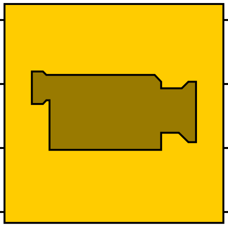
PHY2505S
Atmospheric Radiative Transfer and Remote Sounding

Lecture 2

- Blackbody Radiation
- Additional Laws of Radiation

Radiant Flux Density and Radiance

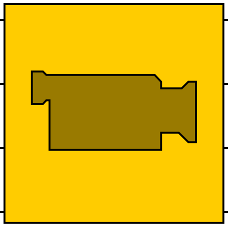


Blackbody Radiation

- Now, let's move from the definitions of radiant flux density and intensity/radiance, to look at some simple radiation fields.
- Consider a large insulated spherical cavity. Observations will show that
  1. There is a homogeneous isotropic radiation field inside the cavity.
  2. This field is independent of the exact size and construction of the cavity.
  3. The strength of the field depends on the temperature of the cavity.
- Next, consider a section of the cavity's surface. Because the field is isotropic, the energy per unit area incident on the surface is $F_\lambda = \pi I_\lambda$.

  → Let's look at what happens to this radiation…

Blackbody Radiation



Blackbodies

- A blackbody is a perfect emitter - it emits the maximum possible amount of radiation at each wavelength.
- A blackbody is also a perfect absorber, absorbing at all wavelengths of radiation incident on it. Therefore, it looks black.

Blackbody energy distribution

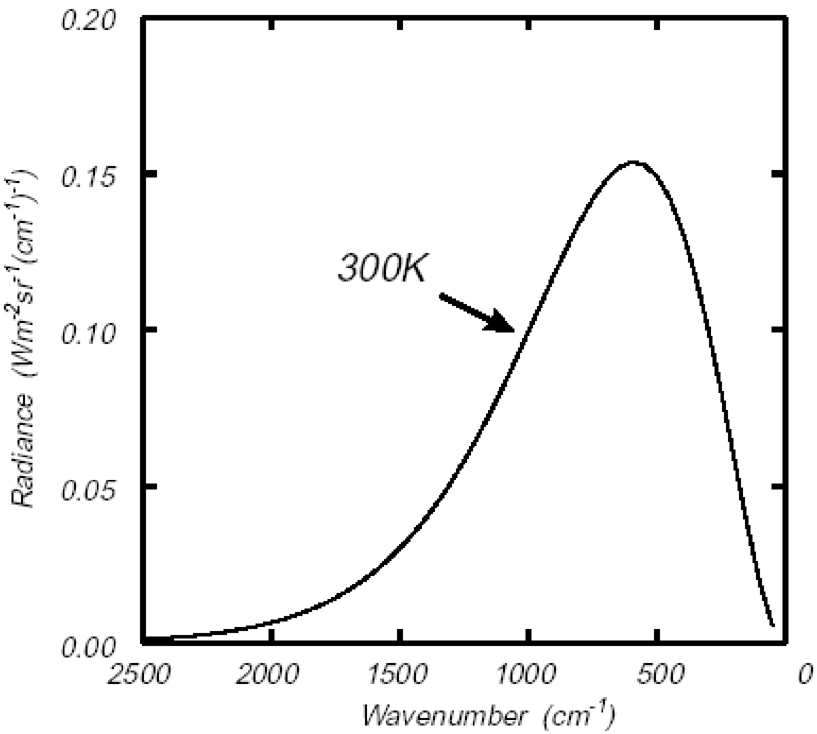


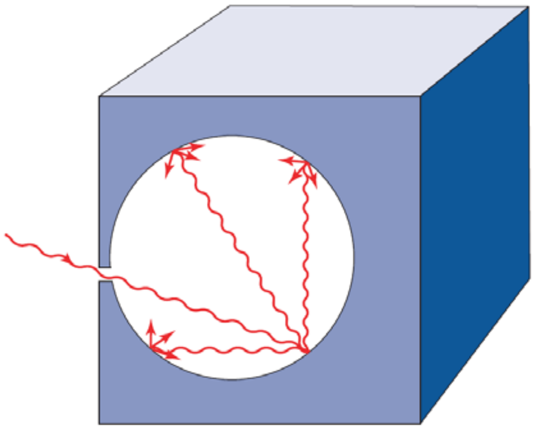
Figure 9: Blackbody Emission Curve

- Independent of the type of material
- Isotropic in nature (i.e., the same in all directions)
- Total energy is proportional to T^4
- Peak emission is given by

$$\bar{\nu} \approx 2T$$

Planck's Blackbody Function

- No real materials are perfect blackbodies. However, the radiation inside a cavity (whose walls are opaque to all radiation) is the radiation that would be emitted by a hypothetical blackbody at the same temperature. The cavity walls emit, absorb, and reflect radiation until equilibrium is reached.



Wallace and Hobbs, Figure 4.5

- Planck postulated that atoms oscillating in the walls of the cavity have discrete energies given by: $E = n\,h\,\nu$

  where

  → n = integer (quantum number), h = Planck's constant, ν = frequency

- A quantum of energy emitted when an atom changes its energy state is then

$$\Delta E = h\,\nu \quad (\Delta n = 1).$$

Planck's Blackbody Function

- Using these two assumptions, Planck derived the <u>blackbody function</u>, describing the <u>radiance</u> emitted by a blackbody:

$$B_\lambda(T) = \frac{2hc^2\lambda^{-5}}{\exp\left(\frac{hc}{\lambda kT}\right) - 1}$$

  where
  - → B_λ = monochromatic radiance (W m^{-2} sr^{-1} μm^{-1})
  - → k = Boltzmann's constant
  - → T = absolute temperature

- This can be written as:

$$B_\lambda(T) = \frac{c_1\lambda^{-5}}{\exp\left(\frac{c_2}{\lambda T}\right) - 1}$$

  where
  - → c_1 = first radiation constant (1.191 × 10^{-16} W m^2 sr^{-1})
  - → c_2 = second radiation constant (1.439 × 10^{-2} m K)

Note: some textbooks have π in denominator so different value for c_1

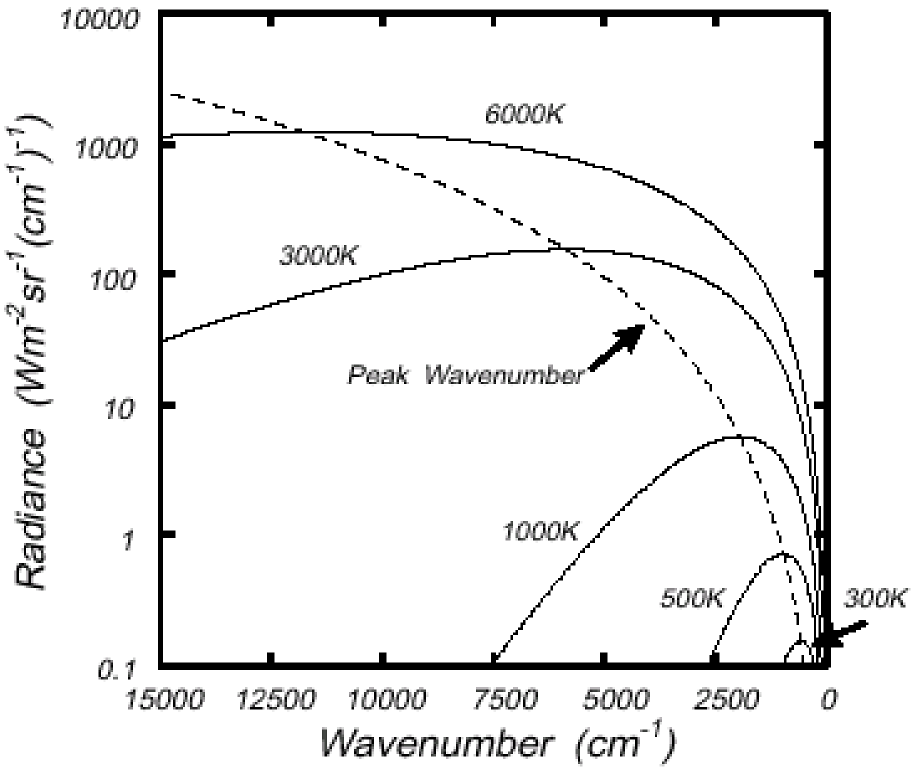
- ∴ The radiance emitted by a blackbody depends only on λ and T.
  - → $B_\lambda(T)$ increases with temperature
  - → the λ of maximum $B_\lambda(T)$ decreases with temperature

Planck's Blackbody Function

We can show that the equivalent form of Planck's function as function of wavenumber is:

$$B_{\bar{\nu}}(T) = \frac{2hc^2\bar{\nu}^3}{\exp\left(\frac{hc\bar{\nu}}{kT}\right)-1} = \frac{c_1\bar{\nu}^3}{\exp\left(\frac{c_2\bar{\nu}}{T}\right)-1}$$

Peak wavenumber
for blackbody emission

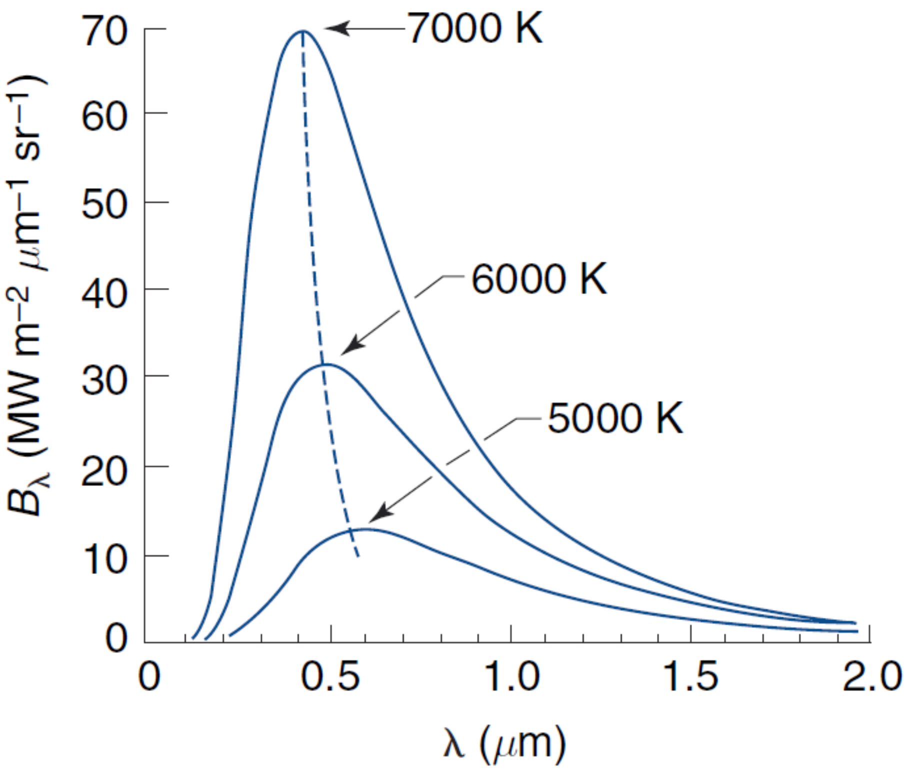


Note : $B_{\bar{\nu}}d\bar{\nu} = B_{\lambda}d\lambda = B_{\nu}d\nu$

Additional Laws of Radiation

Wien's Displacement Law

- The hotter the object, the shorter the λ of its maximum intensity (e.g., element on a stove).
- This law can be used to determine the T of a blackbody from the position of the maximum monochromatic radiance.



Wallace and Hobbs, Figure 4.6

Wien's Displacement Law

- The wavelength at which $B_\lambda(T)$ is a maximum is determined using: $$\frac{\partial B_\lambda(T)}{\partial \lambda} = 0$$
- This gives: $\lambda_m = \dfrac{2897.9}{T}$ µm (T in K) → Wien's Displacement Law.
- Or for wavenumbers:

$$\bar{\nu}_m = 1.962T \cong 2T \quad \text{cm}^{-1}$$

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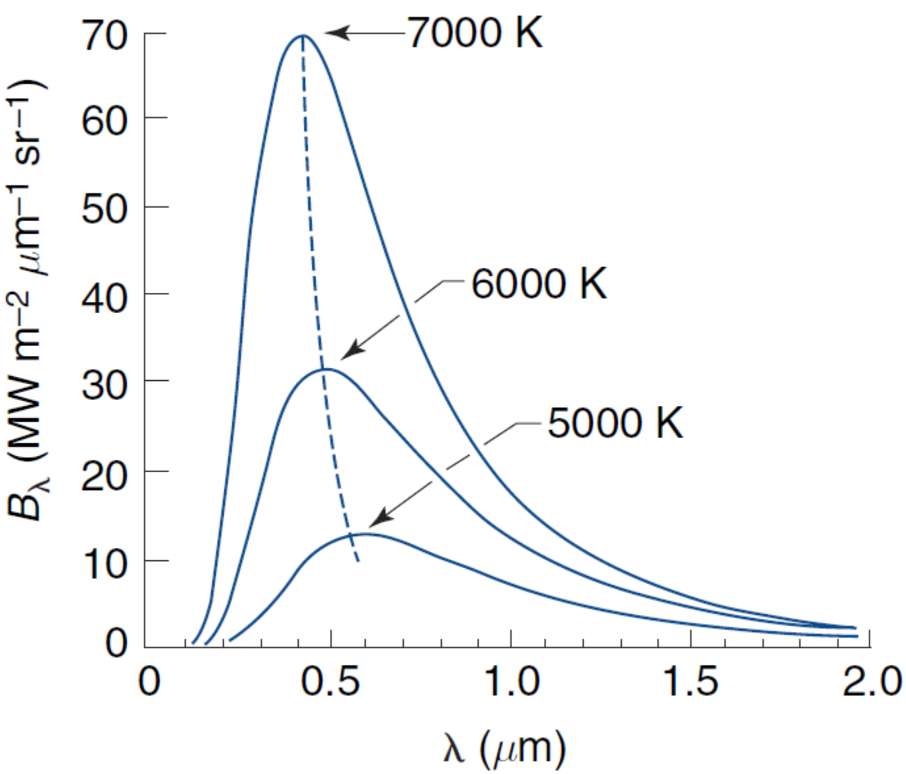
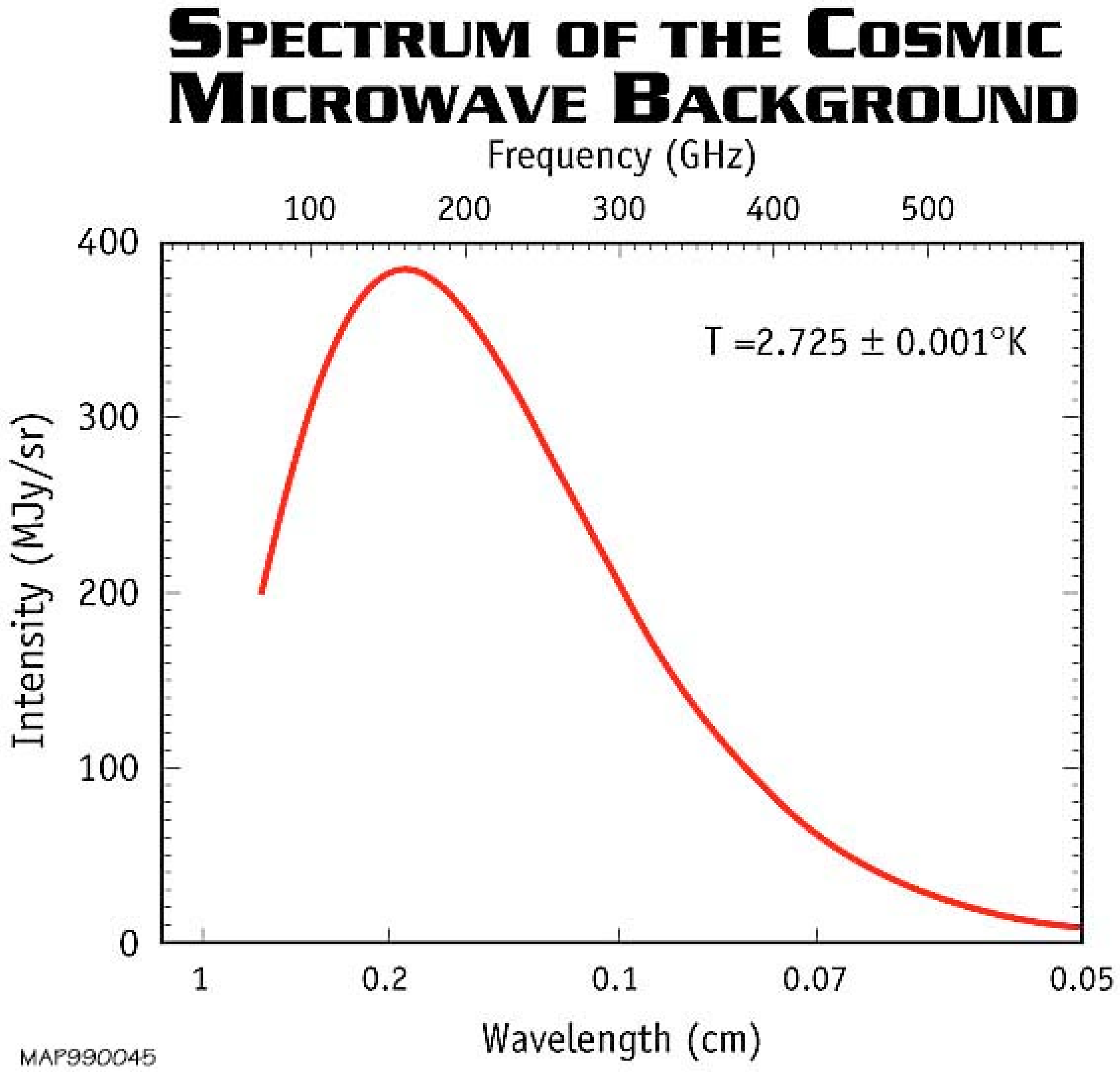


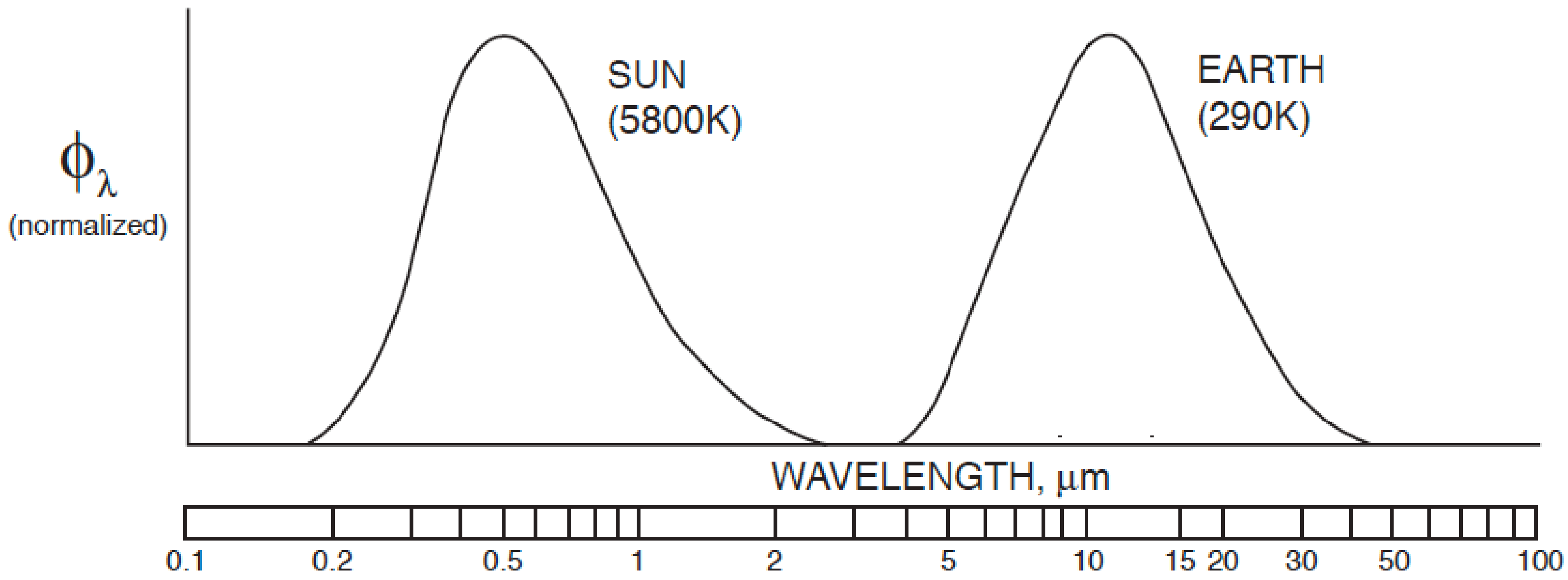
Wallace and Hobbs, Figure 4.6

Blackbody Radiation



http://map.gsfc.nasa.gov/universe/bb_tests_cmb.html

Solar and Terrestrial Radiation



Stefan-Boltzmann Law

- The monochromatic radiant exitance is simply: $M_\lambda(T) = \pi\, B_\lambda(T)$ because the blackbody radiance is isotropic (independent of dir'n).
- We can use this to determine the total radiant exitance from a blackbody

$$M(T) = \int_0^\infty M_\lambda(T)d\lambda = \int_0^\infty \pi B_\lambda(T)d\lambda = \ldots = \frac{\pi^5}{15}\frac{c_1}{c_2^4}T^4$$

$$\therefore \quad M_{BB}(T) = \sigma T^4 \quad \text{or} \quad B(T) = \frac{\sigma T^4}{\pi} \qquad \rightarrow \text{Stefan-Boltzmann Law}$$

where

→ σ = Stefan-Boltzmann constant = 5.67×10^{-8} W m^{-2} K^{-4}

- This states that the total amount of radiation emitted from a given surface area is proportional to T^4.
- If M_{BB} (=F) is known, this can be used to calculate an equivalent blackbody temperature, or effective emission temperature, T_E

Rayleigh Jeans Approximation

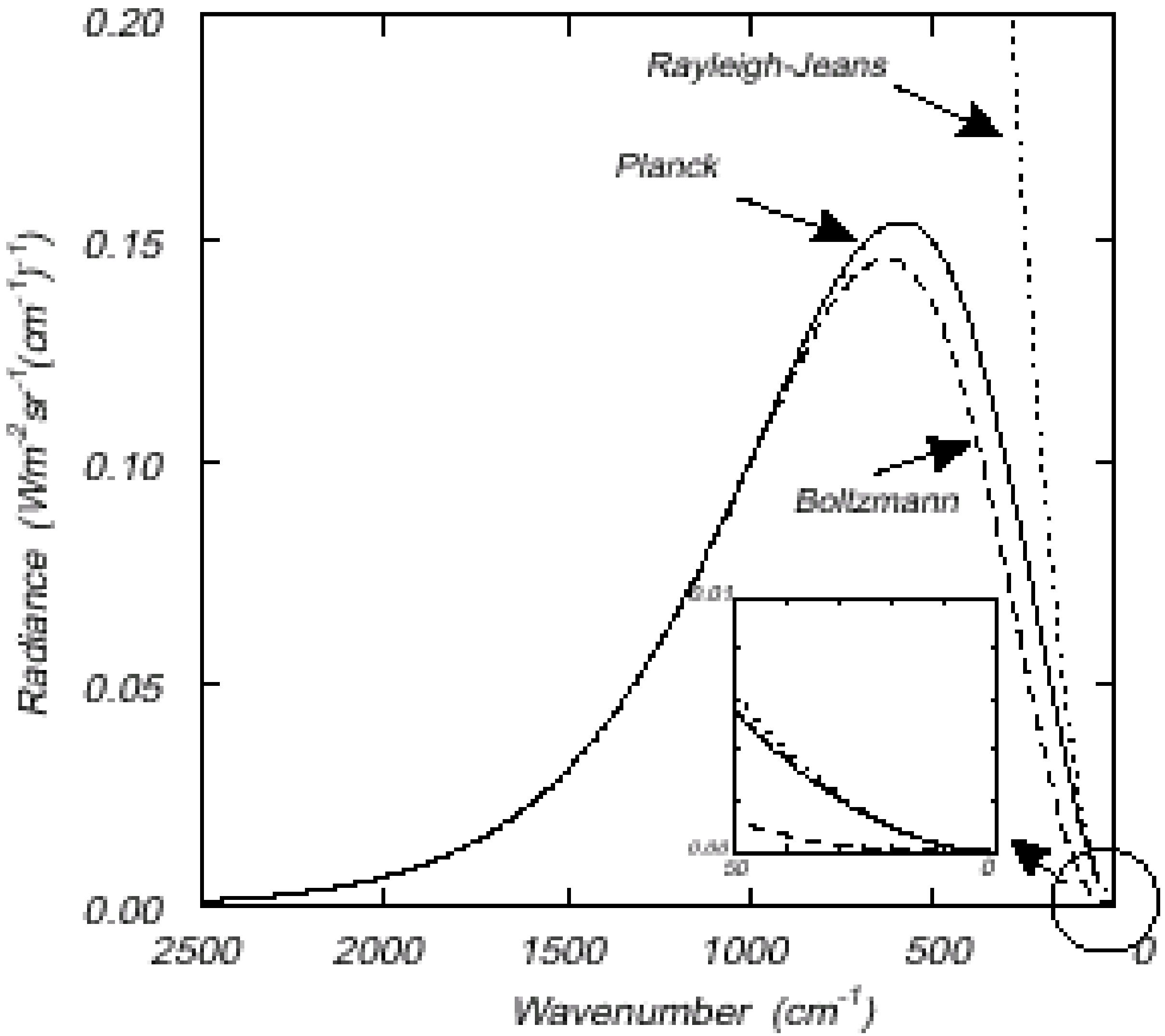
- At longer λ, the Planck blackbody function can be simplified.
- For λ in the microwave region, $c_2 / \lambda T << 1$ for T relevant to Earth.

$$\therefore \exp\left(\frac{c_2}{\lambda T}\right) \approx 1 + \frac{c_2}{\lambda T} \quad \text{giving} \quad B_\lambda(T) \approx \frac{c_1}{c_2}\frac{T}{\lambda^4} \rightarrow$$

Rayleigh Jeans Approximation.

- It states that in the microwave region, the radiance is simply linearly proportional to T.
- Often the radiance will be scaled by $(c_1/c_2)\lambda^{-4}$ to get brightness temperature, T_b, which is radiance in units of temperature.
- T_b is the temperature required to match the measured intensity to the Planck blackbody function at a given λ.
- T_b is also used in the infrared (referred to as the equivalent brightness temperature), but it must be derived from the blackbody function.
- T_b is a wavelength-dependent equivalent blackbody temperature.

Equations for Blackbody Radiation



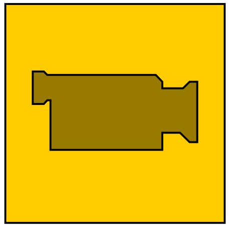
The Blackbody Spectrum

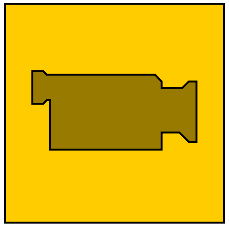
PhET Interactive Simulations, University of Colorado,

- https://phet.colorado.edu/en/simulation/blackbody-spectrum

Interactive blackbody animation:

- https://phet.colorado.edu/sims/html/blackbody-spectrum/latest/blackbody-spectrum_en.html





Grey Bodies

- Thus far, we have been discussing only blackbody radiance. In order to quantify how closely a material approximates a blackbody, the <u>emissivity</u> of the material is defined.

$$\varepsilon_\lambda \equiv \frac{\text{emitted radiance at } \lambda}{\text{blackbody radiance at } \lambda} = \frac{I_\lambda}{B_\lambda(T)}$$

Wavelength-dependent

- For a blackbody, $\varepsilon_\lambda = 1$.
- For other materials, called <u>grey bodies</u>, $0 \leq \varepsilon_\lambda < 1$.
- Can also define:
  - → <u>absorptivity</u> $\alpha_\lambda \equiv$ absorbed radiance at λ / incident radiance at λ
  - → <u>reflectivity</u> R_λ, $\rho_\lambda \equiv$ reflected radiance at λ / incident radiance at λ
  - → <u>transmittivity</u> $\tau_\lambda \equiv$ transmitted radiance at λ / incident radiance at λ
- These three quantities describe the three possibilities for incident radiation. All have values between 0 and 1.

Kirchoff's Radiation Law

- By the conservation of energy, the sum of the absorptance, the reflectance, and the transmittance must be unity:

$$\alpha_\lambda + R_\lambda + \tau_\lambda = 1.$$

- Kirchoff's radiation Law states that for a body in local thermodynamic equilibrium (LTE): $\boxed{\alpha_\lambda = \varepsilon_\lambda}$

  → so the monochromatic emittance must be equal to the monochromatic absorptance, and

  → a medium may absorb radiation of a particular λ, and at the same time also emit radiation of the same λ.

- The rate at which emission takes place is a function of T and λ.
- Note: LTE is characterized by uniform T and isotropic radiation → true in the atmosphere below ~40 km (Liou; 100 km K&VH).

Infrared Emittance

- Any planetary surface looks reasonably "black" when considering thermal (i.e., infrared) radiation emitted from it.

Some examples:

Material	Infrared Emissivity
Sand (wet)	0.962
Sand (dry)	0.949
Peat (dry)	0.970
Grass (thick, green)	0.986
Fresh Snow	0.986
Dirty Snow	0.969

- So we will not be too much in error if we treat planetary surfaces as blackbody surfaces.
- In contrast, many polished surfaces have $\varepsilon_\lambda \sim 0$ in the infrared (e.g., gold has $\varepsilon_\lambda = 0.01$ despite its "brightness" in the visible).

Planetary Radiation Balance

- A planet radiates energy at a rate of $M_{BB} = \sigma T^4$ per unit area (W/m^2).
- For Earth (mean radius R = 6371 km), 1.2×10^{17} W are emitted continuously at an effective radiating temperature of 255 K.

Consider what would happen if this were the only energy flux.

- Earth would cool at a rate given by the simple Newtonian formula:

$$A_{surface} M_{BB} = 4\pi R^2 \sigma T^4 = \frac{4\pi R^3}{3} C \frac{dT}{dt} \left(= V_{Earth} C \frac{dT}{dt} \right)$$

- $C = \rho\, c_p$ is the volumetric heat capacity of the Earth in J m^{-3} K^{-1}. Let's use the value for granite (2.2×10^6 J m^{-3} K^{-1}).
- Cooling rate solution: $\frac{dT}{dt} = \frac{4\pi R^2 \sigma T^4}{\frac{4}{3}\pi R^3 C} = \frac{3\sigma T^4}{RC} = 5.3 \times 10^{-11}\ \mathrm{K\ s^{-1}}$

- This is equivalent to 1.6 K per 1000 years.

Planetary Radiation Balance

- A planet radiates energy at a rate of $M_{BB} = \sigma T^4$ per unit area (W/m^2).

Consider what would happen if this were the only energy flux.

- Over the geological age of the Earth (~4.5 billion years), it would lose all its heat if there were no other source!

Thus:

- Any long-term model of the climate of a planet must have an overall energy balance for the system as a boundary condition.
- The time scales for the radiation problem can be very long. In the short term there are heat flows in the land, ocean and atmosphere – but in the long time only the overall heat balance will matter as everything else averages out.

Later, we'll look at the "other source" of radiation – the Sun.

- Virtually all the energy that the Earth receives and that sets the Earth's atmosphere and oceans in motion comes from the Sun.