PHY2505S
Atmospheric Radiative Transfer and Remote Sounding

Lecture 1

- Course Overview
- Introduction to Radiative Transfer
  - Electromagnetic Radiation, Definitions, Beer's Law

Course Information – 1

See handout.

LECTURER: Prof. Kimberly Strong
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Phone: 416-978-5205 (for MP323)

LECTURES: 10:10 – 12:00 PM, Tuesday, Room MP606

OFFICE HOURS: These will vary from week to week, all in MP323.
3:00 – 4:00 PM, Tuesday, on Jan 7, Jan 21, Feb 18, Feb 25, March 3, March 10, March 24, March 31, April 7, April 14
2:00 – 3:00 PM, Thursday, on Jan 16, Jan 30, Feb 13
4:00 – 5:00 PM, Wednesday on March 18
TBA, week of Feb 3-7
Also, feel free to drop by or make an appointment.

WEBSITE: Quercus – coming soon (https://portal.utoronto.ca/)
Lectures and supplementary material will be posted on the course homepage.

Course Information – 2

REFERENCES:

There is no required textbook for this course. In addition to the lecture notes, the following texts and other works on atmospheric physics and radiation may be useful references. These books are available on short-term loan or electronic form from UofT libraries:

- An Introduction to Atmospheric Radiation (Second Edition), K.N. Liou (Academic Press, 2002) – online, Physics and Gerstein libraries.
- A First Course in Atmospheric Radiation, (Second Edition), G.W. Petty (Sundog Publishing, 2004) – Physics and Gerstein libraries
- Spectroscopy and Radiative Transfer of Planetary Atmospheres, (First Edition), K.V. Chance and R.V. Martin (Oxford University Press, 2017) – Physics library

Also: Atmospheric Science, An Introductory Survey (Second Edition), J. Wallace and P. Hobbs (Academic Press, 2006)

MARKING:

20%	Problem sets: #1 will be handed out on Jan 21 and due in class on Feb 4 #2 will be handed out on March 10 and due in class March 24
15%	Mid-term test (time TBA, tentatively the week of Feb 24-28)
30%	Term paper (discuss topic with instructor by Feb 14, outline and bibliography due March 3, paper due Tuesday, April 7) and presentation (time TBA)
35%	Final exam

Course Assignments

PROBLEM SETS: There will be two problems sets, due two weeks after they are assigned. While you may discuss the assignment with your classmates, you must prepare your answers to the problems independently. Marks will be given for showing workings as well as for final answers. Further information will be provided with each problem set.

TERM PAPER, OUTLINE/BIBILIOGRAPHY AND SEMINAR PRESENTATION: The theme/topic for your paper/presentation will be a recent remote sounding mission/instrument and examples of atmospheric phenomena that it has been used to study. There are four deadlines for this assignment: By Feb 14, before Reading Week, you must discuss the topic with the instructor. An outline for the paper (including an "annotated" bibliography) is due on March 3. The paper is due on April 7, and the presentations will be arranged soon after that. Guidelines for the outline, term paper, and presentation will be provided later in the term.

PENALTIES FOR LATE WORK: Unless otherwise stated, there will be a late penalty of 5% per day, up to seven days, after which material will not be accepted. Requests for exemptions have to be made at least 24 hours before the deadline and may or may not be granted.

SUBMISSION OF WORK: Paper copies of work must be handed to the instructor in person. Work put in the instructor's mailbox, slid under the office door, or submitted by e-mail will not be accepted. Accommodations will be made for students who are in the field.

Course Topics

This course will provide a survey of the interaction of electromagnetic radiation fields with the Earth's atmosphere. Emphasis will be on the physical basis of interactions and calculations. The goal is to develop an understanding of atmospheric radiative transfer, the physics responsible for atmospheric spectra, the information content of these spectra, and how atmospheric properties can be derived using remote sounding methods.

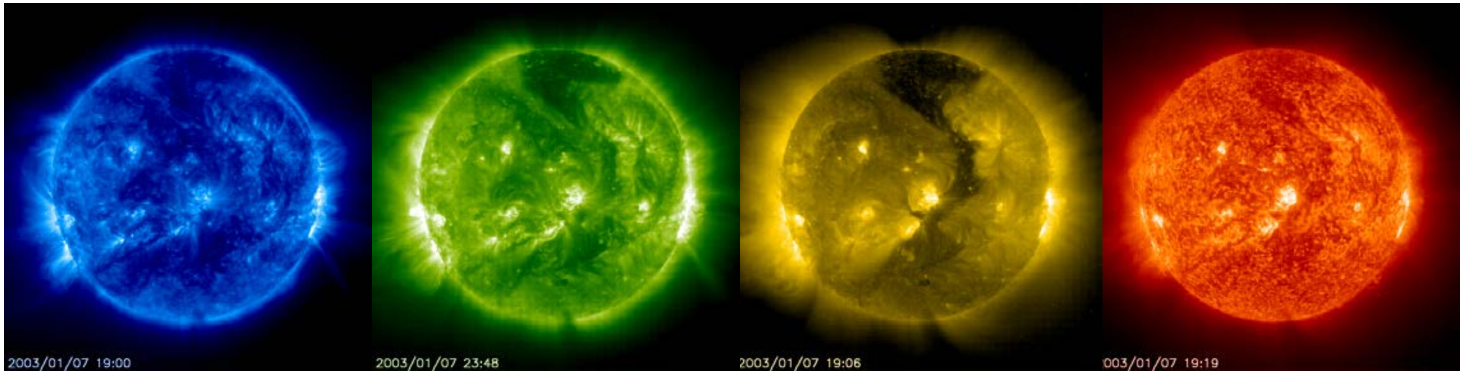
- Review of radiative transfer
- Radiative absorption and emission in general
- Solar variability and determination of the solar constant
- The spectrum of the atmosphere
- The role of minor constituents and the greenhouse effect
- Introduction to molecular spectroscopy (rotational and ro-vibrational)
- Spectral line shapes; approximations and modeling
- Techniques and methodologies for atmospheric remote sounding
- Uses of satellite instrumentation for atmospheric remote sensing
- Scattering of radiation; applications to the atmosphere (as time permits)

Introductory Remarks: Radiative Transfer

- Radiative transfer deals with the interactions between matter and electromagnetic radiation.
- In this course, we will examine radiative processes that occur in Earth's atmosphere.
- The transfer of solar and infrared radiation is the primary physical process driving the circulation of the atmosphere and the ocean currents.
- Radiative processes and the radiative balance between Earth and its atmosphere are fundamental to understanding the climate and climate change.
- The principles of radiative transfer are also essential to the interpretation of data from meteorological and remote sounding satellite instruments – for retrieval of temperature, trace gas concentrations, cloud properties, aerosols.
- *Let's begin with some fundamental concepts, definitions, and units.*

Radiation

- The primary source of energy that drives the Earth's climate system is the Sun.
- The Sun's energy comes to us mostly in the form of electromagnetic radiation.
- Understanding climate requires understanding the nature of electromagnetic radiation and how it interacts with a planetary atmosphere and surface.



$\lambda = 171Å$ $\lambda = 195Å$ $\lambda = 284Å$ $\lambda = 304Å$

Sun – SOHO observations 07/01/03, from http://sohowww.nascom.nasa.gov/

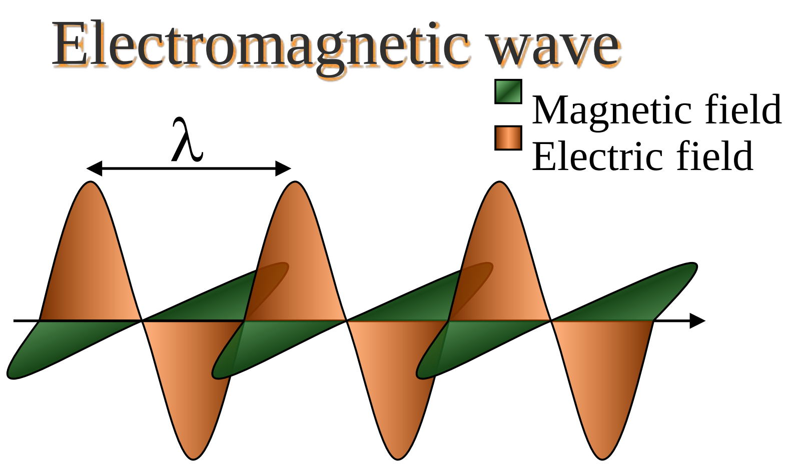
Electromagnetic Radiation – 1

The term electromagnetic radiation refers to a phenomenon that moves energy from one place to another, and carries with it an electric and a magnetic field.

- Defined by solutions of Maxwell’s Eqns.
- As they move, photons carry electric and magnetic fields that oscillate at a certain frequency, giving rise to an electromagnetic wave.
- Can be thought of as particles (photons) that carry energy at speed c
- Usually, we can approximate the speed of light in the atmosphere using that in a vacuum: 2.99792458×10^8 m/s
- However, the change in density and humidity with altitude can cause significant refraction (bending of EM rays), which must be taken into account in satellite pointing.

The electric and magnetic fields are mutually perpendicular to each other and to the direction of propagation.

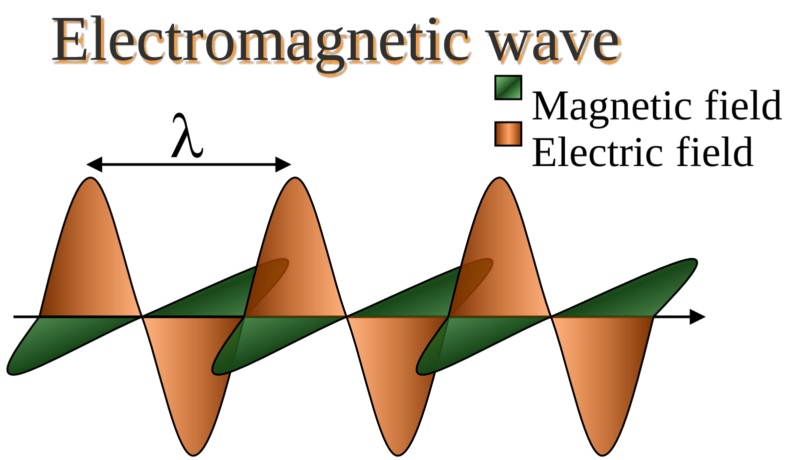
E $\perp$ **H** $\perp$ direction of propagation



Electromagnetic Radiation – 2

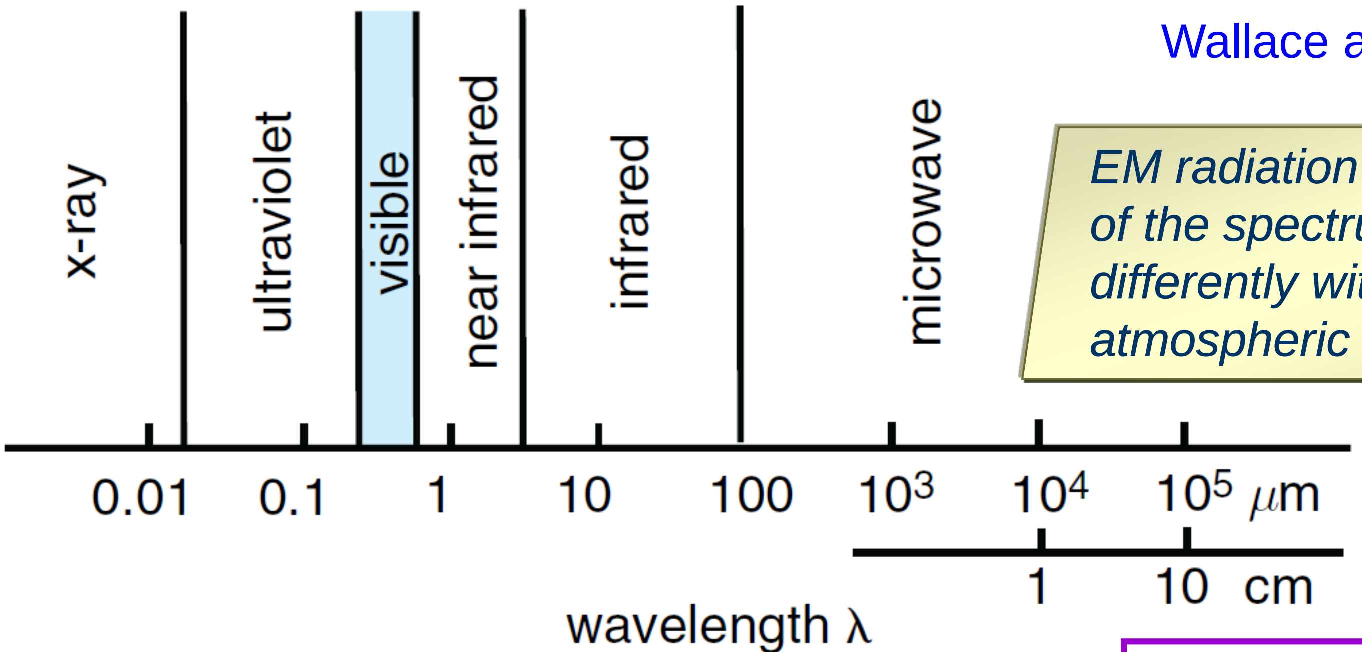
EM waves are usually specified by:

- **E** $\perp$ **H** $\perp$ direction of propagation
- Wavelength (λ) = distance between crests (units of nm, μm, mm, m, etc.)
- Frequency (f *or* ν) = number of oscillations per second = c/λ (units of Hz, MHz, GHz)
- Wavenumber (ν or $\bar{\nu}$ or $\tilde{\nu}$) or Kayser (K, with 1 K = 1 cm^{-1}) = number of crests per unit length = 1/λ (units of m^{-1}, cm^{-1})
- Note: the unit cm^{-1} is often called wavenumber (which is the physical quantity), but more correctly, the unit cm^{-1} is inverse or reciprocal cm



The Electromagnetic Spectrum

Wallace and Hobbs, Figure 4.1



EM radiation in different regions of the spectrum interacts differently with matter: useful for atmospheric remote sounding

- Ultraviolet – 100 to 400 nm
- Visible – 400 to 700 nm
- Near infrared – 700 nm to ~4μm
- Infrared – 700 nm to 1 mm
- Microwave – 1 mm to 1 m
- Radio – includes microwave to > 100 m

Shortwave = UV, vis, near-IR < 4 μm (solar)

Longwave = infrared > 4 μm (terrestrial)

An Aside: Radiation as Particles

- The radiation can also defined in terms of particles: both wave and particle descriptions can be useful in atmospheric applications
- A radiation beam is defined as a flux of massless photons all travelling at the speed of light c, and all carrying an energy:

$$\text{Energy} = hc\bar{\nu} = hf = \frac{hc}{\lambda}$$

Planck's constant: h = 6.6262x10^{-34} Js

- Each photon has momentum: from the de Broglie theory of wave-particle duality.

$$p = \frac{h}{\lambda}$$

- de Broglie's hypothesis: λ = h/p for all matter. This is a generalization of Einstein's photoelectric effect (E = hf) since p = E/c for a photon and λ = c/f.

Radiation as Wave Motion – 1

An EM wave with a single frequency f will have a sinusoidal form:

$$A(x,t) = a\sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right) = a\sin(kx - \omega t)$$

where we have defined

- wave vector $k = 2\pi/\lambda = 2\pi\bar{\nu}$
- angular frequency $\omega = 2\pi f$

The minus sign is due to the direction of propagation, from x = 0 to ∞.

The energy transferred by the wave is the same for all x and t.

$$E(x,t) = E_o \sin(kx - \omega t)$$

$$|E(x,t)|^2 = |E_o|^2$$

This energy can be detected by sensors.

Radiation as Wave Motion – 2

The energy per unit area per unit time flowing perpendicularly into a surface is given by the Poynting vector :

$$\vec{S} = c^2 \varepsilon_o \vec{E} \times \vec{H}$$

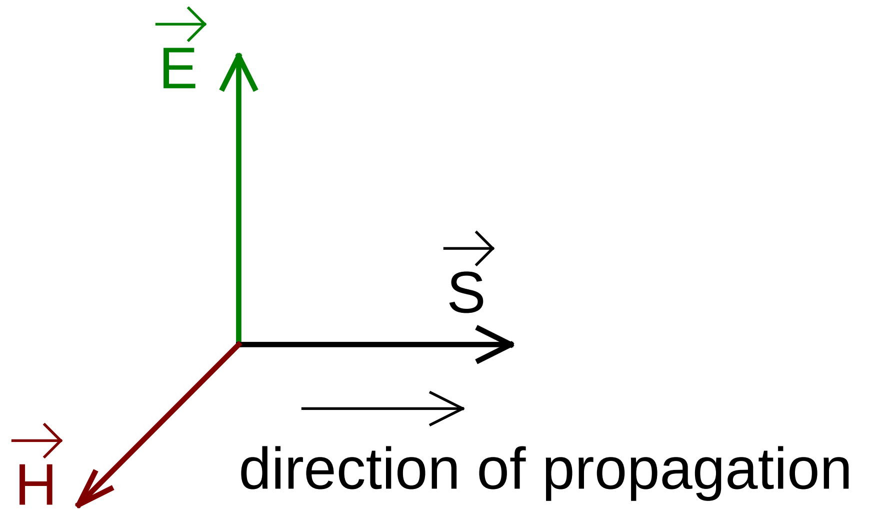
units of W/m^2

where

ε_o = vacuum permittivity

$\vec{E}$ = electric field

$\vec{H}$ = magnetic field



The energy transport is proportional to E^2 because

the electric and magnetic fields are related by a constant of proportionality dependent on medium in which propagation occurs.

Detectors measure the time average of the Poynting vector.

Radiometric Quantities

We will now define the fundamental quantities used to describe radiation and radiative transfer:

- radiant flux density
  - → radiant exitance
  - → emittance
- radiance = intensity

See Bohren and Clothiaux, Fundamentals of Atmospheric Radiation, page 204!

4.1.8 A Few Words about Terminology and Units

By conscious design, not accident, we gradually introduced radiometric and photometric terms, as few as possible and only as needed. The tragedy of radiometry (and photometry) is that an inherently interesting subject – the best scientific instrument we carry with us everywhere is our eyes, and we are constantly immersed in a world of ever-changing brightness and color – has been made dreadfully boring by wallowing in a mire of terminology and units. This harsh view is shared by others, expressed with biting wit by R. C. Hilborn, who defined radiometry as an acronym for revulsive, archaic, diabolical, invidious, odious, mystifying, exotic terminology regenerating yawns. Terms are multiplied without end. Units in photometry border on the fantastic (foot-candle, talbot, nit, troland, candela, lux, lumen, stilb, foot-Lambert, nox, skot, and so on *ad nauseam*). Radiometry comes across as the science of terminology, its seeming objective being to multiply distinctions endlessly and thereby coin as many terms as possible. To make matters worse, the symbols are ghastly, quantities that are not derivatives yet written as derivatives, and strange-looking ones at that. Mass density rarely, if ever, is written as the derivative of mass with respect to volume [see Eq. (4.16)], but this sort of thing is done all the time in radiometry even though radiometric quantities are distribution functions fundamentally no different from mass density.

RADIOMETRY = acronym for Revulsive, Archaic, Diabolical, Invidious, Odious, Mystifying, Exotic Terminology Regenerating Yawns!

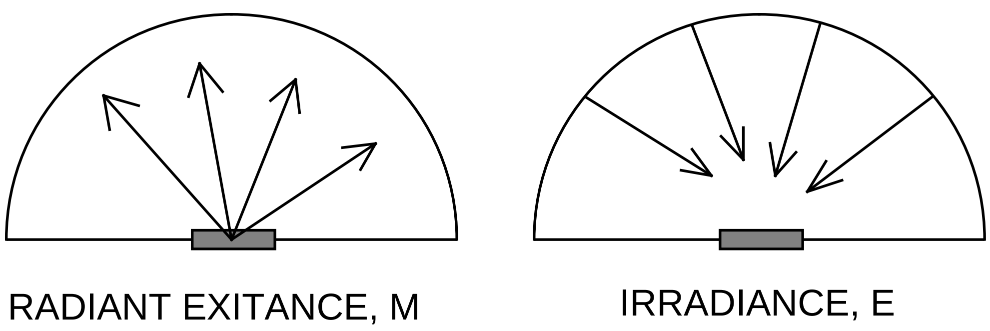
Radiant Flux Density

- The electric and magnetic fields and the Poynting vector $(\vec{E}, \vec{H}, \vec{S})$ of an EM wave oscillate rapidly, so are difficult to measure instantaneously.
- Usually measure the average magnitude over some time interval:

$$F = \langle \vec{S} \rangle$$

- This is the radiant flux density ($W\ m^{-2}$).

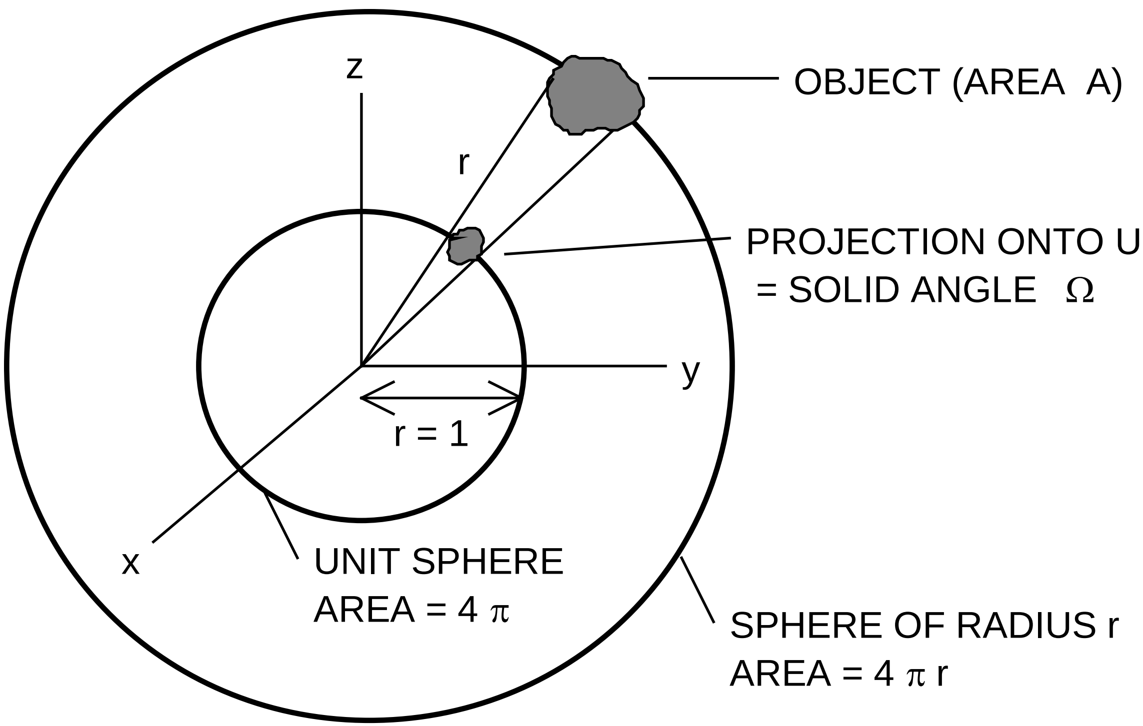
- The radiant flux density is redefined based on the direction of energy travel:
  - → radiant exitance (M) = radiant flux density emerging from an area
  - → irradiance (E) = radiant flux density incident on an area

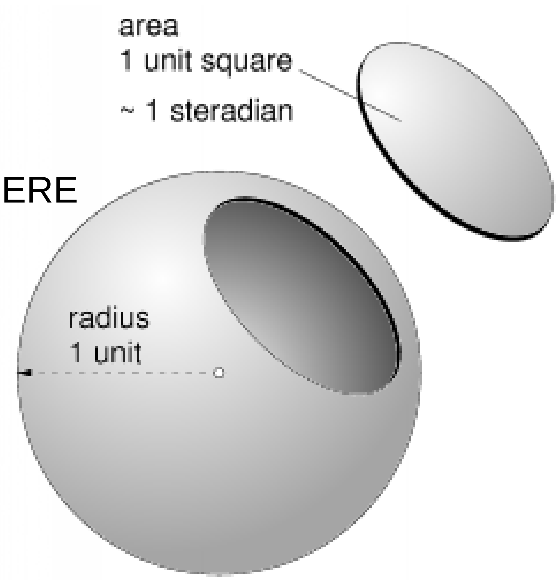


Solid Angle – 1

Radiation depends on direction. To deal with this, we define the solid angle, which also accounts for divergence of the radiation beam.

<u>Solid angle:</u> the area of the projection onto a unit sphere of an object, where lines are drawn from the centre of the sphere to every point on the surface of the object





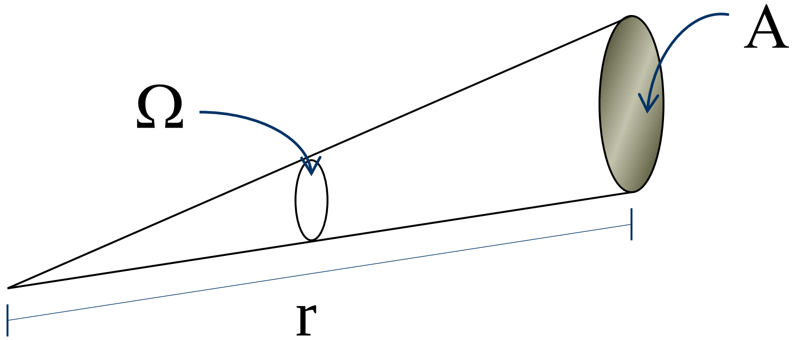
http://www.schorsch.com/kbase/glossary/solid_angle.html

Solid Angle – 2

The solid angle is numerically equal to the ratio of the area A of a spherical surface intercepted to the square of the radius, r.

$\Omega = A/r^2$ steradian (sr) [ω in some textbooks]

Mathematically, the solid angle is unitless, but for practical reasons, the steradian is assigned.



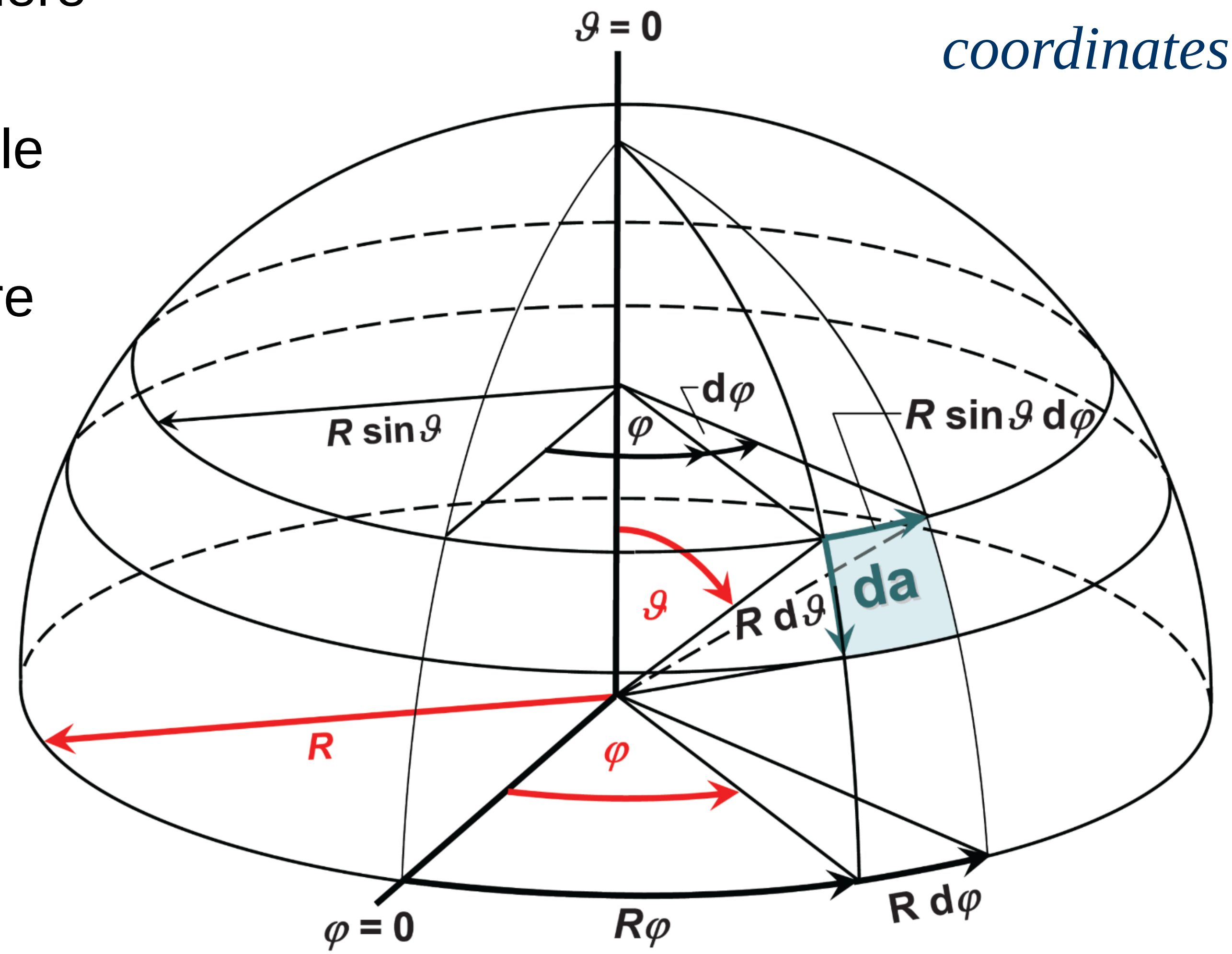
Examples:

- solid angle of a sphere whose surface area is $4\pi r^2$: $\Omega = 4\pi$ sr (note: this is the maximum possible solid angle ~12.57)
- solid angle of any object that completely surrounds a point: $\Omega = 4\pi$ sr
- solid angle of an infinite plane: $\Omega = 2\pi$ sr

Solid Angle – 3

- r = radius of sphere
- θ = zenith angle
- ϕ = azimuth angle
- da = area on surface of sphere

Polar coordinates



https://seos-project.eu/laser-rs/images/solid-angle-1.png

Solid Angle – 4 *(written notes)*

- To obtain a <u>differential element of solid angle</u>, construct a sphere of radius r whose central point is denoted by *O.*
- Differential area on the surface located at r from point *O* is:

$$dA = (r\, d\theta)(r \sin\theta \;\; d\phi)$$

Polar coordinates

- Then the differential solid angle is:

$$d\Omega = d\left(\frac{A}{r^2}\right) = \sin\theta\, d\theta\, d\phi = d\mu\, d\varphi$$

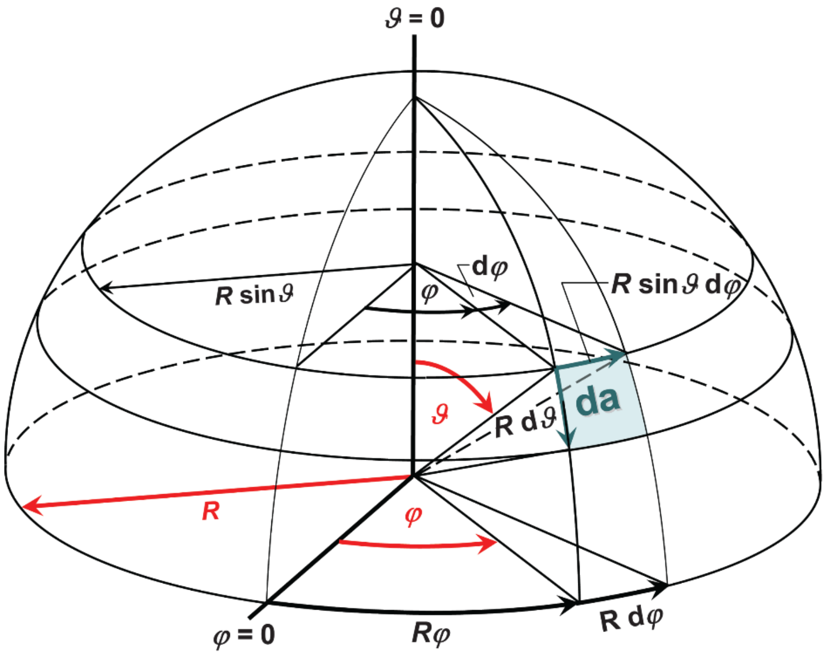
where

$\rightarrow$ $\mu = \cos\theta$

$\rightarrow$ θ = zenith angle
= 90° – elevation angle

$\rightarrow$ ϕ = azimuth angle

[d(cosθ)=-sinθ; dropped - sign]



Intensity (or Radiance) – 1

Now we can combine energy and direction to define:

- Intensity or radiance (I or L) = radiant flux density per unit solid angle (W m^{-2} sr^{-1}).

$$I = \frac{\langle \vec{S} \rangle}{d\Omega}$$

- The intensity is the radiation flow in a particular direction at a particular point.
- Strictly, the intensity represents the EM radiation leaving or incident upon an area perpendicular to the beam. For other directions, it must be weighted by cos θ.
- The radiation may be wavelength dependent, so we prefix the energy-dependent terms by "monochromatic" or "spectral".
- Symbols are subscripted accordingly, e.g., I_λ, I_ν, $I_{\bar{\nu}}$ refers to intensity per unit λ, ν, $\bar{\nu}$: $I_\lambda = I/d\lambda$, , $I_\nu = I/d\nu$, $I_{\bar{\nu}}/d\bar{\nu}$
- Units for monochromatic radiance:
  - $\rightarrow$ I_λ in W m^{-2} sr^{-1} nm^{-1}, I_ν in W m^{-2} sr^{-1} Hz^{-1}, $I_{\bar{\nu}}$ in W m^{-2} sr^{-1} $(cm^{-1})^{-1}$

Total Radiance

$$I = \int_0^\infty I_\lambda \, d\lambda = \int_0^\infty I_\nu \, d\nu = \int_0^\infty I_{\tilde{\nu}} \, d\tilde{\nu}$$

We also have

$$I_\lambda = -\frac{d\nu}{d\lambda} I_\nu = \frac{\nu^2}{c} I_\nu = \tilde{\nu}^2 I_{\tilde{\nu}}$$

$$\lambda I_\lambda = \nu I_\nu = \tilde{\nu} I_{\tilde{\nu}}$$

Radiance is useful for satellite meas. because it is independent of distance from an object as long as the viewing angle and amount of intervening matter don't change.

Both F and Ω decrease with R^2 so I is constant.

Total Radiance

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Radiance is useful for satellite meas. because it is independent of distance from an object as long as the viewing angle and amount of intervening matter don't change.

Both F and Ω decrease with R^2 so I is constant.

Intensity (or Radiance) – 2

- Consider the differential amount of radiant energy dE_λ that flows through a small area dA within a small cone of solid angle $d\Omega$ oriented at angle θ to the normal to dA, in a time interval dt and in a wavelength interval λ to $\lambda + d\lambda$.
- This energy is expressed in terms of the monochromatic intensity I_λ by:

$$dE_\lambda = I_\lambda \cos\theta \, dA \, d\Omega \, d\lambda \, dt \quad \text{and} \quad I_\lambda = \frac{dE_\lambda}{\cos\theta \, dA \, d\Omega \, d\lambda \, dt}$$

- Then the intensity is: $I = \int_0^\infty I_\lambda d\lambda = \int_0^\infty I_\nu d\nu = \int_0^\infty I_{\bar{\nu}} d\bar{\nu}$ W m^{-1} sr^{-1}
- If the intensity is a function only of the direction but not position, then the field is <u>homogeneous</u>.
- If the intensity is independent of all spatial and directional parameters, then the field is <u>homogeneous</u> and <u>isotropic</u>.

Radiant Flux Density – Revisited

- The radiant flux density, F_λ, is the total or net energy flow in a particular direction, with

$$dE_\lambda = F_\lambda dA\, d\lambda\, dt \qquad F_\lambda = F / d\lambda \quad \text{etc.}$$

- The monochromatic flux density (or the monochromatic irradiance) is defined by the normal component of I_λ integrated over the entire hemispheric solid angle: $F_\lambda = \int_\Omega I_\lambda \cos\theta \, d\Omega$

- In polar coordinates: $F_\lambda = \int_0^{2\pi} \int_0^{\pi/2} I_\lambda(\theta,\phi) \cos\theta \sin\theta \, d\theta \, d\phi$

- Note, for isotropic radiation (independent of direction): $F_\lambda = I_\lambda \int_0^{2\pi} \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta \, d\varphi = \pi I_\lambda$

- Total flux density (or irradiance): $F = \int_0^\infty F_\lambda d\lambda$

- The total flux or radiant power (energy per time): $f = \int_A F \, dA$

Summary of Radiometric Quantities

Symbol	Quantity	Dimension	Unit
E	Energy	ML^2T^{-2}	Joule (J)
f	Flux (luminosity)	ML^2T^{-3}	Joules per second (W)
F, M, E	Flux density (irradiance) Emittance	MT^{-3}	Joules per second per square meter (W m^{-2})
I, **B** , L	Intensity (radiance) Brightness (luminance)	MT^{-3}	Joules per second per square meter per steradian (W m^{-2} sr^{-1})

Interaction of Radiation with the Atmosphere

- Now that we have defined some of the basic units of radiation, let's look at the interaction of radiation with the atmosphere.
- We have the monochromatic intensity I_λ (or I_ν or $I_{\overline{\nu}}$) in units of energy / unit area / unit solid angle / unit spectral interval.
- If we pass a monochromatic beam of radiation I(0) through a medium that interacts with the beam, then the beam will undergo an extinction that reduces the intensity.
  - $\rightarrow$ This phenomenon is described by "Beer's Law".

"Beer's Law"

$$I_\lambda(x) = I_\lambda(0) + \Delta I_\lambda$$

where

ΔI_λ is negative $\longrightarrow$ extinction.

In approx where ΔI_λ is small, this interaction is linear.

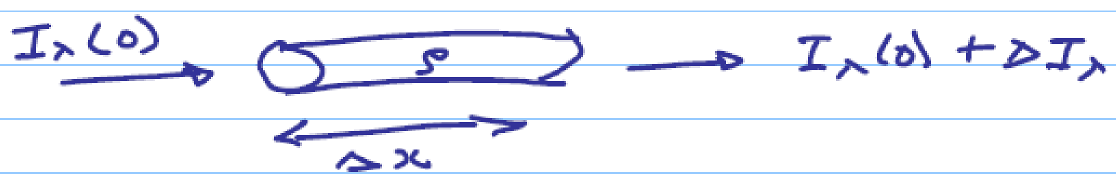
$$\Delta I_\lambda = -\text{coupling term} \times \text{amount of material} \times I_\lambda(0)$$

$$\Delta I_\lambda = -k_\lambda \times \rho \Delta x \times I_\lambda(0)$$

where

k_λ = mass absorption coefficient or cross-section (area/mass)

ρ = density of material



"Beer's Law"

Beer – Bouguer – Lambert Law

In differential form:

$$\frac{\Delta I_\lambda}{\Delta x} = \frac{dI_\lambda}{dx} = -k_\lambda \rho I_\lambda = -\sigma_\lambda I_\lambda$$

where σ_λ = volume absorption coeff. (unit of 1/length).

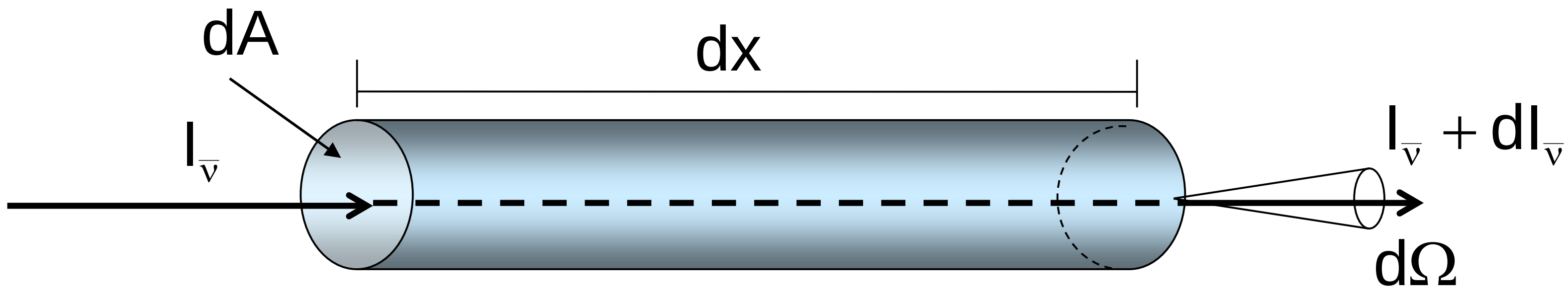
If k_λ and ρ are constant, integrate to get:

$$I_\lambda(x) = I_\lambda(0) \exp(-k_\lambda \rho x).$$

So the intensity decays exponentially as it traverses the medium.

Radiative Transfer in the Atmosphere

Consider the general interaction of radiation with gaseous material.

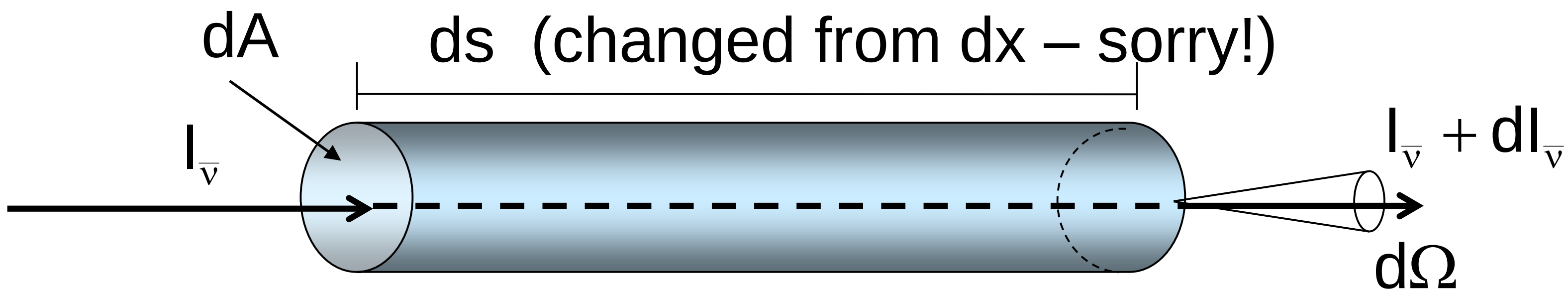


- The monochromatic intensity at a point **p** in a direction **s** is $I(\mathbf{p},\mathbf{s},\bar{\nu},t)$ W m^{-2} sr^{-1} (cm^{-1})$^{-1}$
  - → It is the amount of energy per unit time flowing within a solid angle $d\Omega$ about the direction **s** through a small area dA perpendicular to **s** (therefore weight by cos θ) in a wavenumber interval $d\bar{\nu}$:

$$dE(\bar{\nu}) = I(\mathbf{p},\mathbf{s},\bar{\nu},t)\cos\theta \, dA \, d\Omega \, d\bar{\nu} \, dt$$

*For simplicity, we will omit the dependencies of I on **p**, **s**, and t.*

Beer-Bouguer-Lambert Law



- For a plane-parallel beam travelling through an absorbing medium, over small distances, the behaviour is linear and follows the Beer-Bouguer-Lambert Law (or Beer's Law).
  - → See Perrin, Whose Absorption Law?, *J. Optical Society of America*, 38(1), 72-74, 1948.

- Loss of the beam intensity *dI* is proportional to the **amount of material traversed**, $\rho\,ds$, where ρ is the density of the absorber:

$$dI_{\bar{\nu}} = - I_{\bar{\nu}}\, k_{\bar{\nu}}\, \rho\, ds$$

coupling coefficient

Beer-Bouguer-Lambert Law

- For the case of:
  - $\rightarrow$ monochromatic radiation
  - $\rightarrow$ an absorber of uniform density

  we can integrate Beer's Law over a finite distance from 0 to s:

transmission τ

$$I_{\bar{\nu}}(s) = I_{\bar{\nu}}(0)\exp\left(-\int_0^s k_{\bar{\nu}}\,\rho\,ds\right) = \underbrace{I_{\bar{\nu}}(0)\,e^{-k_{\bar{\nu}}\,\rho\,s}}_{\text{familiar (?) exponential decay law of transmission through uniform material}} = I_{\bar{\nu}}(0)\,e^{-\chi_{\bar{\nu}}} = I_{\bar{\nu}}(0)\,\tau_{\bar{\nu}}$$

optical depth χ

- This is the sort of thing we see in the laboratory as well as the atmosphere.

- **Note: Some textbooks use τ_λ for optical depth and T_λ for transmission.**

The Extinction Coefficient

What is k ?

extinction coefficient – in the sense that it extinguishes the beam through scattering and/or absorption processes

- $\boldsymbol{k\,\rho\,dx}$ must be a "number" - meaning no units.
  - $\rightarrow$ There are several sets of units which can be used …

- Note that the product $\boldsymbol{\rho\,dx}$ is often used - this is the area density unit or column, for example, molecules cm^{-2}. It represents the amount of material in the path and usually has symbol **u** or **du**.

Mass and Volume Extinction Coefficients

- Most common units for the extinction coefficient, k:
  - → **k in cm^2 molecule^{-1}** and ρ **in molecules cm^{-3}** ("number density")
  - → or in SI units, with length dx in m: **k in m^2 kg^{-1}** and ρ **in kg m^{-3}**

- Cross-section σ (units of cm^2 molecule^{-1}) is also used to describe the strength of absorption, i.e., σ = k as we have used it.
  - → ***There is a lot of inconsistency in the use of σ and k! Be careful!***

- Whichever symbols are used, there are two absorption coefficients:
  - → the **mass absorption coefficient** or cross section has units of m^2 kg^{-1} or cm^2 molecule^{-1}
  - → the **volume absorption coefficient** has units of (m^{-1} or cm^{-1}) and equals the mass absorption coefficient $\times$ density

Extinction Processes – 1

There are a number of processes that can remove energy from the beam of radiation – roughly divided into four types:

- Simple scattering
  - → Most easily described as "billiard ball" dynamics (however complex form)
  - → Very short interaction, on the order of time for photon and object to collide
  - → Zero energy exchange between the photon and object
  - → Some spatial relation between incoming and outgoing photon
  - → Single photon in produces single photon out
- Resonant scattering
  - → Longer interaction time because an excited complex is formed
  - → $M + h\nu \rightarrow M^* \rightarrow M + h\nu$
  - → No correlation between direction of incoming and outgoing photons
  - → May be some shift in ν of the outgoing photon (energy exchange)
  - → Single photon in produces single photon out

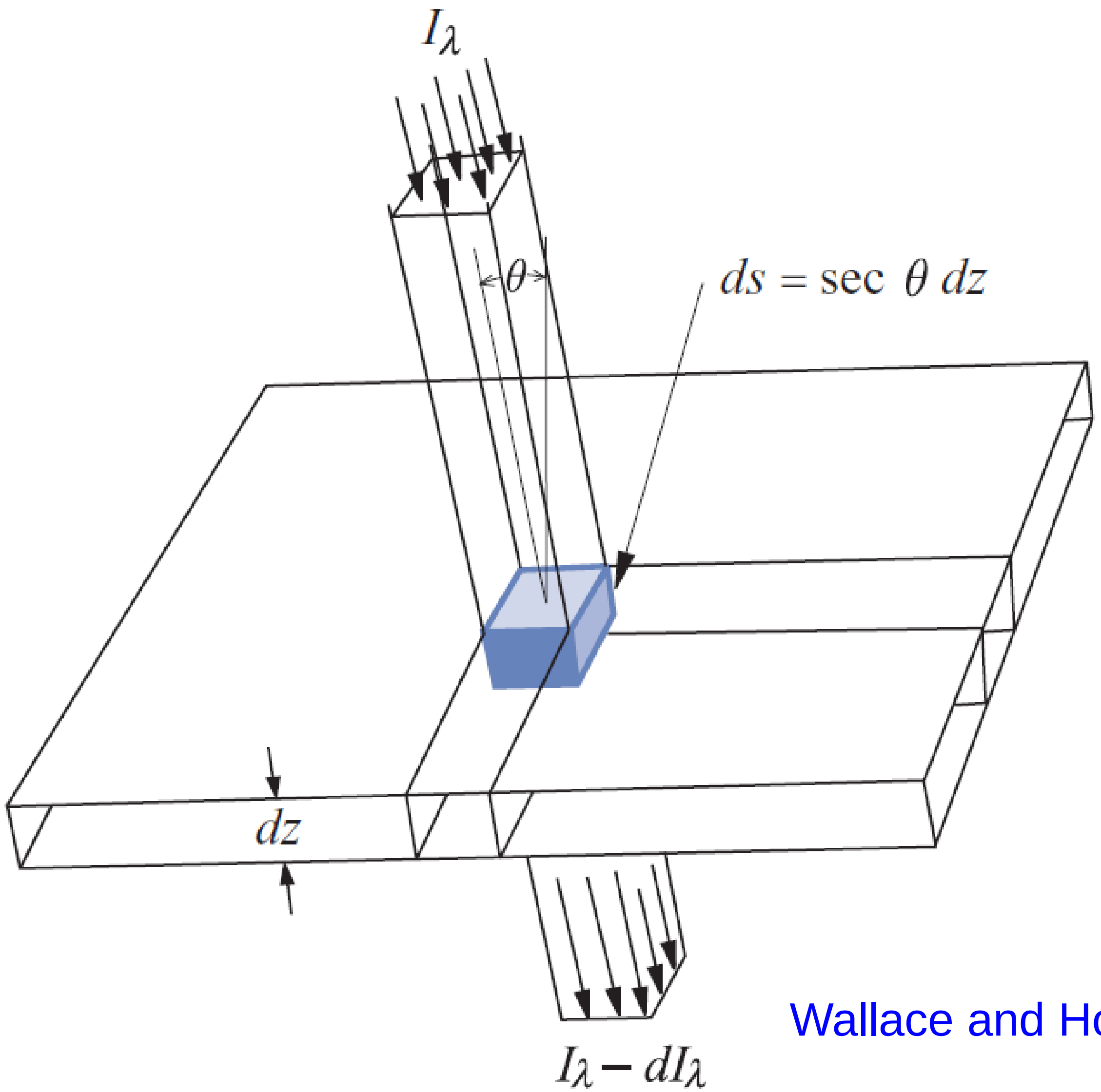
Extinction Processes – 2

- Multiprocess scattering
  - → Long interaction time because a complex is formed
  - → Decay of complex emits several photons, where the sum of energy of emitted photons is approximately that of incoming $h\nu \geq \Sigma\, h\nu_i$
  - → Single photon in produces multiple photons out
- Absorption
  - → No photons are produced
  - → After a short time, photon energy is converted into kinetic energy of the molecule

More on Beer's Law

See blackboard notes.

Beer's Law Applied to Atmosphere



Wallace and Hobbs, Figure 4.10

Beer's Law Applied to Atmosphere

- Let's integrate Beer's Law from the top of the atmosphere ($z=\infty$) down to any level z to determine what fraction of the incident radiation is absorbed or scattered, and what fraction transmitted.
- We have ds = sec θ dz, so: $dI_\lambda = -I_\lambda\, k_\lambda\, \rho \sec\theta\, dz$
- Integrate:

$$\int_{I_\lambda(\infty)}^{I_\lambda(z)} \frac{dI_\lambda}{I_\lambda} = \ln\frac{I_\lambda(z)}{I_\lambda(\infty)} = -\int_\infty^z k_\lambda\, \rho \sec\theta\, dz$$

Atmospheric paths are generally <u>inhomogeneous</u> - varying in some parameter(s) such as temperature and pressure, so need to keep the integral.

$$I_\lambda(z) = I_\lambda(\infty)\exp\left(-\int_\infty^z k_\lambda\, \rho \sec\theta\, dz\right) = I_\lambda(\infty)\, e^{-\chi_\lambda \sec\theta} = I_\lambda(\infty)\, \tau_\lambda$$

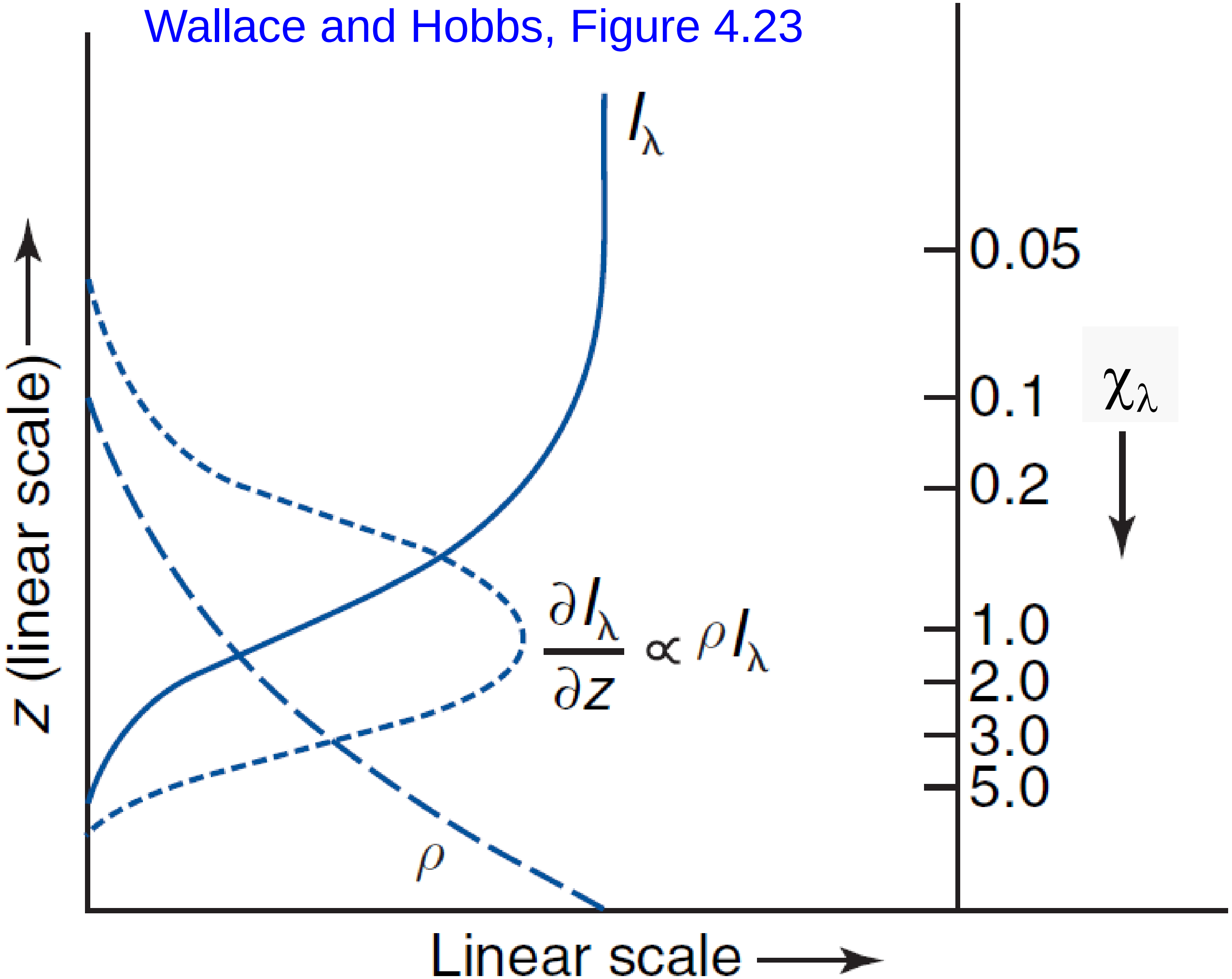
where

- χ_λ = <u>**normal** optical depth</u> or <u>optical thickness</u>
- τ_λ = transmissivity

If no scattering, absorptivity is: $\alpha_\lambda = 1 - \tau_\lambda = 1 - e^{-\chi_\lambda \sec\theta}$

Level of Unit Optical Depth

Wallace and Hobbs, Figure 4.23



See Wallace and Hobbs, solution to Exercise 4.44 for derivation.

The level of unit optical depth (χ_λ = 1) is the altitude at which the intensity of vertically incident radiation (and atmospheric transmission) decrease most rapidly – absorption is a maximum.

- Above χ_λ = 1, density is low.
- Below χ_λ = 1, intensity is low.