PHY 2505S
ATMOSPHERIC RADIATIVE TRANSFER AND REMOTE SOUNDING
Spring Term, 2020

Problem Set #2

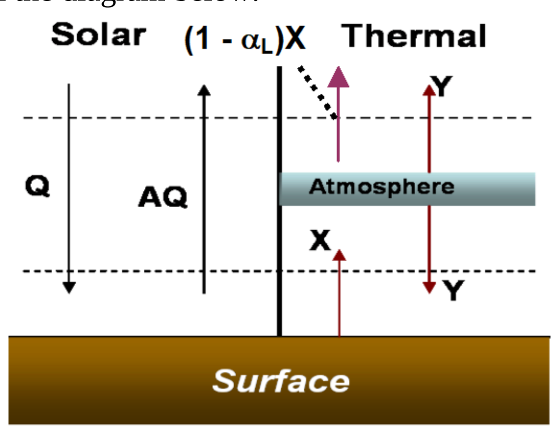
DUE: Handed out on March 10 and due in class March 24.

NOTES: Marks, shown in brackets, will given for showing your work, as well as for final answers. Total = 90 marks. Show all workings and units. Remember to define parameters and provide enough explanation to convince the marker that you know what the equations mean and where they come from. Submitted problem sets must be typed or legibly hand-written. While you may discuss the assignment with your classmates, you must prepare your answers to the problems independently.

LATE PENALTY: 5% per day, up to seven days, after which material will not be accepted.

QUESTIONS:

1. Consider a two-tone radiation budget model of the Earth as shown below, which has an albedo A in the visible and a perfectly "black" surface in the infrared, and an infinitely thin atmosphere which does not absorb in the visible (solar), but has an absorptivity $\alpha_L=\varepsilon$ for outgoing longwave infrared (terrestrial) radiation. Assume that the solar constant is 1370 W/m^2, and $Q = S_o/4$ in the diagram below.



(a) Derive an expression for the effective radiating temperature (T_e) of the Earth in terms of the temperature of the surface (T_g). ***[3 marks]***

(b) If ε = 1 and T_e = 255 K, then what is the temperature of the surface? ***[2 marks]***

(c) If ε = 1 and T_e = 255 K, then how much radiation (W m^{-2}) does Earth's atmosphere prevent from escaping to space from the surface? If the atmospheric CO_2 concentration is doubled and the atmosphere absorbs an extra 4.0 Wm^{-2}, estimate the resulting change in the surface temperature. ***[4 marks]***

(d) If $\varepsilon = 0$ and $T_e = 255$ K, then what is the temperature of the surface? ***[2 marks]***

(e) If $T_e = 255$ K and $T_g = 288$ K, then what is the absorptivity of the atmosphere ($\alpha_L = \varepsilon$)? ***[4 marks]***

2. Marshall and Plumb, Chapter 2, Problem 5.
Consider an atmosphere that is completely transparent to shortwave (solar) radiation, but very opaque to infrared (IR) terrestrial radiation. Specifically, assume that it can be represented by N slabs of atmosphere, each of which is completely absorbing of IR, as depicted in the following schematic figure (not all layers are shown). Assume blackbody radiation.

(a) By considering the radiative equilibrium of the surface, show that the surface must be warmer than the lowest atmospheric layer. ***[5 marks]***

(b) By considering the radiative equilibrium of the nth layer, show that, in equilibrium:

$$2T_n^4 = T_{n+1}^4 + T_{n-1}^4$$

where T_n is the temperature of the n^{th} layer, for $n > 1$. Hence, with Te being the planetary emission temperature, show that the equilibrium surface temperature is: $T_s = (N+1)^{\frac{1}{4}} T_e$. ***[5 marks]***

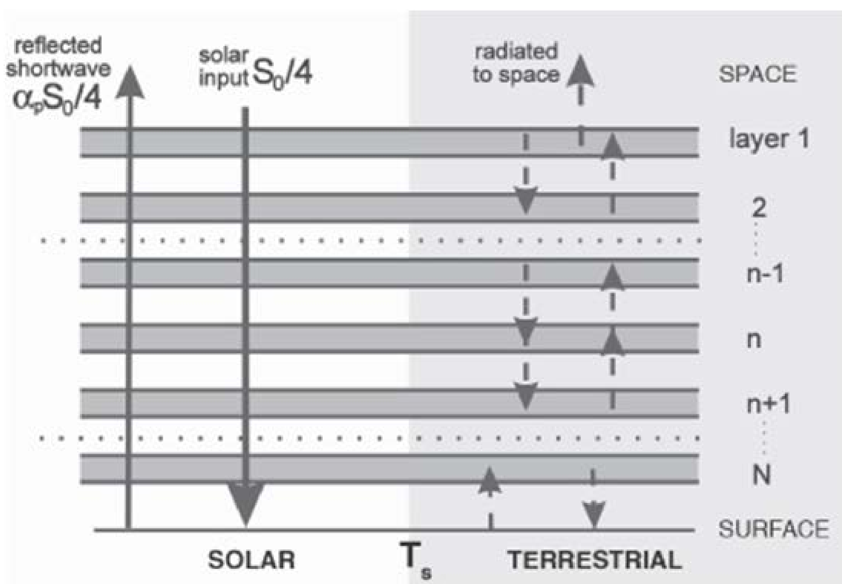


Figure 2.12: An atmosphere made up of *N* slabs, each of which is completely absorbing in the IR spectrum.

3. In the n-layer greenhouse model, $T_s = (n+1)^{1/4} T_E$, the increase of the surface temperature with the number of layers reflects the influence of increased abundances of greenhouse gases in the atmosphere on the surface temperature. Let's assume that the number of layers is a power of the surface temperature

$$n(T_s) = \alpha T_s^{\beta}.$$

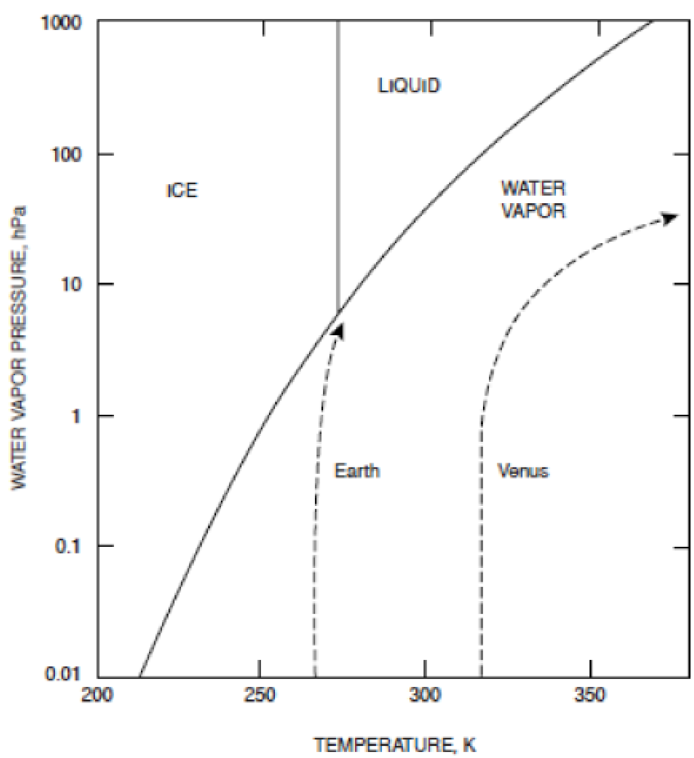
This captures, for example, the rapid increase of the saturation vapour pressure of water in the atmosphere with temperature.

(a) Determine the values of a appropriate for current conditions $T_s = 288\text{K}$ and $T_E = 255\text{K}$, for b = 2 and b = 4. ***[3 marks]***

(b) Using these values of a, plot T_S as a function of T_E (over the range 150 – 350 K) for b = 2 and b = 4. Would you characterize the dependence of T_s on T_E for b = 2 as an example of a runaway greenhouse effect? What about for b = 4? The saturation vapour pressure of water actually increases exponentially with temperature. ***[3 marks]***

(c) The high surface temperature on Venus is often described as an example of a runaway greenhouse. Calculate the emission temperature of Venus assuming a planetary albedo of 0.15, which might be expected in the absence of clouds early in the planet's history. You may assume the following: the mean radius of Venus' orbit is 0.72 times that of the Earth's orbit; and the solar flux density decreases with the square of the distance from the sun and has a value of 1367 W m^{-2} at the mean Earth orbit. ***[3 marks]***

(d) The figure below shows the evolution of temperatures in the early atmospheres of Venus and Earth, together with the phase diagram for water. Using this figure, explain why Earth did not experience a runaway greenhouse as water accumulated in the atmosphere from outgassing from the planet's interior. ***[3 marks]***



4. Assuming an absorber has uniform absorption coefficient and is uniformly mixed with height, a ground temperature of 280K, a tropopause temperature of 220 K at 250 mbar, and radiative equilibrium conditions, determine (i) the net upward flux ϕ, (ii) the modified optical depth at the surface, χ_o^*, and (iii) the temperature discontinuity at the surface. ***[6 marks]***

5. Show that the J value of the rotational energy level with the maximum population is related to temperature and rotational constant by: $(2J+1)^2 = (2kT)/(Bhc)$. Given that the $H^{81}Br$ molecule has a rotational constant of 8.119 cm^{-1}, determine the J values of the rotational levels with the maximum population at 300 and 2500 K. ***[6 marks]***

6. The linear molecule CO_2 has three vibration modes:

↑ (on C of $\bar{\nu}_2$)

$\bar{\nu}_1$: ← O — C — O → $\bar{\nu}_2$: O — C — O $\bar{\nu}_3$: ← O — C → ← O

↓ ↓ (on both O of $\bar{\nu}_2$)

The $\bar{\nu}_1$ mode does not induce a dipole moment and so is inactive in absorption. Therefore, CO_2 has two fundamental vibration modes (v changes from 0 to 1): $\bar{\nu}_2$ at 667 cm^{-1} (~15 μm) and $\bar{\nu}_3$ at 2349 cm^{-1} (~4.3 μm).

(a) Given that the rotational constant B = 0.3925 cm^{-1}, determine the wavenumber of the absorption lines in the *P* and *R* branches of the asymmetric stretch vibrational band for which the initial rotational quantum number is: *J* = 1, 3, 5, 7, 9. ***[5 marks]***

(b) Determine the relative intensities (S_{rel}) of the same rotational lines of CO_2 as used in part (a), normalized to P(1). Assume the sample of gas is at room temperature. ***[5 marks]***

7. Assume that the atmosphere follows the ideal gas law, with the density decreasing as $\rho(z) = \rho(0) \exp(-z/H)$, with scale height H=8 km. If the Lorentz line width is 100 times larger than the Doppler width at ground level, at what altitude are these widths equal? ***[5 marks]***

8. Prove that the Doppler half-width at half maximum, γ_D, is related to the Doppler width, α_D, by $\gamma_D = \alpha_D \sqrt{(\ln 2)}$. ***[4 marks]***

9. (a) Derive the approximation for the equivalent width of a Lorentz line in the weak absorption limit. ***[2 marks]***

(b) Derive the approximation for the equivalent width of a Lorentz line in the strong absorption limit. (Note: line centre is black, so $\alpha_L << \bar{\nu} - \bar{\nu}_o$ for this case.) ***[4 marks]***

(c) Sketch the curve of growth (W vs. u) and sketch the transmission (τ vs. $\bar{\nu} - \bar{\nu}_o$) for a typical Lorentz line for which $S = 10^{-19}$ $cm^{-1}/(molec\ cm^{-2})$, $\alpha_L^o = 0.06$ cm^{-1}, and given u (cm^2/molec) = 3×10^{-21} (case A), 2×10^{-19} (case B), and 4×10^{-19} (case C). Indicate the weak and strong limits in both figures. ***[6 marks]***

10. Given the Goody line strength distribution $N(S) = (N_o/\sigma) \exp(-S/\sigma)$:

(a) Derive an expression for the sum of the equivalent widths of these lines, $\sum W_i$, assuming no line overlap. ***[2 marks]***

(b) Using the equation $k(\bar{\nu}) = S \times f(\bar{\nu})$, integrate over S to simplify the expression for $\sum W_i$. ***[2 marks]***

(c) Assuming a Lorentz line, integrate over $\bar{\nu}$ to further simplify $\sum W_i$. ***[2 marks]***

(d) Determine the values of $\sum W_i$ in the weak and strong limits (i.e. $\sigma\rho x / \pi\alpha_L$<<1 and >>1. ***[2 marks]***

(e) Compare the values of $\sum W_i$ obtained for the Goody model with $\sum W_i$ derived from the general case in problem 9; match the results in the weak limit and the results in the strong limit to find expressions for the Goody model parameters, N_o and σ. ***[2 marks]***