PHY 2505S
ATMOSPHERIC RADIATIVE TRANSFER AND REMOTE SOUNDING
Spring Term, 2020

Problem Set #1

DUE: Handed out on January 21 and due in class on February 4.

NOTES: Marks, shown in brackets, will given for showing your work, as well as for final answers. Total marks = 70. Show all workings and units. Remember to define parameters and provide enough explanation to convince the marker that you know what the equations mean and where they come from. Submitted problem sets must be typed or legibly hand-written. While you may discuss the assignment with your classmates, you must prepare your answers to the problems independently.

LATE PENALTY: 5% per day, up to seven days, after which material will not be accepted.

QUESTIONS:

1. Wallace and Hobbs, Chapter 4, Problems 14 and 15.

   (a) A body is emitting radiation with the following idealized spectrum of monochromatic flux density:

$\lambda < 0.35\ \mu m$	$F_\lambda = 0$
$0.35\ \mu m < \lambda < 0.50\ \mu m$	$F_\lambda = 1.0\ W\ m^{-2}\ \mu m^{-1}$
$0.50\ \mu m < \lambda < 0.70\ \mu m$	$F_\lambda = 0.5\ W\ m^{-2}\ \mu m^{-1}$
$0.70\ \mu m < \lambda < 1.00\ \mu m$	$F_\lambda = 0.2\ W\ m^{-2}\ \mu m^{-1}$
$\lambda > 1.00\ \mu m$	$F_\lambda = 0$

   Calculate the flux density of the radiation. ***[2 marks]***

   (b) An opaque surface with the following absorption spectrum is subjected to the radiation described in part (a).

$\lambda < 0.70\ \mu m$	$\alpha_\lambda = 0$
$\lambda > 0.70\ \mu m$	$\alpha_\lambda = 1$

   How much of the radiation is absorbed? How much is reflected? ***[4 marks]***

2. Starting with Planck's blackbody function, $B_\lambda(T)$, derive Wien's Displacement Law, $\lambda_{max} = 2897.9 / T$, where λ is in μm and T is in K. Use this expression to calculate the maximum wavelengths of the Planck function in interstellar space at 3 K, at room temperature (25°C), in a flame (2200°C), and in the photosphere of the Sun (5780 K). ***[8 marks]***

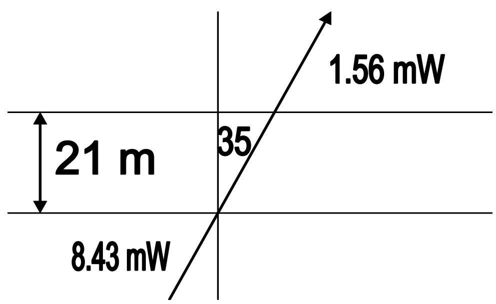
3. Starting with Planck's blackbody function, $B_\lambda(T)$, derive the Stefan-Boltzmann Law, $M(T) = \sigma T^4$, where $\sigma = (\pi^5 / 15)(c_1 / c_2^4)$. ***[5 marks]***

4. Convert the monochromatic Planck blackbody function, $B_\lambda(T)$, to the alternative forms B_ν (T) and $B_{\bar{\nu}}(T)$, and show that all three expressions are equivalent. ***[6 marks]***

5. A land surface at a temperature of 25°C, and with an emissivity of 0.7, emits radiation at all frequencies. Calculate and compare the emitted radiances at 550 nm (visible), 2000 cm^{-1} (infrared) and 27 GHz (microwave) <u>using appropriate units</u>. ***[6 marks]***

6. Given the values of the solar constant (1370 W/m^2), the mean Earth-Sun distance (1.5×10^8 km), and the Sun's radius (0.7×10^6 km), calculate
(a) the solid angle subtended by the Sun at the Earth,
(b) the solar radiance incident on the top of the atmosphere, and
(c) the Sun's equilibrium temperature, assuming it behaves as a blackbody. ***[6 marks]***

7. A helium-neon laser beam at 0.6328 μm with an output intensity of 8.43 $mW/m^2/sr$ is directed through a cloud layer 21 m thick at an angle of 35° from the normal to the layer, as shown in the figure below. The measured intensity at the top of the cloud is 1.56 $mW/m^2/sr$. Neglecting emission and scattering into the beam, calculate the following:
(a) <u>volume</u> extinction coefficient σ (in units of m^{-1}) ***[2 marks]***
(b) transmission along the slant path and the transmission along the vertical path. ***[4 marks]***



8. The direct solar flux density, F, at the Earth's surface in the 1.8 to 1.9 μm wavelength interval is measured on a clear day, yielding the following data:

SZA (°)	40	50	60	70
F (W/m^2)	14.0	12.6	10.5	7.67

Determine the solar flux density at the top of the atmosphere, and the transmission of the atmosphere for normal incidence in this spectral interval. ***[6 marks]***

9. An infrared radiometer on board a meteorological satellite measures the outgoing radiation from the Earth in the 10 μm spectral region. The observed radiance at 10 μm is 0.100 $W/m^2/cm^{-1}/sr$.
(a) Assuming no atmospheric absorption, what is the temperature of the Earth?
(b) What would the radiance be at 30 GHz (assuming 300 K and no absorption)?
(c) If the observed radiance at 10 μm was 0.100 $W/m^2/cm/sr$ (assuming no absorption), calculate the temperature of the Earth and comment on your result. ***[6 marks]***

10. On a clear day, an experiment is being conducted to measure the solar constant by the "long" method. The measurements of the solar intensity I(θ) at the Earth's surface (including instrumental noise) in two spectral regions are as follows:

Solar Zenith Angle	cm^{-1} = 22,200 cm^{-1}	cm^{-1} = 15,400 cm^{-1}
0	0.0702	3.532
10	0.0657	3.477
20	0.0540	3.311
30	0.0362	3.025
40	0.0189	2.599
50	0.0065	2.024
60	0.0017	1.300
70	2.3×10^{-5}	0.517
80	2.0×10^{-10}	0.029

(a) Using a spreadsheet or a real piece of graph paper, plot on the same graph ln[I(θ)] vs. sec θ for both wavenumbers. This is called a Langley plot. ***[5 marks]***

(b) Hence calculate the value of the extra-terrestrial solar intensity at these wavenumbers and the value of k_aM for both wavenumbers. ***[4 marks]***

(c) Can the solar constant (flux density) be obtained from these data? Explain why or why not. ***[3 marks]***

(d) Show that for these two cases, the vertical optical depth, k_aM, is approximately proportional to the fourth power of the wavenumber. ***[3 marks]***