PHY2505S
Atmospheric Radiative Transfer and Remote Sounding

Lecture 7

- Terrestrial Fluxes: Schwarzchild's Equation Revisited
- The Two-Stream Model
- Including Clouds in the Two-Stream Model

Terrestrial Fluxes: Schwarzchild's Equation Revisited - 1

- We now want to consider the radiative transfer of flux density in the atmosphere.
- But the terrestrial FLUX DENSITY is not the INTENSITY!
  - $\rightarrow$ *Schwarzchild's Equation applies to intensity, NOT to flux density.*

$$\frac{dI_{\bar{\nu}}}{k_{\bar{\nu}}\,\rho\,dx} = -I_{\bar{\nu}} + J_{\bar{\nu}}$$

F = radiant flux density (W m^{-2})
I = radiance or intensity (W m^{-2} sr^{-1})

Let's consider a simple flux density problem.

- First, note that for a vertical beam of intensity I_o incident on the bottom of a thin layer having absorption coefficient k, the transmitted beam will just be $I_o \exp(-k\,\rho\,\Delta z)$.
- Now, if we assume an isotropic flux density incident on the bottom of the layer, what flux density will emerge from the top?

Terrestrial Fluxes: Schwarzchild's Equation Revisited - 2

- The total energy (flux density) incident on the bottom of the layer is:

$$F_o = I_o \int d\Omega = I_o 2\pi \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta \; = \; \pi I_o$$

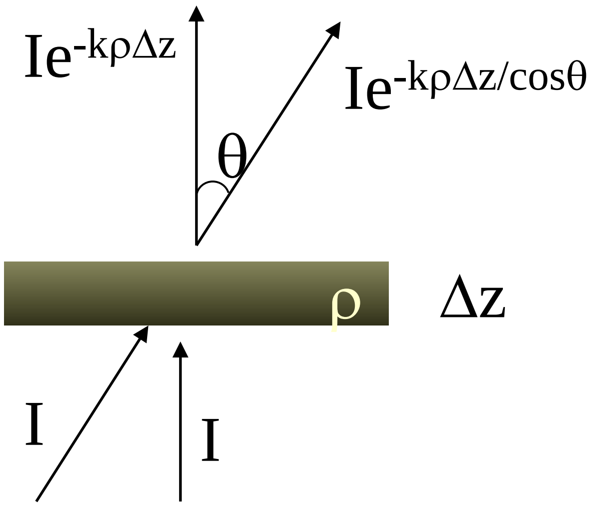
- The energy (flux density) emerging from the top of the layer:

$$F = I_o 2\pi \int_0^{\pi/2} \exp(-k\rho\Delta z / \cos\theta) \cos\theta \sin\theta \, d\theta$$

$$= 2\pi I_o \int_1^{\infty} \frac{e^{-k\rho\Delta z y}}{y^3} dy \quad = \quad 2\pi I_o E_3(k\rho\Delta z)$$

where y is $1/\cos\theta$

the third exponential integral - non-analytic



Flux transmitted through a thin layer with an isotropic flux incident on the bottom.

$$y = \frac{1}{\cos\theta} \longrightarrow dy = \frac{\sin\theta}{\cos^2\theta} d\theta$$

$$dy = \frac{\sin\theta\cos\theta}{\cos^3\theta} d\theta$$

$$\frac{1}{y^3} dy = \cos\theta \sin\theta \, d\theta$$

Terrestrial Fluxes: Schwarzchild's Equation Revisited - 3

Thus the emerging flux contains a non-analytic integral and is also not isotropic. We need some approximations to help:

(1) We are only interested in UPWARD and DOWNWARD fluxes.

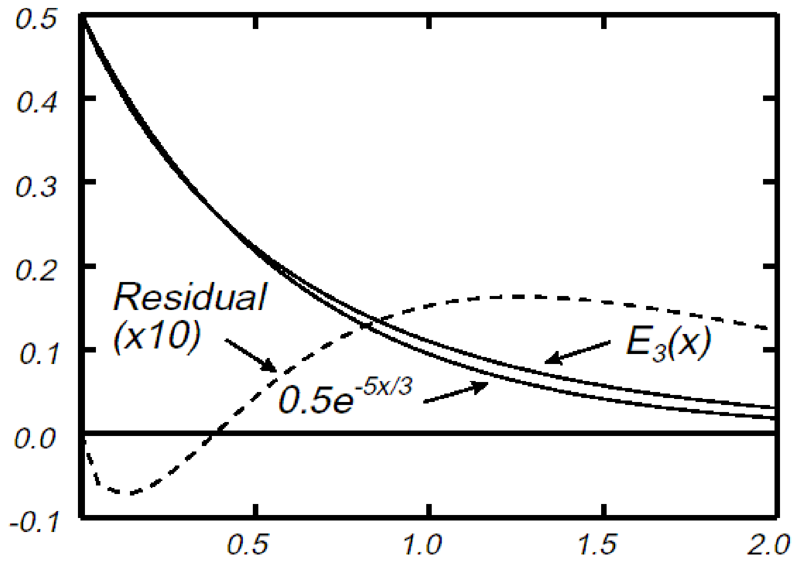
→ Therefore we can ignore the fact that the energy is no longer isotropic.

(2) Next, the third exponential integral can be approximated as:

$$E_3(k\rho\Delta z) \cong \exp\left(-\tfrac{5}{3}k\rho\Delta z\right)$$

→ This means we can write the attenuation of the upward flux density in the same way as intensity by multiplying the amount of material in the path by 5/3.

→ This factor 5/3 is called the <u>diffusivity factor</u> (r). In practice, r varies from 1.2 to 2 depending on the circumstances.



Terrestrial Fluxes: Schwarzchild's Equation Revisited - 4

- With these approximations, we can write a "Beer's Law for Fluxes":

$$dF_{\bar{\nu}} = -F_{\bar{\nu}} k_{\bar{\nu}} \rho r \, dz$$

- We can also write a "Schwarzchild's Equation for Fluxes":

$$\frac{dF_{\bar{\nu}}}{k_{\bar{\nu}} \rho r \, dz} = -F_{\bar{\nu}} + \pi \bar{J}_{\bar{\nu}}$$

where $\bar{J}_{\bar{\nu}}$ is an average value of the source function in the upward direction.

→ In many cases, J can be replaced by B (the blackbody function).

→ The π in the equation is necessary to ensure that it is valid in a blackbody cavity - keeps units as flux density - W/m^2.

→ Note that the diffusivity factor, *r*, is a constant for the equation, i.e. it does not depend upon *z*.

Important: this is NOT Schwarzchild's Equation because it applies to flux density. It is an ***approximation*** *and should be tested before use in any particular case. It is also called the* ***Diffuse Approximation****.*

Thermal Fluxes

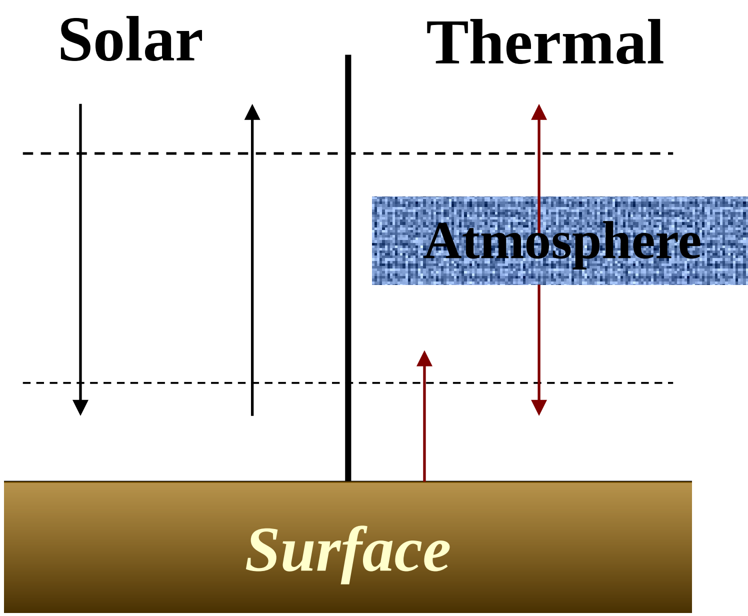
Now let's consider
the problem of
how to describe the
thermal flux in the atmosphere.

We will use "Schwarzchild's Equation for Fluxes"
in our simple greenhouse atmospheric model.

Two-Tone Atmosphere

Recall that for the two-tone atmospheric model:

- Absorption is divided into solar and thermal regimes.
- The atmosphere has different properties for each.
- Restriction: the atmosphere does not absorb solar radiation
  - → It may scatter, but there is no heating due to the solar beam.
- Solar radiation incident upon the ground does heat the atmosphere indirectly because it emerges as thermal radiation and the atmosphere is "heated from below".
- There is no overall heating or cooling of any region and so the atmosphere is in "local radiative equilibrium"
  - → No layer of the atmosphere gains or loses heat due to the passage of either the solar or the thermal beam.



Extension of Two-Tone Atmosphere

- Extension: include a continuous medium as the atmosphere, instead of the discrete layers we have been using.
- There are two general streams of radiation:
  - → one directed upwards, taking heat to the top, F_{up}
  - → one directed downward taking heat to the bottom, F_{down}
- Note: this is still an approximation because energy also flows sideways in the real atmosphere.

- Consider the boundary conditions on F_{up} and F_{down}:
  - → At the top of the atmosphere:

    $F_{down} = 0$ → *The sun and deep space contribute little thermal radiation.*

    $F_{up} = \sigma T_e^4$ → *Determines the effective radiating temperature*

  - → At the bottom of the atmosphere:

    $F_{up} = \sigma T_g^4$

    $F_{down} = ?$ → *We know little about the downward flux at this level…*

Two-Stream Model - 1

- Now we consider a layer of:
  - → thickness Δz
  - → absorption/emission coefficient k (we will ignore scattering)
  - → density ρ
- For brevity, we will drop the subscripts $\bar{\nu}$
- For such a layer we can apply "Schwarzchild's Equation for Fluxes":

$$\frac{dF}{k\rho r\, dz} = -F + \pi B$$

- By looking at altitude z in the atmosphere, we can derive expressions for $F_{up}(z)$ and $F_{down}(z)$.

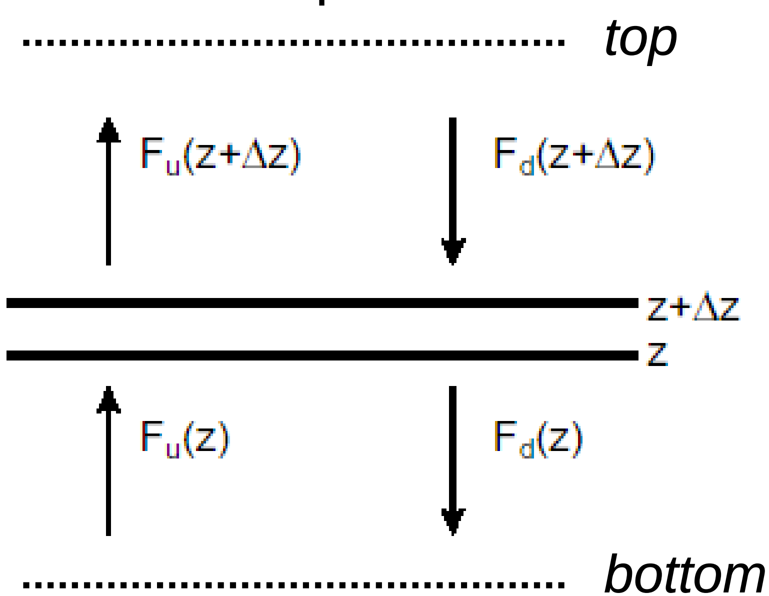


Figure 62: Two-stream Model Diagram

Two-Stream Model - 2

- The downward flux density at z is:

$$F_{down}(z) = F_{down}(z+\Delta z) - F_{down}(z+\Delta z)k\rho r\Delta z + \pi Bk\rho r\Delta z$$

$$= F_{down}(z) + \frac{dF_d}{dz}\Delta z - F_{down}(z+\Delta z)k\rho r\Delta z + \pi Bk\rho r\Delta z$$

- The upward flux density is:

$$F_{up}(z+\Delta z) = F_{up}(z) + \frac{dF_{up}}{dz}\Delta z$$

$$= F_{up}(z) - F_{up}(z)k\rho r\Delta z + \pi Bk\rho r\Delta z$$

- Rearranging gives:

$$-\frac{dF_{down}}{k\rho r dz} = -F_{down} + \pi B$$

$$\frac{dF_{up}}{k\rho r dz} = -F_{up} + \pi B$$

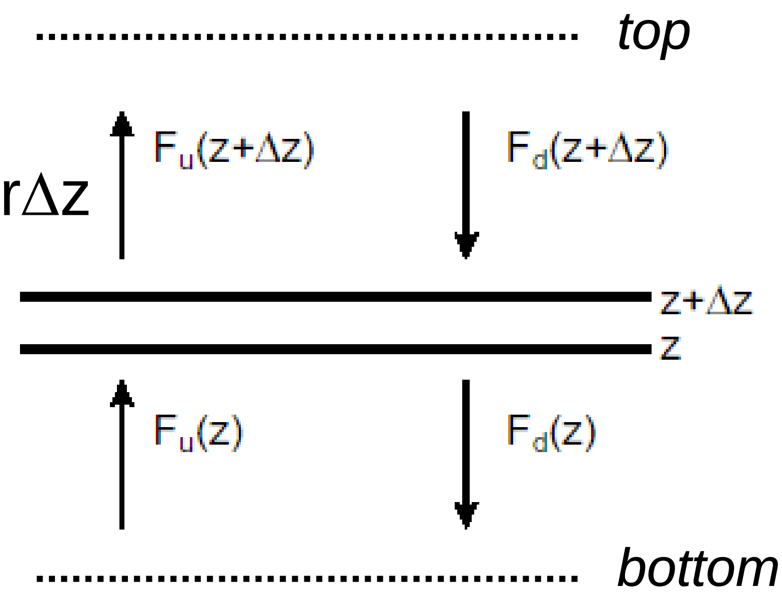


Figure 62: Two-stream Model Diagram

Two-Stream Model - 3

- We now replace altitude z by optical depth χ to further simplify these equations (recall that $d\chi = - k \rho dz$).
- But because the diffusivity factor is now included, we will use $d\chi^* = - k \rho r dz.$
- This gives: $\frac{dF_{down}}{d\chi^*} = -F_{down} + \pi B$ and $-\frac{dF_{up}}{d\chi^*} = -F_{up} + \pi B$
- Now the atmosphere is in radiative equilibrium, so changes in the fluxes must balance. So in any layer: $dF_{up} - dF_{down} = 0$
- Integrating: $F_{up} - F_{down} = \phi$
- This implies that there is a net upwards transport of energy from the bottom of the atmosphere to the top $\rightarrow$ energy is being transported from the ground to space and not lost along the way.

Two-Stream Model - 4

- Now let's subtract our two equations:

$$\frac{dF_{down}}{d\chi^*} = -F_{down} + \pi B$$

$$-\frac{dF_{up}}{d\chi^*} = -F_{up} + \pi B$$

$$\frac{d(F_{up} + F_{down})}{d\chi^*} = F_{up} - F_{down} = \phi \qquad (1)$$

- Integrate: $F_{up} + F_{down} = \phi\chi^* + c \qquad (2)$
- Combining equations (1) and (2) gives:

$$F_{up} = (\phi/2)(\chi^* + 1) + c/2$$

$$F_{down} = (\phi/2)(\chi^* - 1) + c/2$$

- At the top of the atmosphere, $\chi^* = 0$, so $F_{down} = 0$, giving $c = \phi$.
- Also at the top, we know that $F_{up} = \sigma T_e^4$, giving $\phi = \sigma T_e^4$.

Two-Stream Model Solution - 1

- We have thus solved for the upward and downward flux densities:

$$F_{up} = \sigma T_e^4 (\chi^* + 2)/2$$
$$F_{down} = \sigma T_e^4 \chi^*/2$$

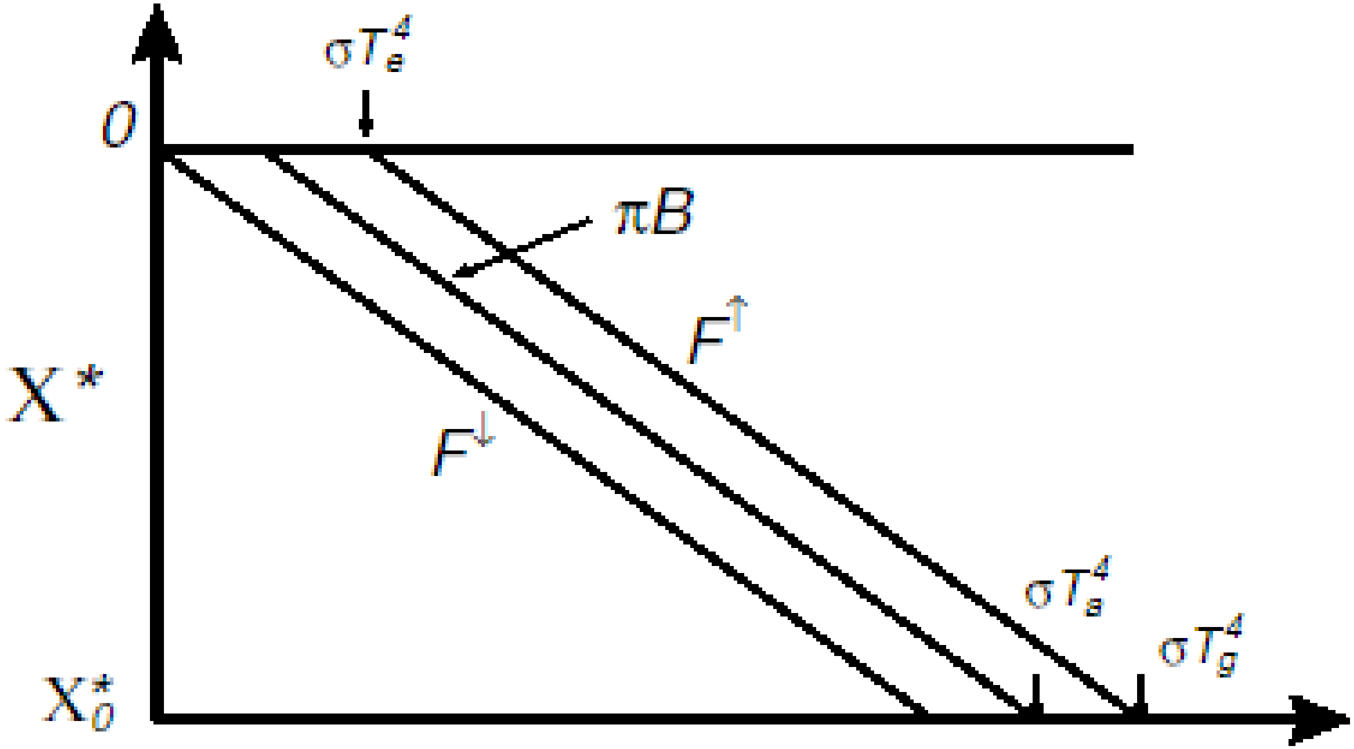
- Finally, we can use the original equations to solve for:

$$\pi B = \sigma T_a^4 = \frac{\sigma T_e^4 (\chi^* + 1)}{2}$$

where T_a is the atmospheric temperature.

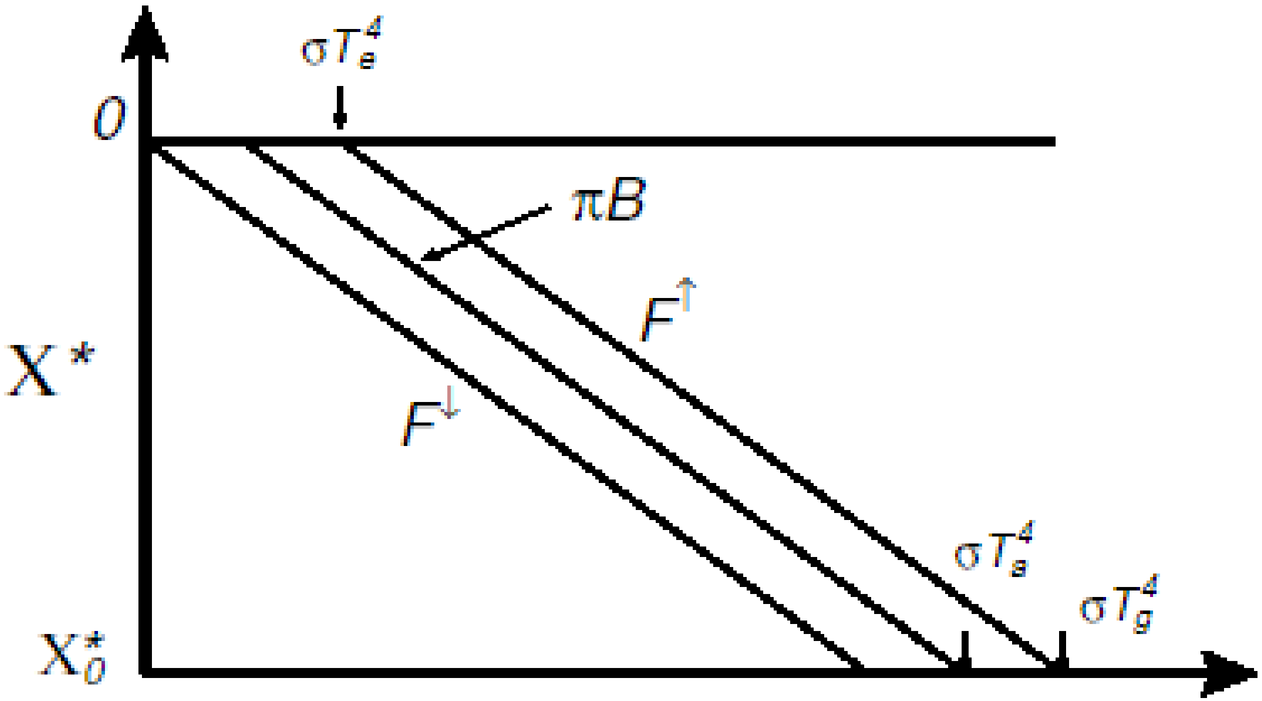
- These three solutions can be plotted against χ^*.
  - → Gives three parallel lines that show how the flux densities and the blackbody function vary with χ^* and hence with altitude.

Two-Stream Model Solution - 2



- Note the constant net up-flux (F_{up} - F_{down}), which accounts for the observed equivalent radiating temperature T_e, and increases to the bottom of the atmosphere.
- T_g increases to account for the optical depth of the atmosphere.

Two-Stream Model Solution - 3



- If the central line represents the atmospheric temperature and the line for F_{up} must match the ground temperature, then there is a temperature discontinuity at the ground.
- This is a common feature of this type of model and can sometimes be observed, e.g. hot sidewalks on summer days.

Application of the Two-Stream Model

We can now apply this model to the Earth's atmosphere.

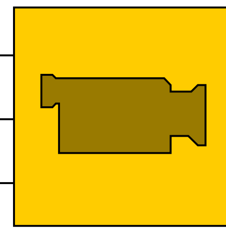
- Let the effective radiating temperature be T_e = 255 K.
- Let the average surface temperature be T_g = 286 K.

Results:

- χ_o* (the total optical depth including diffusivity) is about 1.16.
- The vertical optical depth without the diffusivity factor, χ_o, is about 0.70.
- Thus, the Earth's atmosphere absorbs a significant amount of the outgoing infrared flux - we need to consider the source of this absorption.

In comparison, Venus (T_e = 230 K, T_g = 700 K) gives χ_o = 170 and χ_o = 100. CO_2 cannot account for all of this large optical depth.*

Application of the Two-Stream Model



Two-Stream Model Temperature Profile

- Can we derive a temperature profile from the two-stream model?
  - → Yes - replotting the graph as z vs T is a non-trivial job that requires detailed knowledge of the atmosphere, but it can be done.
- The resulting profile is as shown.
  - → This is a “super-adiabatic” lapse rate and is unstable to vertical convection.
  - → Therefore the Earth’s atmosphere cannot be in a radiative equilibrium because that state is unstable.
- It must show a tendency towards that state which is countered by the vertical motions which try to keep the profile sub-adiabatic (convection).
- The atmosphere tends to “hang-around” the adiabatic lapse rate because it is trying to balance these two forces.

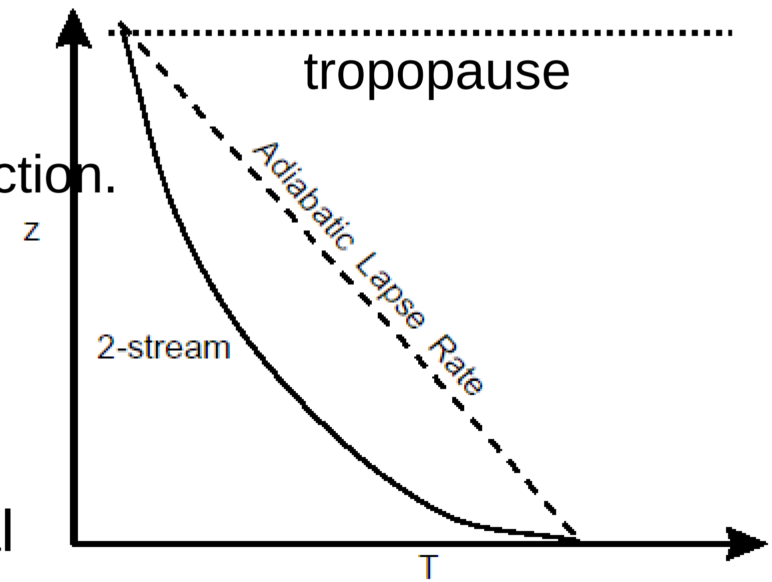
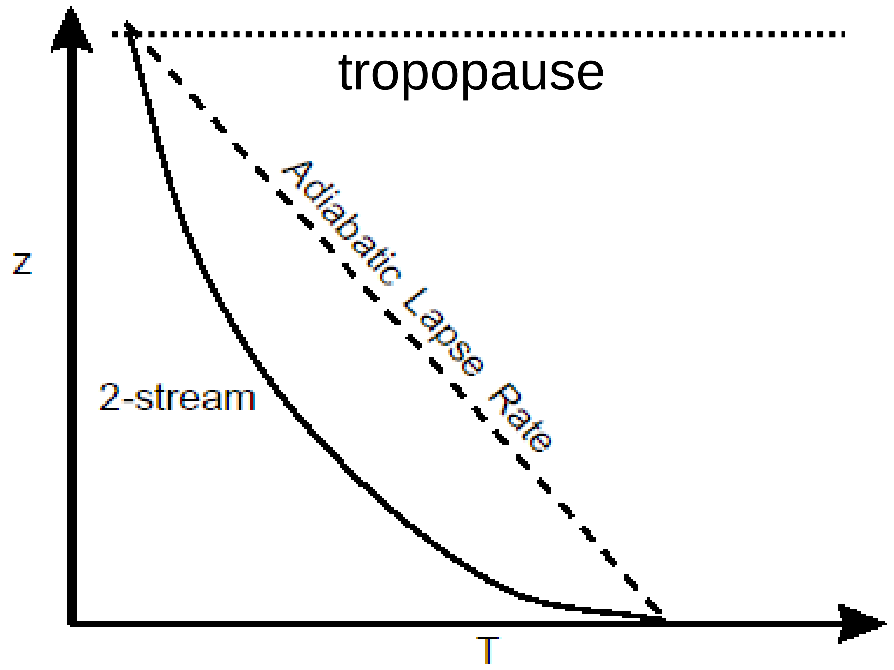


Figure 64: Two-stream Model Temperature Profile

Two-Stream Model and the Tropopause

- The <u>tropopause</u> is the surface situated at a height of about 10 km in mid latitudes which divides:
  - → the region below (the troposphere or turning-sphere), in which convection is dominant mechanism of vertical heat,
  - → from the region above (the stratosphere) which is much more stably stratified and where radiative transfer is dominant.



Clouds

- Inclusion of Clouds in a the Two-Stream Model
- The Visible Approximation
- A Scattering Model for Clouds

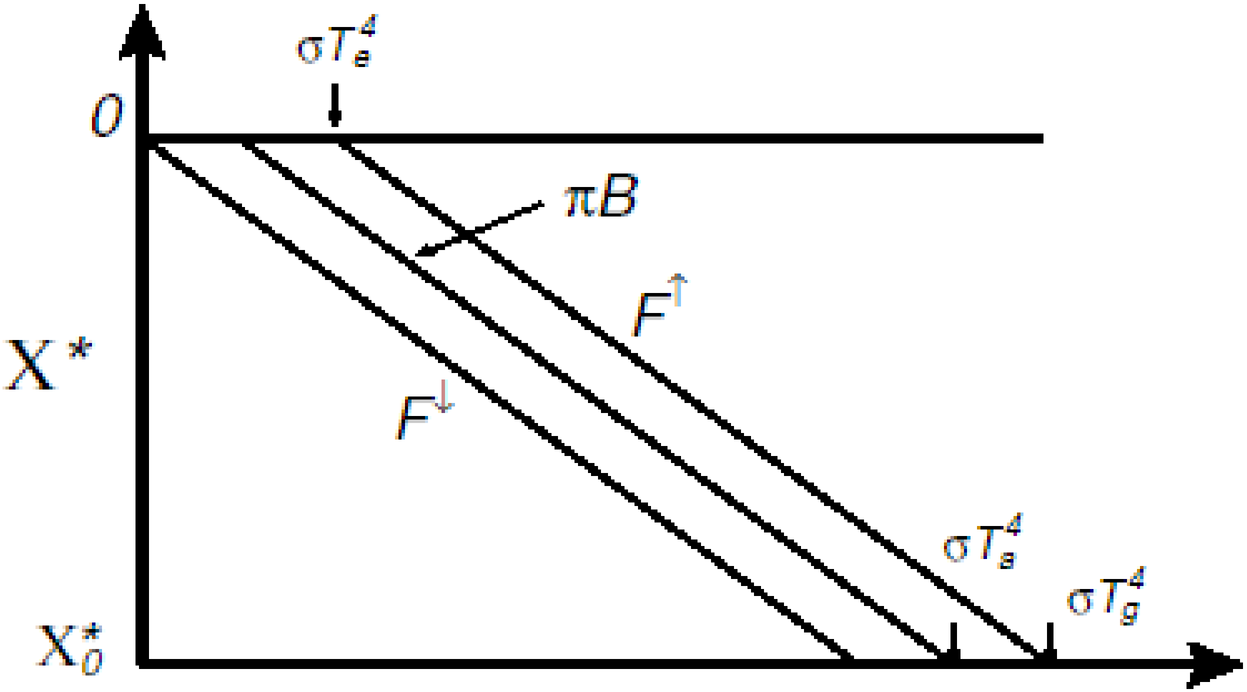
The Two-Stream Model

We solved for the upward and downward flux densities and πB:

$$F_{up} = \sigma T_e^4(\chi^* + 2)/2$$

$$F_{down} = \sigma T_e^4 \chi^*/2$$

$$\pi B = \sigma T_a^4 = \frac{\sigma T_e^4(\chi^* + 1)}{2}$$

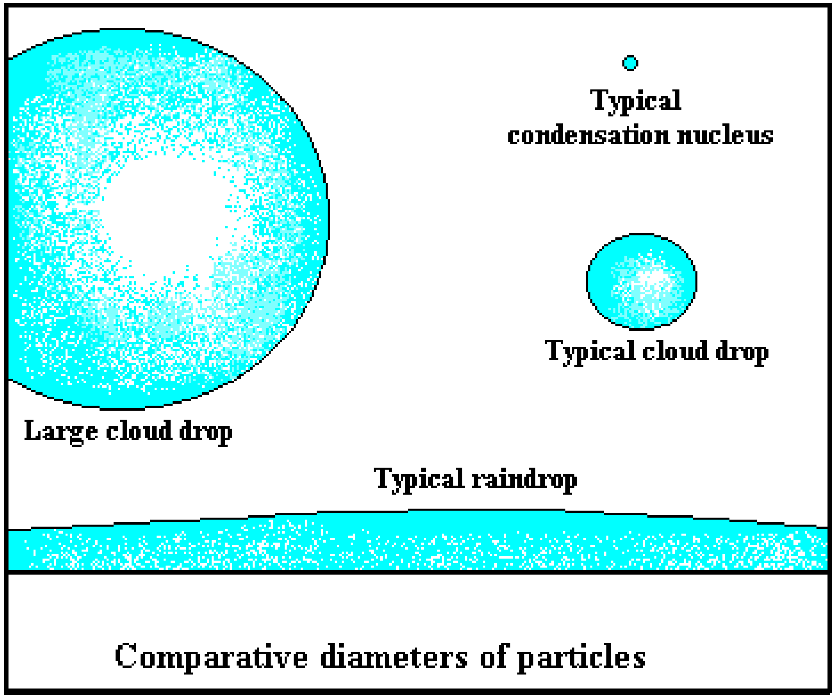


Now Let's Consider Clouds

Clouds - consist of water drops and ice crystals of radius ~10 μm and are classified on the basis of size:

- cloud drops r = 10 μm
- drizzle r = 100 μm
- rain drops r = 1000 μm (1 mm)

Cloud drops grow by diffusion of water vapour to cloud particles and subsequent condensation, and by collision and coalescence of particles



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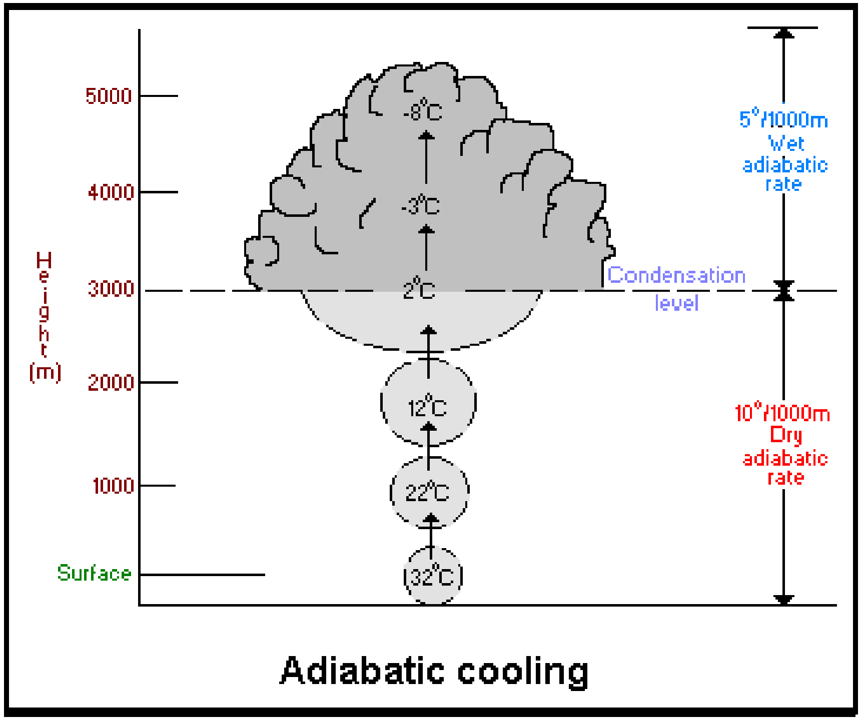

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Why Do We Care About Clouds?

- Clouds cover ~50% of the Earth.
- Clouds are important because they:
  - → influence Earth's radiation budget
  - → influence climate
  - → influence daily weather
  - → interfere with radiometric observations of the atmosphere and surface

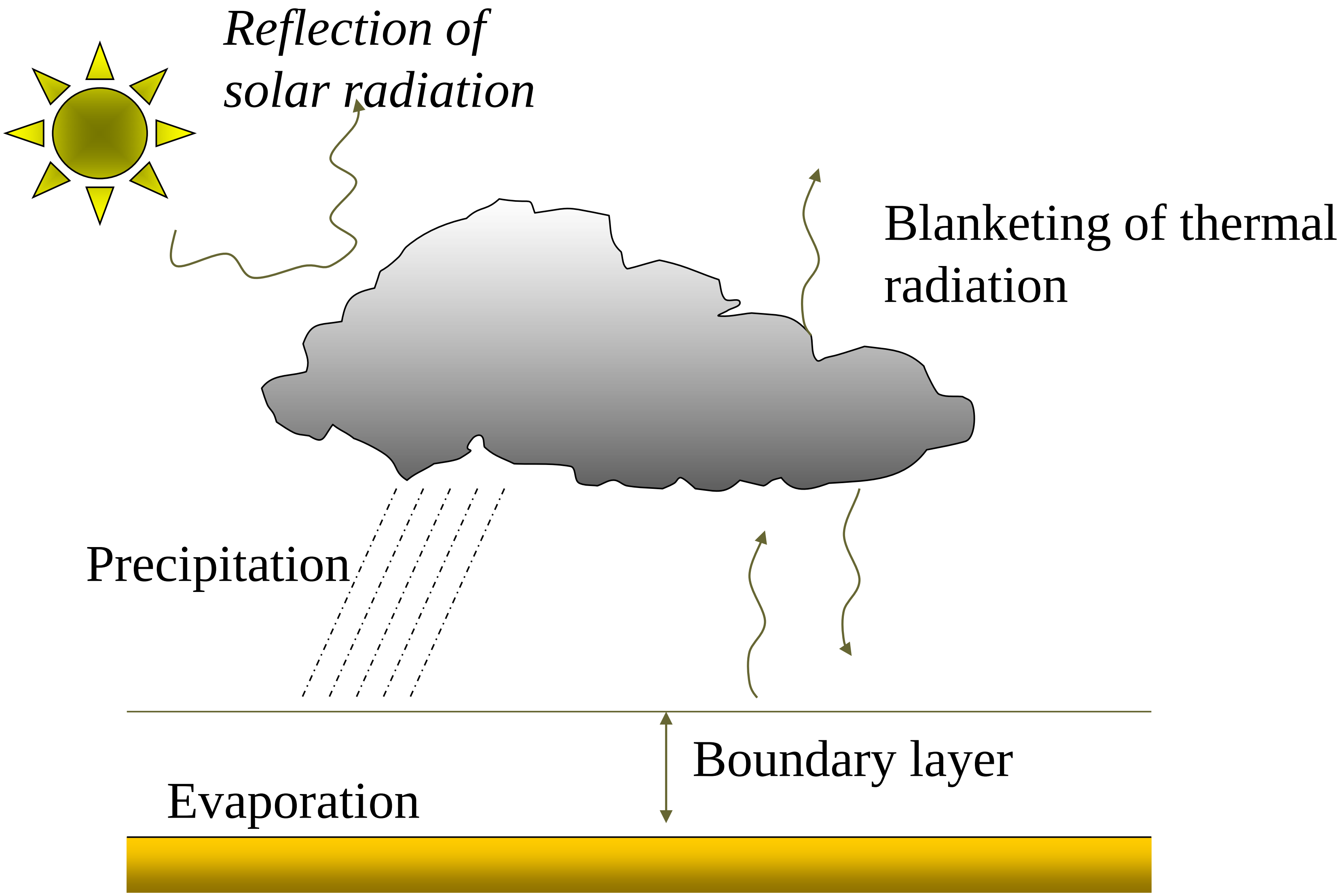
Clouds influence the energetics of the atmosphere in two ways:

(1) By their role in the atmospheric water cycle.

→ latent heat is released on condensation and liquid water is removed from the atmosphere on precipitation.

(2) By scattering, absorption, and emission of solar and terrestrial radiation.

Radiative Effects of Clouds



How Do We Include Clouds in a Radiative Balance Model?

The Infrared Approximation:

- The simplest IR model is a horizontally infinite layer of absorber.
  - → Cloud droplets are liquid water, which absorbs IR radiation.
  - → Any scattering in the cloud will just increase the absorption path.
- Treat the cloud as a black layer in the two-stream model.

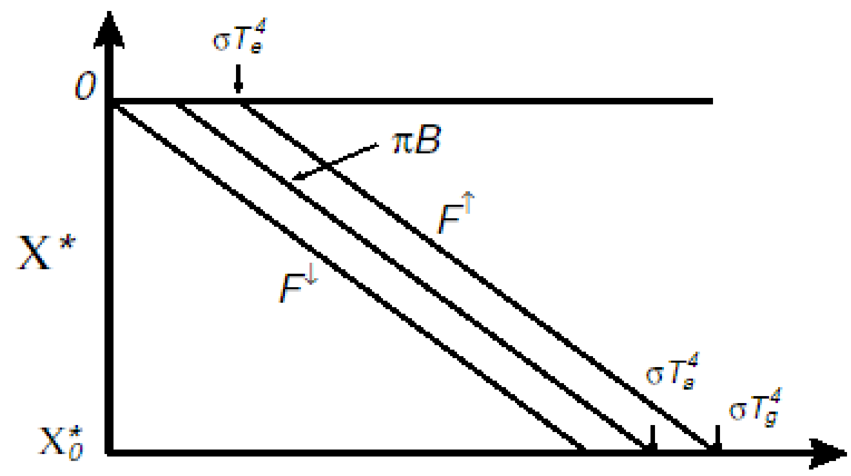
- Recall our equations for the upward and downward flux densities in the two-stream model:

$$F_{up} = \sigma T_e^4 (\chi^* + 2)/2$$

$$F_{down} = \sigma T_e^4 \chi^* / 2$$

where χ^*_o = the total optical depth

T_e = the effective radiating temperature of the planet



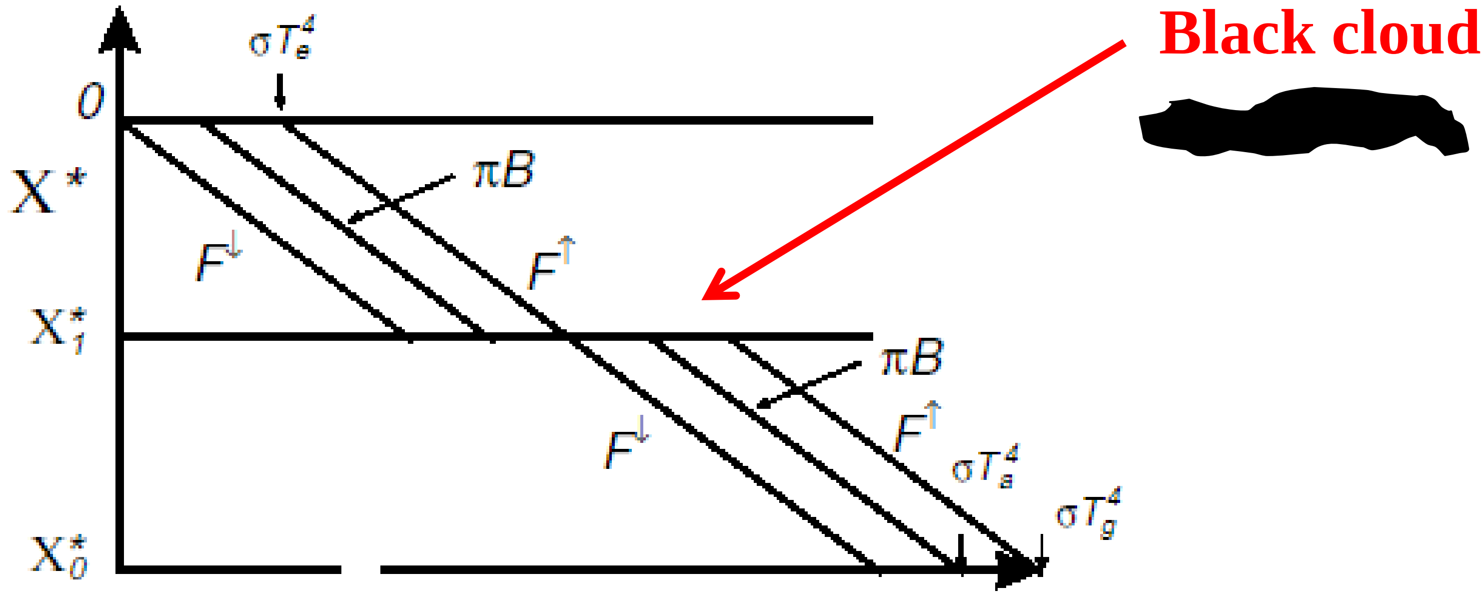
Clouds in the Two-Stream Model - 1

Now add a black layer in the two-stream model to represent cloud.

- If instead of the ground at χ_o* we place a cloud at χ_1*, then at the **<u>upper</u>** cloud surface (and above the cloud), the fluxes are:

$$F_{up} = \sigma T_e^4 (\chi_1^* + 2)/2$$

$$F_{down} = \sigma T_e^4 \chi_1^* / 2$$



Clouds in the Two-Stream Model - 2

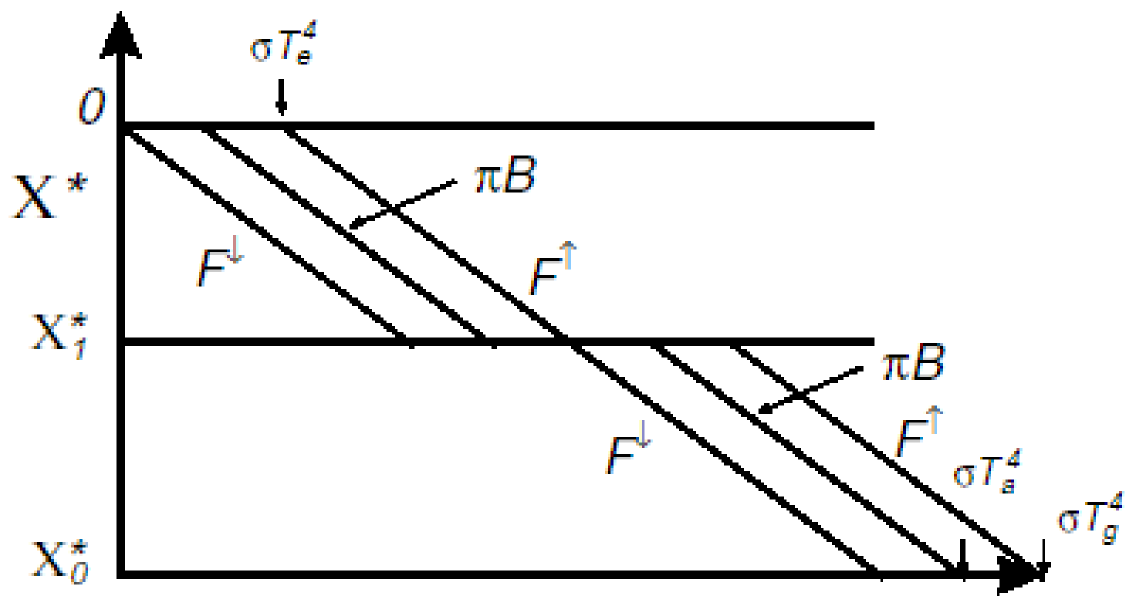
Making the following assumptions, we get equations for F_{up} & F_{down}:

- The flux from the top surface of the layer, F_{up}, must equal the flux from the bottom surface, F_{down}
- Radiative equilibrium
- No solar absorption in the atmosphere or the cloud

Below the cloud:

$$F_{up} = \sigma T_e^4(\chi^* + 4)/2$$

$$F_{down} = \sigma T_e^4(\chi^* + 2)/2$$



Clouds in the Two-Stream Model - 3

- These equations imply that the ground temperature T_g' is:

$$\sigma\left(T_g'\right)^4 = \sigma T_e^4(\chi^* + 4)/2$$

- In terms of the original surface temperature T_g, this becomes:

$$\left(T_g'\right)^4 = T_g^4 + T_e^4$$

Thus, including the cloud in the two-stream has caused a significant **heating of the surface**.

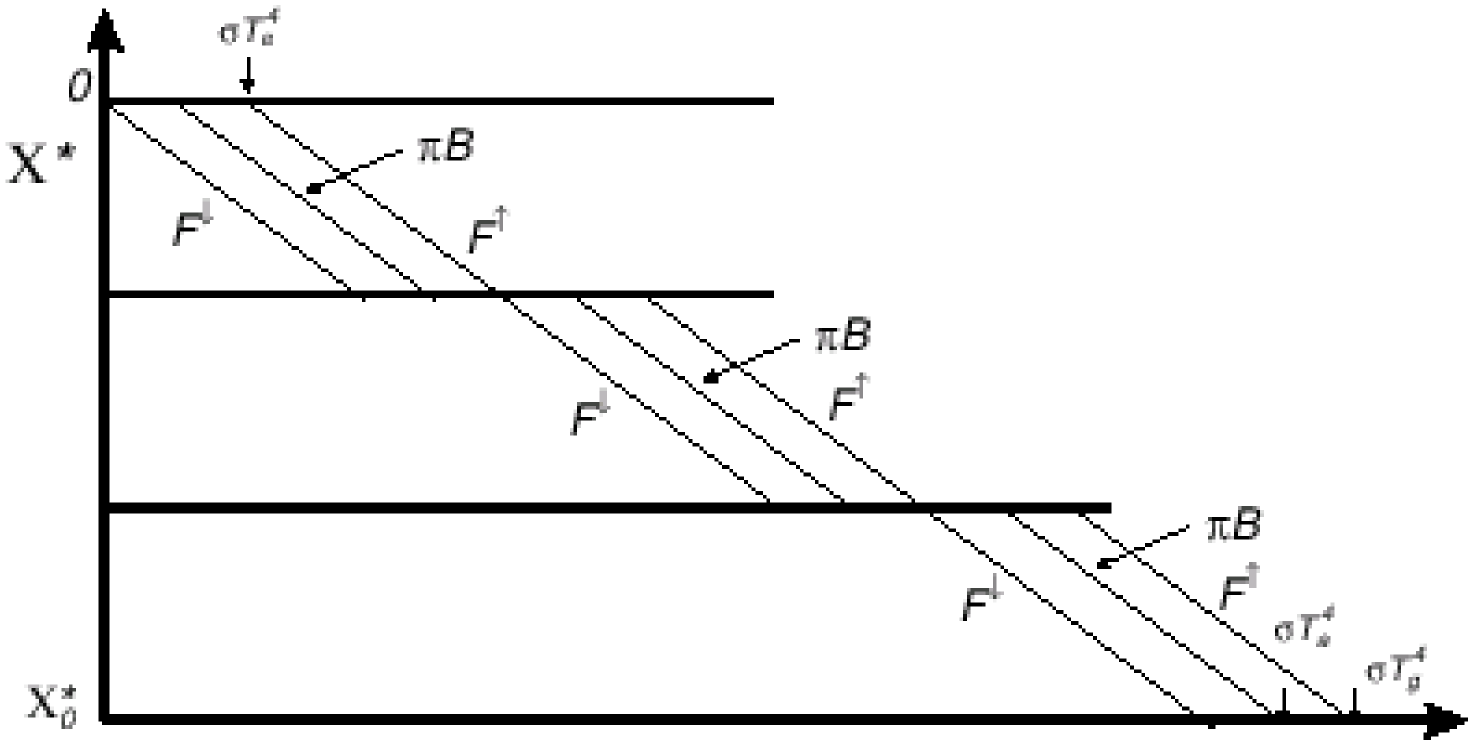
For Earth: T_g = 286 K and T_e = 255 K

→ these give T_g' = 323 K

→ a substantial increase of 37 K in the ground temperature!

Clouds in the Two-Stream Model - 4

- For multiple cloud layers, this heating effect is increased.
- If the clouds do not absorb perfectly, then there will be a smaller shift in the line and the heating effect will be smaller.



The Visible Approximation - 1

- So far, we have assumed that clouds do not change the albedo of the planet.
- However, clouds are visible and so they must interact with incoming solar radiation.

How?

- Almost no solar UV reaches the troposphere, and so the interaction must be with the visible and near-infrared.
- Clouds do not absorb much at these wavelengths, but do <u>scatter</u> solar radiation in the visible and near-infrared.
- Clouds thus increase the backscatter to space
  $\rightarrow$ increases the albedo
  $\rightarrow$ reduces the amount of energy absorbed by the ground
  $\rightarrow$ leads to <u>**cooling of the surface**</u>
- This is opposite of the infrared effect which heats the surface.

The Visible Approximation - 2

This impacts calculations of the greenhouse effect in the future for the following reason: If you increase the surface temperature, you increase evaporation which increases water vapour in the atmosphere. This evidently increases the infra-red absorption which increases the surface temperature which increases evaporation, i.e. the feedback is positive and the process accelerates. However clouds form from the water vapour and therefore there is the infra-red heating and solar cooling from them to take into account. Is the final result positive feedback or negative feedback - to accenuate or suppress the greenhouse effect? The answer is not known at the present time.

Let's do a simple calculation to look at these opposing effects.

- Consider the natural state of the earth as cloudless, with an albedo of 0.3 and a diffusive optical depth of 1.2.

  Question: If we add a cloud which produces infrared heating and albedo cooling, what increase in albedo is required to produce no change in the surface temperature?

- Recall the radiation balance equation: $F(1-A) = 4\sigma T_e^4$
- So we can use: $T_e^4 = \dfrac{F(1-A)}{4\sigma}$

The Visible Approximation - 3

- The equation for the surface temperature without the cloud is:

$$T_g^4 = T_e^4(\chi_o^* + 2)/2 = \left[\frac{F(1-A)}{4\sigma}\right](\chi_o^* + 2)/2$$

- The equation for the surface temperature with the cloud is:

$$(T_g')^4 = T_e^4(\chi_o^* + 4)/2 = \left[\frac{F(1-A')}{4\sigma}\right](\chi_o^* + 4)/2$$

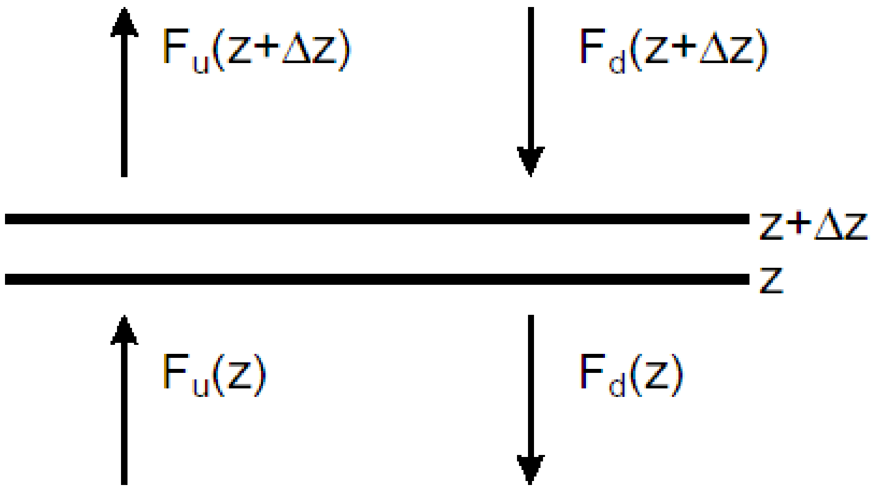
- This leads an equation for the new albedo:

$$A' = \frac{2 + A(2 + \chi_o^*)}{4 + \chi_o^*}$$

- With A = 0.3 and X^*_o = 1.2, this gives A' = 0.57.

A Scattering Model for Clouds - 1

- In the visible region clouds act to scatter radiation in all directions.
  - → This converts the incident solar beam into a flux which passes through the cloud and out the bottom.
- We can treat this as a flux problem and use the two-stream model set up for pure scattering instead of absorption.
  - → This means that energy is conserved and the extinguished energy must be included in full in the transfer equations.
  - → Although there is no emission from the layer, the scattered radiation provides a source function.



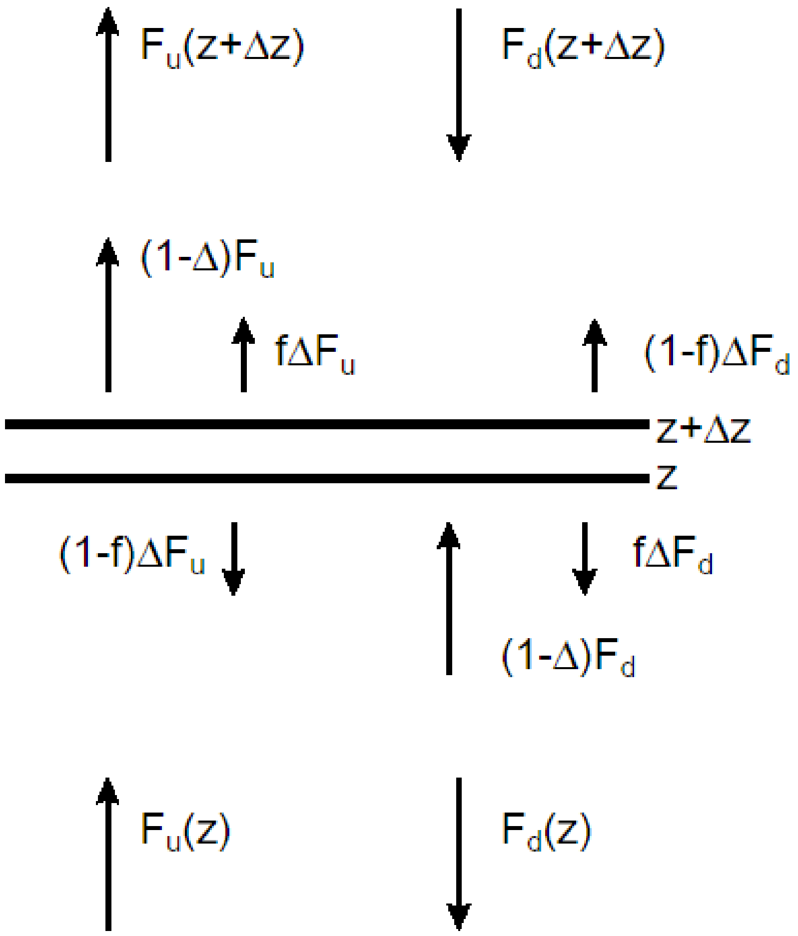
A Scattering Model for Clouds - 2

Need to determine how the energy is distributed between F_{up} & F_{down}.

Divide the total scattered radiation into:

- a fraction f, scattered forwards (back into the original flux)
- fraction (1-f) backscattered

Note: for isotropic scattering, f = 1/2.



A Scattering Model for Clouds - 3

The equations for the two-stream model become:

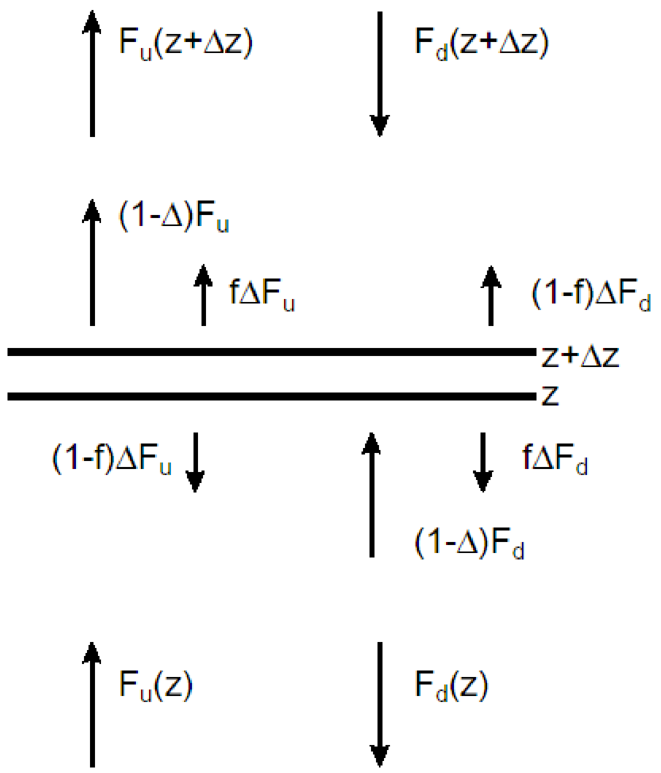
$$F_d(z) = F_d(z+\Delta Z) - F_d(z)k\rho r\Delta z + \big(fF_d(z) + (1-f)F_u(z+\Delta z)\big)k\rho r\Delta z$$

$$F_u(z+\Delta z) = F_u(z) - F_u(z+\Delta z)k\rho r\Delta z + \big(fF_u(z+\Delta z) + (1-f)F_d(z)\big)k\rho r\Delta z$$

These can be "tidied up" to get:

$$\frac{dF_{down}}{d\chi^*} = \frac{dF_{up}}{d\chi^*} = (1-f)(F_u - F_d)$$

which becomes: $F_{up} - F_{down} = \phi$



A Scattering Model for Clouds - 4

Integrating the equations and remembering the constraint on (F_{up} – F_{down}) for all levels gives:

$$F_{down} = (1-f)\phi\chi^* + c$$

$$F_{up} = (1-f)\phi\chi^* + \phi + c$$

Boundary conditions:

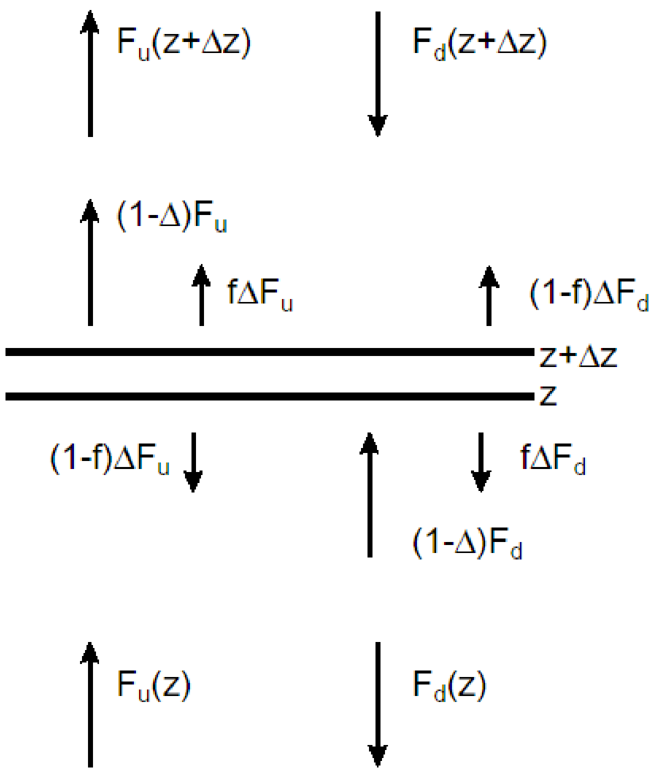
1) $F_{down} = F_s$ for solar radiation incident on the top of the cloud at $\chi^* = 0$

2) $F_{up} = 0$ at χ^*_0

These give:

$$c = F_s$$

$$-\phi = \frac{F_s}{1+(1-f)\chi_o^*}$$



A Scattering Model for Clouds - 5

From this, the two most useful properties of the cloud can be found.

The albedo is given by:

$$A = \frac{F_{up}(\chi^* = 0)}{F_s} = \frac{\chi_o^*(1-f)}{1+\chi_o^*(1-f)}$$

The transmission is:

$$\tau = \frac{F_{down}(\chi^* = \chi_o^*)}{F_s} = \frac{1}{1+\chi_o^*(1-f)}$$

- *Note that for this non-absorbing cloud, $A + \tau = 1$. As the optical depth tends to infinity, the albedo of a perfectly scattering cloud tends to unity and the transmission tends to zero.*
- *But if there is a small amount of absorption in the cloud, then the large increase in pathlength that accompanies the scattering produces a large amount of absorption.*

Non-Water Clouds

- Our model of a cloud as an infrared absorber and solar scatterer really only applies to water clouds. Things get more complicated for other types of clouds.

- There are three types of non-water clouds:
  - → Dust cloud - rare on Earth except on small scales, common on Mars
  - → Aerosol layer - very common on Earth
  - → Haze layer - strictly, this defines an aerosol layer which exists in a nearly-saturated water-vapour environment and therefore any hygroscopic particles in the aerosol layer are surrounded by water and take on some of the properties of “dirty” water drops

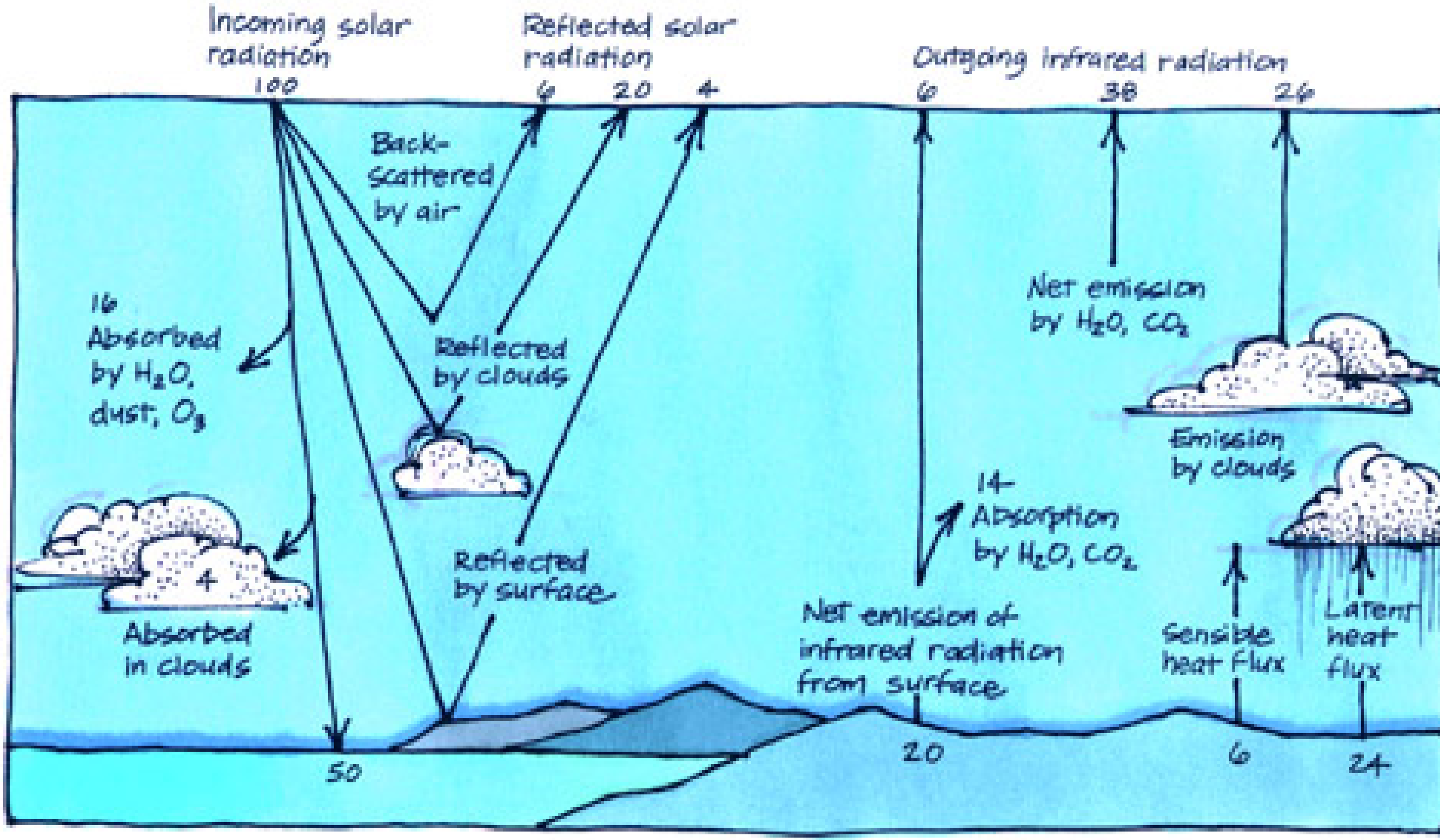
Clouds and Climate

- Clouds are important to climate because they strongly modulate incoming solar and outgoing thermal radiation.
- Clouds are the source of precipitation therefore they are a key element in the hydrologic cycle.
- We need a better understanding of clouds and climate for better predictions of climate change, to guide policy in ameliorating or adjusting to change, and to provide better stewardship of our water resources.

Aerosols and Clouds?

- Aerosol particles are very important to climate:
  - → directly by scattering light
  - → indirectly by serving as cloud condensation nuclei and by altering cloud properties

Energy can be absorbed, reflected or transmitted by clouds



http://www.etl.noaa.gov/eo/notes/clouds_and_climate.html