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# PHY2505S

## Atmospheric Radiative Transfer and Remote Sounding

### Lecture 3

- Emission in Continuous Media
- Extinction and Absorption Processes
- Schwarzschild's Equation

# Proposed Change to Assignments

## Original

- 20% Problem sets:
  - #1 will be handed out on Jan 21 and due in class on Feb 4
  - #2 will be handed out on March 10 and due in class March 24
- 15% Mid-term test (time TBA, tentatively the week of Feb 24-28)
- 30% Term paper (discuss topic with instructor by Feb 14, outline and bibliography due March 3, paper due Tuesday, April 7) and presentation (time TBA)
- 35% Final exam

## Proposed

- 20% Problem sets:
  - #1 will be handed out on Jan 21 and due in class on Feb 4
  - #2 will be handed out on March 10 and due in class March 24
- 15% **Journal club presentation/discussion facilitation (two-hour slot outside class time, at a time to be agreed in late March/early April)**
- 30% Term paper (discuss topic with instructor by Feb 14, outline and bibliography due March **10**, paper due Tuesday, April **14**) and presentation (time TBA)
- 35% Final exam

**Class agreed to proposed change**

# “Journal Club” Presentation

## “JOURNAL CLUB” PRESENTATION/DISCUSSION FACILITATION

You will be responsible for presenting a journal article related to the course material (chosen by your lecturer) and leading a class discussion of this paper. These presentations will be during the term (a two-hour slot outside of class time, at a time to be agreed). You will be evaluated on how you present your paper and lead the discussion on it and on how you participate in discussion of the other papers.

### PRESENTATION AND DISCUSSION GUIDELINES:

You will have 20 minutes for your presentation and discussion. In the first ~10 minutes (no more than half of the time), you will present a summary/synthesis of the results of the paper and provide any background necessary for the class to have a clear understanding of the article. In the rest of your time (~10 minutes), you will lead a discussion of the paper focusing on the aspects that relate to the course.

You should include in your presentation and discussion:

What is the motivation for this study?

- What was new and why was it published?

- What is next step or implication for this work?

Highlight the radiative transfer and/or remote sounding aspects of the work

- How does this relate to what we have learned in the course?

- What are the advantages and challenges of using this technique?

- Can these results be obtained by another method?

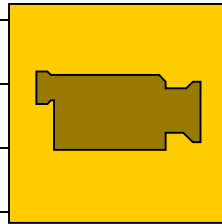
Remember to formulate some questions for the class to help lead the discussion part

### MARKING:

Marks will be given for content and comprehension of material, clarity, presentation method and use of diagrams or other aids. For the discussion component, marks will be given for your ability to lead the discussion of your paper and to provide topics and questions to facilitate this discussion. You will also be graded on your participation in the discussions of the other papers.

# Emission in Continuous Media

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# **Radiative Transfer Within the Atmosphere: Schwarzchild's Equation**

**What processes will change the intensity of electromagnetic radiation as it passes through a volume of atmosphere?**

**Let's re-examine Schwarzchild's Equation.**

# Atmospheric Extinction Processes

- Recall that we briefly described four processes that can cause extinction of radiation.

Extinction of the beam:

- Absorption ( $k_a$ )
- Simple scattering ( $k_s$ )
- Resonant scattering ( $k_r$ )
- Multiprocess scattering

$$k = k_a + k_s + k_r \quad (\text{ignoring multiprocess scattering})$$

  
*total extinction coefficient*

# Absorption

- The extinction mechanism for absorption of radiation:

transformation of the energy in the beam to molecular motion



overall heating of the gas



“transfer to the kinetic energy reservoir”

*Energy is permanently lost to the radiation field unless some other process converts kinetic energy to radiative energy.*

# Simple Scattering

- Simple scattering is the deflection of energy from its current direction into another direction (recall intensity has direction).
- The energy is redirected rather than lost.
- There is usually very little interaction with the kinetic energy reservoir and so little heating or cooling is associated with simple scattering.

Consequences:

- Need to pay attention to the angular properties of the energy redistribution
- Energy is also scattered into the beam from other directions



# Resonant Scattering

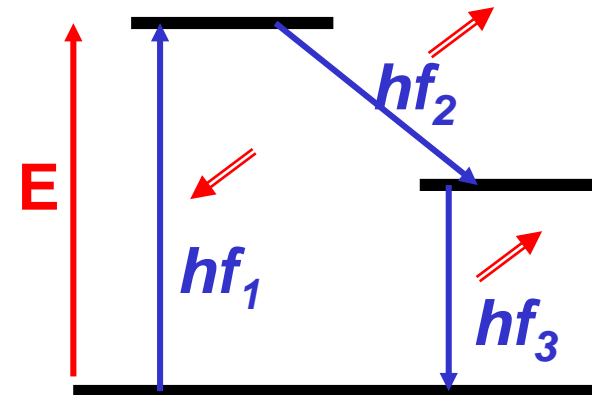
- Resonant scattering combines some of the properties of both absorption and single scattering.
- Energy is first absorbed by the molecule, and then re-emitted some time later.

The re-emission may be at:

- same
- nearly the same
- very different

depending upon the quantum levels involved

- The re-radiation is usually isotropic.
- Some degree of interaction between the process and the kinetic energy field.
- The only atmospheric process by which radiation may be directly changed in frequency.
- It is less important in the lower atmosphere.



$$hf_1 \sim hf_2 + hf_3$$

(f = frequency)

# Atmospheric Source Processes

- There are also three atmospheric processes that can cause augmentation of radiation
  - i.e., they add energy to the beam.

Augmentation of the beam: {  
- Emission  
- Simple scattering  
- Resonant scattering

# Emission

- Emission is the process by which energy is transferred from the kinetic energy reservoir to the radiation field.
- For a blackbody (cavity), there is no net change in kinetic energy, and so the absorption and emission must balance (Kirchoff's Law).
  - Therefore the absorption and emission coefficients must be equal.
- The emitted intensity is thus:

$$dl_{\text{emitted},\bar{\nu}} = B_{\bar{\nu}}(T) k_{a,\bar{\nu}} \rho dx$$

*Absorption coefficient  
= emission coefficient  
by Kirchoff's Law*

*Monochromatic blackbody function for T of gas*

# Simple Scattering

- This is the same process as we considered for extinction.
- Now, it is the intensity due to scattering into the direction of the beam from all other directions, including this direction.

$$dI_{\text{simple scattering}, \bar{v}} = \left[ \int_{\Omega} I(\Omega') P_{\bar{v}}(\Omega, \Omega') \frac{1}{4\pi} d\Omega' \right] k_s \rho dx$$

*Enables the scattering function to have a smooth form and not be constrained to be zero in the forward direction.*

where


- $(\Omega, \Omega')$  is the scattering angle ( $\psi_s$ ) between the incident radiation (at  $\mathbf{s}$ , defined by  $\Omega$ ) and any general direction (defined by  $\Omega'$ )
- $P(\Omega, \Omega')$  is the phase function for scattering as a function of the angle between the input and output beams
- $1/4\pi$  normalizes  $P$  with respect to  $k_s$ , so that  $P$  contains the angular distribution and  $k_s$  contains the total energy, so that for an isotropic radiation field:

$$\int_{\Omega} \frac{P(\Omega, \Omega')}{4\pi} d\Omega' = 1$$

*This is correct if  $P = 1$  for an isotropic scatter.*

# Resonant Scattering

- Again, this is the same process as we considered for extinction.
- It is (almost) always isotropic:

$$dI_{\text{resonant scattering}, \bar{\nu}} = \left[ \int_{\bar{\nu}} \int_{\Omega} I(\bar{\nu}', \Omega') Q(\bar{\nu}, \bar{\nu}') d\Omega' d\bar{\nu}' \right] k_r \rho dx$$


*Q – the coupling between the wavenumber of the incident radiation and some other wavenumber.*

- So, to summarize the three source terms:
  - $B_{\bar{\nu}}(T) k_{a, \bar{\nu}} \rho dx$  → *emission*
  - $[ ] k_s \rho dx$  → *simple scattering*
  - $[ ] k_r \rho dx$  → *resonant scattering*

# The Atmospheric Source Function

- We can now collect all of the source terms and include them in the equation for the change of intensity across some path  $dx$ :

$$\begin{aligned} dl_{\bar{\nu}} &= -l_{\bar{\nu}} k_{\bar{\nu}} \rho dx + j_{\bar{\nu}} \rho dx \\ &= -l_{\bar{\nu}} k_{\bar{\nu}} \rho dx + \underbrace{J_{\bar{\nu}} k_{\bar{\nu}} \rho dx}_{\text{Source terms}} \end{aligned}$$

where

- $j$  refers to the summation of the three source terms
  - $J = j/k$  has units of intensity and makes the equation symmetric
- This can be rearranged to give:

$$\frac{dl_{\bar{\nu}}}{k_{\bar{\nu}} \rho dx} = -l_{\bar{\nu}} + J_{\bar{\nu}}$$

**Schwarzchild's Equation**

- This is a fundamental equation for radiative transfer in planetary atmospheres - contains practically all the physics of the problem.
  - Note: applies to monochromatic radiation.

# Schwarzschild's Equation - 1

- Although Schwarzschild's Equation does not have a simple general solution, it can be integrated as follows:
- Multiple by both sides by an integrating factor of  $e^{k_{\bar{v}} \rho x}$  :

$$\frac{dl_{\bar{v}}}{dx} e^{k_{\bar{v}} \rho x} = -k_{\bar{v}} \rho l_{\bar{v}} e^{k_{\bar{v}} \rho x} + k_{\bar{v}} \rho J_{\bar{v}} e^{k_{\bar{v}} \rho x}$$

$$\frac{dl_{\bar{v}}}{dx} e^{k_{\bar{v}} \rho x} + l_{\bar{v}} k_{\bar{v}} \rho e^{k_{\bar{v}} \rho x} = \frac{d}{dx} \left( l_{\bar{v}} e^{k_{\bar{v}} \rho x} \right) = k_{\bar{v}} \rho J_{\bar{v}} e^{k_{\bar{v}} \rho x}$$

- Now integrate from  $x = 0$  to  $X$ :

$$l_{\bar{v}}(X) e^{k_{\bar{v}} \rho X} - l_{\bar{v}}(0) = \int_0^X k_{\bar{v}} \rho J_{\bar{v}} e^{k_{\bar{v}} \rho x} dx$$

$$l_{\bar{v}}(X) = l_{\bar{v}}(0) e^{-k_{\bar{v}} \rho X} + \int_0^X k_{\bar{v}} \rho J_{\bar{v}} e^{k_{\bar{v}} \rho (x-X)} dx$$

# Schwarzchild's Equation - 2

Let  $x' = X - x$ : 
$$I_{\bar{\nu}}(X) = I_{\bar{\nu}}(0)e^{-k_{\bar{\nu}} \rho X} + \int_0^X k_{\bar{\nu}} \rho J_{\bar{\nu}} e^{-k_{\bar{\nu}} \rho x'} dx'$$

intensity at  $x=0$  scaled  
by trans. from  $x = 0$  to  $X$

intensity emitted by gas at each  
level, integrated from  $x = 0$  to  $X$

- This is the integral form of Schwarzchild's Equation.
- We can further simplify by introducing transmission:  $\tau_{\bar{\nu}}(x) = e^{-k_{\bar{\nu}} \rho x}$

$$I_{\bar{\nu}}(X) = I_{\bar{\nu}}(0)\tau_{\bar{\nu}}(X) + \int_{\tau_{\bar{\nu}}(X)}^1 J_{\bar{\nu}} d\tau$$

- Note: This is not a solution, just a different form of the equation.
- We will look at two applications to illustrate the usefulness of Schwarzchild's Equation.



# Schwarzschild's Equation - Summary

- The derivative form of Schwarzschild's Equation:

$$\frac{dl_{\bar{v}}}{k_{\bar{v}} \rho dx} = -l_{\bar{v}} + J_{\bar{v}}$$

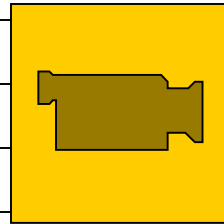
- And for local thermodynamic equilibrium:  $\frac{dl_{\bar{v}}}{k_{\bar{v}} \rho dx} = -l_{\bar{v}} + B_{\bar{v}}$

- The integral form of Schwarzschild's Equation, using  $J = B$ :

$$l_{\bar{v}}(X) = l_{\bar{v}}(0)e^{-k_{\bar{v}} \rho X} + \int_0^X k_{\bar{v}} \rho B_{\bar{v}} e^{-k_{\bar{v}} \rho x'} dx'$$

$$l_{\bar{v}}(X) = l_{\bar{v}}(0)\tau_{\bar{v}}(X) + \int_{\tau_{\bar{v}}(X)}^1 B_{\bar{v}} d\tau \quad \tau_{\bar{v}}(x) = e^{-k_{\bar{v}} \rho x}$$

# Atmospheric Application of Emission in Continuous Media



# Solar and Terrestrial Blackbodies

## Plot sketched in class

Note this is monochromatic blackbody intensity per unit wavenumber

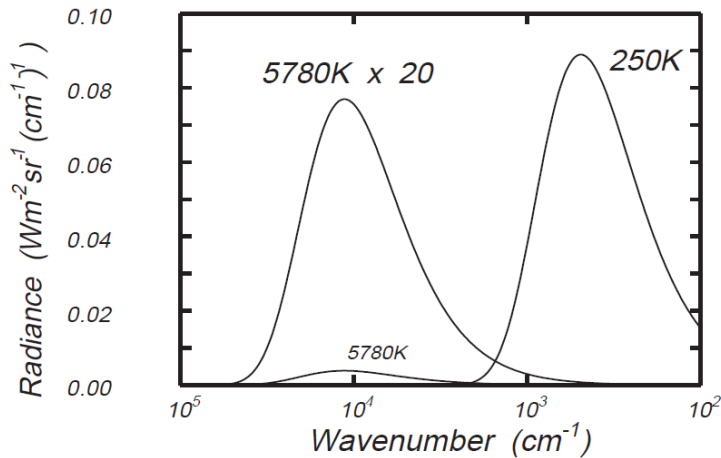
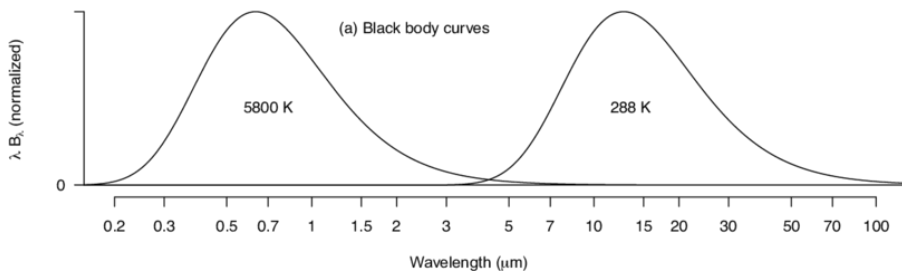


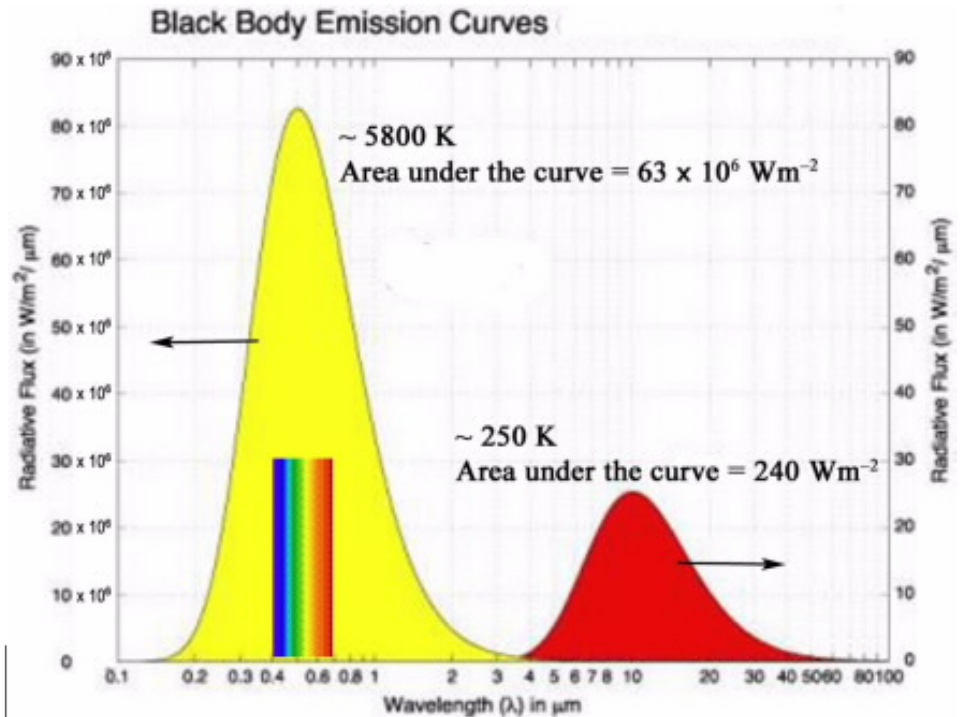
Figure 17: Solar and Thermal Blackbody Curves

## Normalized blackbody function



## Alternate plot

Note this is monochromatic blackbody flux density per unit wavelength



<https://www.acs.org/content/acs/en/climatescience/energybalance.html>