(m^2 - 1)^2}{N^2} f(\delta)$$

$$(m_r - 1) \times 10^8 = 6432.8 + 2949810/(146 - \lambda^{-2}) + 25540/(41 - \lambda^{-2}) \quad (\lambda \text{ in } \mu\text{m}) \qquad \chi \equiv 2\pi r / \lambda$$

$$Q_s = \frac{k_s}{\pi r^2} \quad \int_0^\infty f(\bar{v}) d\bar{v} = 1 \quad \text{and} \quad S = \int_0^\infty k_{\bar{v}} d\bar{v}$$

$$f_L(\bar{v} - \bar{v}_o) = \frac{1}{\pi} \frac{\alpha_L}{(\bar{v} - \bar{v}_o)^2 + \alpha_L^2} \quad \alpha_L(T, p) = \alpha_L^o(T_o, p_o) \frac{p}{p_o} \sqrt{\frac{T_o}{T}} \quad k_a(\bar{v}) = \frac{S}{\pi} \frac{\alpha_L}{(\bar{v} - \bar{v}_o)^2 + \alpha_L^2}$$

$$f_D(\bar{v} - \bar{v}_o) = \frac{1}{\sqrt{\pi} \alpha_D} \exp\left(-\frac{(\bar{v} - \bar{v}_o)^2}{\alpha_D^2}\right) \quad \alpha_D(T) = \sqrt{\frac{2k_B T}{M}} \frac{\bar{v}_o}{c} \quad k_a(\bar{v}) = \frac{S}{\sqrt{\pi} \alpha_D} \exp\left(-\frac{(\bar{v} - \bar{v}_o)^2}{\alpha_D^2}\right)$$

$$\frac{dI_{\bar{v}}}{k_{\bar{v}} \rho dx} = -I_{\bar{v}} + J_{\bar{v}} \quad I_{\bar{v}}(X) = I_{\bar{v}}(0)\tau_{\bar{v}}(X) + \int_{\tau_{\bar{v}}(X)}^1 B_{\bar{v}} d\tau$$

$$I_{\bar{v}}(X) = I_{\bar{v}}(0)e^{-k_{\bar{v}} \rho X} + B_{\bar{v}} \left(1 - e^{-k_{\bar{v}} \rho X}\right)$$

$$dF_{\bar{v}} = -F_v k_{\bar{v}} \rho r dz \quad \frac{dF_{\bar{v}}}{k_{\bar{v}} \rho r dz} = -F_v + \pi \bar{J}_{\bar{v}} \quad h_{\bar{v}} = \frac{dT_{\bar{v}}}{dt} = \frac{I_{\bar{v}}(\infty) \Omega_s k_{\bar{v}}}{c_p} \exp\left[\frac{-k_{\bar{v}} p(z)}{g}\right]$$

$$h_{\bar{v}} = \left(\frac{dT}{dt} \right)_{\bar{v}} = -\frac{\pi}{c_p} k_{\bar{v}} r B_{\bar{v}}(z) \frac{e^{-\tau_{\bar{v}}/\bar{\mu}}}{\bar{\mu}} \quad I_{\lambda}(z_1) = I_{\lambda}(0) \tau_{\lambda}(0, z_1) + \int_{\text{surface}}^{\text{satellite}} B_{\lambda}(T) K_{\lambda}(y) dy$$

$$K_{\lambda}(y) = \frac{d\tau_{\lambda}}{dy} \quad K_{\bar{v}}(p) = 2 \left(\frac{p}{p_{\max}} \right)^2 \exp\left[-\left(\frac{p}{p_{\max}} \right)^2\right] \quad I_{\bar{v}}(z_{\text{tangent}}) = \int_{-\infty}^{\infty} B_{\bar{v}}[T(x)] \frac{\tau_{\bar{v}}(x, z_{\text{tangent}})}{dx} dx$$

$$\tau = M / F \quad C_g = k_H p_g \quad \frac{dN}{d(\log D)} = CD^{-\beta} \quad \mathbf{V} \equiv u \mathbf{i} + v \mathbf{j} \quad \mathbf{V}(\Psi) = \mathbf{k} \times \nabla \Psi$$

$$\left[\frac{D}{Dt} = \right] \frac{d}{dt} \equiv \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} \equiv \frac{\partial}{\partial t} + \bar{\mathbf{V}} \bullet \nabla \quad \frac{d\mathbf{V}}{dt} = \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \bullet \nabla \mathbf{V}$$

	Vectorial	Natural coords.	Cartesian coords.
<i>Shear</i>		$-\frac{\partial V}{\partial n}$	
<i>Curvature</i>		$V \frac{\partial \psi}{\partial s}$	
<i>Diffluence</i>		$V \frac{\partial \psi}{\partial n}$	
<i>Stretching</i>		$\frac{\partial V}{\partial s}$	
<i>Vorticity</i> ζ	$\mathbf{k} \cdot \nabla \times \mathbf{V}$	$V \frac{\partial \psi}{\partial s} - \frac{\partial V}{\partial n}$	$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$
<i>Divergence</i> $\text{Div}_H \mathbf{V}$	$\nabla \cdot \mathbf{V}$	$V \frac{\partial \psi}{\partial n} + \frac{\partial V}{\partial s}$	$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$
<i>Deformation</i>		$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}; \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$	

$$\zeta = 2\omega \quad \frac{dA}{dt} = \delta y \frac{d}{dt} \delta x + \delta x \frac{d}{dt} \delta y = \delta x \delta y \frac{\partial u}{dx} + \delta x \delta y \frac{\partial v}{dy} \quad \frac{1}{A} \frac{dA}{dt} = \frac{\partial u}{dx} + \frac{\partial v}{dy}$$

$$C = \oint V_s ds = \iint \zeta dA = \bar{\zeta} A \quad \oint V_n ds = \iint \text{Div}_H V dA \quad \frac{\partial \psi}{\partial t} = -\mathbf{V} \bullet \nabla \psi = -u \frac{\partial \psi}{\partial x} - v \frac{\partial \psi}{\partial y}$$

$$\mathbf{g} = \mathbf{g}^* + \Omega^2 \mathbf{R}_A \quad \mathbf{C} = -f \mathbf{k} \times \mathbf{V} \quad \mathbf{P} \equiv -\frac{1}{\rho} \nabla p = -\frac{1}{\rho} (g \rho \nabla z) = -g \nabla z = -\nabla \Phi = -g_o \nabla Z$$

$$P_x \equiv -\frac{1}{\rho} \frac{\partial p}{\partial x} \quad P_y \equiv -\frac{1}{\rho} \frac{\partial p}{\partial y} \quad \mathbf{F} \equiv -\frac{1}{\rho} \frac{\partial \mathbf{r}}{\partial z} \quad \boldsymbol{\tau}_s = -\rho C_D V_s V_s$$

$$\frac{d\mathbf{V}}{dt} = -\frac{1}{\rho} \nabla p - f \mathbf{k} \times \mathbf{V} + \mathbf{F} = -\nabla \Phi - f \mathbf{k} \times \mathbf{V} + \mathbf{F} \quad \begin{aligned} \frac{du}{dt} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + fv + F_x \\ \frac{dv}{dt} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} - fu + F_y \end{aligned}$$

$$V_g \equiv \frac{1}{f} (\mathbf{k} \times \nabla \Phi) = -\frac{1}{f} \frac{\partial \Phi}{\partial n} \quad u_g = -\frac{1}{f} \frac{\partial \Phi}{\partial y} = -\frac{1}{f \rho} \frac{\partial p}{\partial y} \quad v_g = \frac{1}{f} \frac{\partial \Phi}{\partial x} = \frac{1}{f \rho} \frac{\partial p}{\partial x}$$

$$\mathbf{n} \frac{V^2}{R_T} = -\nabla \Phi - f \mathbf{k} \times \mathbf{V} \quad V_{gr} = \frac{1}{f} \left(|\nabla \Phi| + \frac{V_{gr}^2}{R_T} \right) \quad (u_g)_2 - (u_g)_1 = -\frac{g_o}{f} \frac{\partial (Z_2 - Z_1)}{\partial y}$$

$$(V_g)_2 - (V_g)_1 = \frac{1}{f} \mathbf{k} \times \nabla (\Phi_2 - \Phi_1) = \frac{g_o}{f} \mathbf{k} \times \nabla (Z_2 - Z_1) = \left(\frac{R}{f} \ln \frac{p_1}{p_2} \right) \mathbf{k} \times \nabla (\bar{T})$$

$$\omega(x, y, p) \equiv \frac{dp}{dt} = \frac{\partial p}{\partial t} + \mathbf{V} \bullet \nabla p + w(x, y, z) \frac{\partial p}{\partial z} = -\rho g w + \frac{\partial p}{\partial t} + \mathbf{V} \bullet \nabla p \equiv -\rho g w$$

$$\frac{dw}{dt} = -\frac{1}{\rho} \nabla p - g - C_z + F_z \quad \frac{dT}{dt} = \frac{\kappa T}{p} \omega + \frac{J}{c_p} \quad \frac{\partial T}{\partial t} = -\mathbf{V} \bullet \nabla T + \left(\frac{\kappa T}{p} - \frac{\partial T}{\partial p} \right) \omega + \frac{J}{c_p}$$

$$\frac{DQ}{Dt} = c_p \frac{DT}{Dt} - \frac{1}{\rho} \frac{Dp}{Dt} \quad \frac{dp}{dt} + \rho \nabla \bullet \mathbf{V} = 0 \quad \frac{\partial \omega}{\partial p} = -\nabla \bullet \mathbf{V}$$

$$\frac{\partial p_s}{\partial t} = -\mathbf{V}_s \bullet \nabla p_s - w_s \frac{\partial p}{\partial z} - \int_0^{p_s} (\nabla \bullet \mathbf{V}) dp$$

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List of Constants for the Final Exam

Universal Constants

Gravitational constant	$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
Standard gravitational acceleration	$g = 9.81 \text{ m s}^{-2}$
Speed of light in vacuum	$c = 3.00 \times 10^8 \text{ m s}^{-1}$
Planck's constant	$h = 6.626 \times 10^{-34} \text{ J s}$
Boltzmann constant	$k_B = 1.381 \times 10^{-23} \text{ J K}^{-1}$
First radiation constant	$c_1 = 1.191 \times 10^{-16} \text{ W m}^2 \text{ sr}^{-1}$
Second radiation constant	$c_2 = 1.439 \times 10^{-2} \text{ m K}$
Avogadro's number	$N_A = 6.022 \times 10^{23} \text{ molecules mole}^{-1}$
Stefan-Boltzmann constant	$\sigma = 5.670 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Loschmidt's number	$n_0 = 2.687 \times 10^{19} \text{ molecules cm}^{-3}$

Earth and Sun

Global mean surface pressure	$p_s = 1.013 \times 10^5 \text{ Pa} = 1013 \text{ mbar}$
Global mean surface temperature	$T_s = 288 \text{ K}$
Global mean effective radiating temperature	$T_e = 255 \text{ K}$
Global mean atmospheric surface density	$\rho_s = 1.235 \text{ kg m}^{-3}$
Earth's rotation rate	$\Omega_{\text{rot}} = 7.27 \times 10^{-5} \text{ s}^{-1}$
Earth's mean radius	$r_{\text{Earth}} = 6370 \text{ km}$
Mean Earth-Sun distance	$d_{\text{Earth}} = 1.496 \times 10^{11} \text{ m}$
Solar constant	$F \text{ or } S_0 = 1367 \text{ W m}^{-2}$
Mean Earth albedo	$A = 0.30$
Radius of the Sun (visible disk)	$r_{\text{sun}} = 6.96 \times 10^8 \text{ m}$
Linear angle subtended by the Sun	$\alpha_{\text{sun}} = 32 \text{ arcminutes}$
Solid angle subtended by the Sun	$\Omega_{\text{sun}} = 6.8 \times 10^{-5} \text{ steradians}$
Effective solar radiating temperature	$T_{\text{sun}} = 5780 \text{ K}$

Properties of Air

Standard pressure	$p_0 = 1013.25 \text{ mbar} = 1.01325 \times 10^5 \text{ N m}^{-2}$
Standard temperature	$T_0 = 273.15 \text{ K}$
Specific heat at constant pressure	$c_p = 1005 \text{ J kg}^{-1} \text{ K}^{-1}$
Specific heat at constant volume	$c_v = 718 \text{ J kg}^{-1} \text{ K}^{-1}$
Dry air density at 273 K, 1013 hPa	$\rho_0 = 1.293 \text{ kg m}^{-3}$
Universal gas constant	$R^* = 8.3143 \text{ J mole}^{-1} \text{ K}^{-1}$
Gas constant for dry air	$R \text{ or } R_d = 287 \text{ J kg}^{-1} \text{ K}^{-1}$
Gas constant for water vapour	$R_v = 461.39 \text{ J kg}^{-1} \text{ K}^{-1}$
Mean molecular weight of dry air	$M_a = 28.97 \text{ g/mole}$