Radiation Balance for Earth

Recap:

- Solar radiation at TOA is ~ blackbody at 5780 K
- Has small solid angle → plane wave
- Integrated energy on a normal to the surface is ~ 1370 W/m^2
- varies with latitude and season

We can calculate effective T of Earth by considering radiative equilibrium between solar and terrestrial radiation.

Energy absorbed by Earth

= energy intercepted × (1 − average reflection coefficient)

$$= \pi R^2 F (1 - A)$$

where
R = Earth's radius
F = solar flux density at Earth
A = planetary albedo (= ratio of reflected flux density to incoming solar flux density)

Energy radiated by Earth

= emissivity × surface area × blackbody flux density

$$= \varepsilon \, 4\pi R^2 \sigma T_e^4$$

$\varepsilon \approx 1$ for infrared

$$\therefore \quad 4\pi R^2 \sigma T_e^4 = \pi R^2 F(1-A)$$

$$T_e = \left[\frac{F(1-A)}{4\sigma} \right]^{1/4}$$

defines the effective radiating temperature of Earth (or any planet)

For Earth, $T_e = 255$ K

The atmosphere has a warming effect that makes the Earth's surface temperature about 33 K warmer than T_e.

Oxygen and Ozone in the Atmosphere

① atomic oxygen created by photolysis of molecular oxygen

$$O_2 + h\nu \ (\lambda < 242\,nm) \rightarrow O + O$$

photolysis rate J_1

② production of ozone

$$O + O_2 + M \rightarrow O_3 + M$$

rate coefficient k_1

③ photolysis of ozone

$$O_3 + h\nu \ (\lambda < 350\,nm) \rightarrow O_2 + O$$

photolysis rate J_2

④ collision of ozone with atomic oxygen

$$O_3 + O \rightarrow O_2 + O_2$$

rate coefficient k_2

⑤ self reaction of atomic oxygen

$$O + O + M \rightarrow O_2 + M$$

very slow, can ignore < 60 Km

These reactions occur simultaneously
→ Chapman Cycle

Reactions 2 and 3 are faster than 1 and 4, so for equilibrium of O and O_3 (odd oxygen):

$$k_1 [O][O_2][M] = J_2 [O_3]$$

$$k_2 [O][O_3] = J_1 [O_2]$$ for overall balance.

Combine these:

$$[O] = J_1 [O_2] / k_2 [O_3]$$

$$[O_3] = \frac{k_1}{J_2} [O][O_2][M]$$

$$= \frac{k_1}{J_2} \frac{J_1 [O_2]}{k_2 [O_3]} [O_2][M]$$

$$\therefore [O_3] = [O_2] \sqrt{\frac{J_1 k_1 [M]}{J_2 k_2}}$$

So $[O_3]$ depends on J_1 / J_2, which increases with height.

$$J = \int_\lambda \sigma_\lambda F_\lambda(\infty)\, \tau_\lambda(z,\infty)\, d\lambda$$

→ dissociating quanta per molecule that are absorbed

$[O_3]$ also depends on $[M]$ and $[O_2]$ which decrease with height.

∴ There is maximum in $[O_3]$ at some height

→ ozone layer

There will also be a maximum in heating of the atmosphere at this height

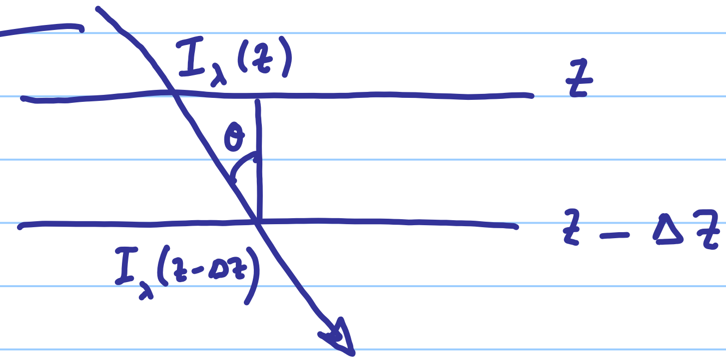
→ stratosphere

Solar Heating of the Atmosphere

Consider an atmospheric layer in a plane-parallel atmosphere.

solar beam incident at angle θ

$\mu = \cos\theta$



$$I_\lambda(z-\Delta z) = I_\lambda(z)\,\tau_\lambda(\Delta z/\mu)$$
$$= I_\lambda(z)\exp\left(-k_\lambda \rho \frac{\Delta z}{\mu}\right)$$

using Schwarchild's Equation for a homogeneous layer with $J=0$.

Energy lost (per unit area)

$$= \left[I_\lambda(z) - I_\lambda(z-\Delta z)\right]\Delta\Omega_s$$
$$= \left[I_\lambda(z) - I_\lambda(z)\,\tau_\lambda(\Delta z/\mu)\right]\Delta\Omega_s$$
$$= I_\lambda(z)\,A(\Delta z/\mu)\,\Delta\Omega_s \quad \text{where } A_\lambda = 1-\tau_\lambda$$
$$= \frac{\Delta E_\lambda}{\Delta t}$$

This energy loss from the beam is also the net energy gain of the atmosphere, so

$$\frac{\Delta E_\lambda}{\Delta t} = \rho\, C_p\, \Delta V \frac{dT}{dt}$$

where ρ = density
C_p = specific heat at constant pressure → J/Kg/K
ΔV = volume of cylinder where heat is absorbed

$\Delta V = \Delta z/\mu$ assuming a cylinder of unit cross-section aligned along the beam

$$\frac{dT}{dt} = \frac{1}{\rho\, C_p\, \Delta z/\mu} \frac{\Delta E}{\Delta t}$$

$$= \frac{\mu}{\rho\, C_p\, \Delta z} I_\lambda(z)\, A_\lambda\, (\Delta z/\mu)\, \Delta\Omega_s$$

From Hydrostatic Eqn $\quad \Delta p = -\rho g \Delta z$

$$\frac{dT}{dt} = -\frac{g}{C_p} \frac{\mu I_\lambda(p)\, A_\lambda\, (\Delta p/\mu)\, \Delta\Omega_s}{\Delta p}$$

$$\frac{dT}{dt} = -\Gamma \frac{d F_\lambda(\mu, p)}{dp} = \underline{\text{heating rate}}$$

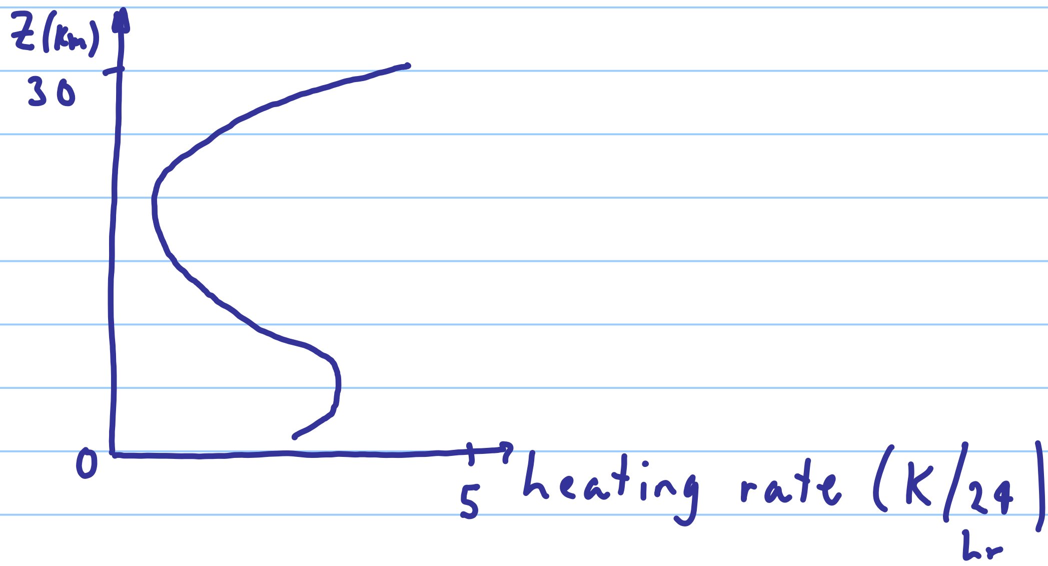
where

$\Gamma = g/C_p =$ adiabatic lapse rate

$F_\lambda = \mu I_\lambda(p)\, A_\lambda\, (\Delta p/\mu)\, \Delta\Omega_s =$ radiant flux density incident on unit area

Since dF_λ/dp is always < 0, so $\frac{dT}{dt}$ is > 0.

This equation allows calculation of the heating rates in the atmosphere due to solar radiation.



Plotting the heating rate for solar radiation in the lower atmosphere in °C/day shows:

- peak at the ground due to H_2O and CO_2 absorption
- then a decrease to a minimum
- above the tropopause, O_3 absorption becomes important, causing a large increase in heating rate

The above calculation of dT/dt is for monochromatic radiation.

Total heating rate due to solar radiation is

$$\left(\frac{dT}{dt}\right)_{total} = \sum_{\lambda} \left(\frac{dT}{dt}\right)_{\lambda}$$

$$= -\Gamma \sum_{\lambda_i} \left.\frac{dF_{\lambda_i}}{dp}\right|_p \Delta\lambda_i$$

sum over all spectral intervals "i"