# PHY 140Y – FOUNDATIONS OF PHYSICS
# 2001-2002
# Term Test #1 – Make-Up Version – Solutions
## Friday, October 26, 2001
## 2:00 PM - 4:00 PM

## QUESTIONS:

1. Give BRIEF answers to each of the following. *[5 marks each for 20 total]*

(a) Define and <u>briefly</u> explain the difference between inertial and noninertial frames of reference. What is a fictitious force and why does it arise?

**<u>Solution:</u>**
Inertial frames of reference move at constant velocity. Non-inertial reference frames are accelerating.

Observers in two inertial reference frames may measure different velocities for an object, but they will measure the same acceleration and force on an object, i.e., velocities can be different in different inertial frames of reference, but accelerations and forces cannot be different in different inertial frames of reference.

An observer in a non-inertial frame of reference will measure velocities, accelerations, and forces on an object that are different from those seen by an observer in an inertial frame of reference.

e.g., $\vec{F}_B = \vec{F}_A + \vec{F}_f$

Observer B (non-inertial) sees a different force applied to object P than the force that observer A (inertial) sees. Observer B is mistaking his/her own acceleration for that of the object.

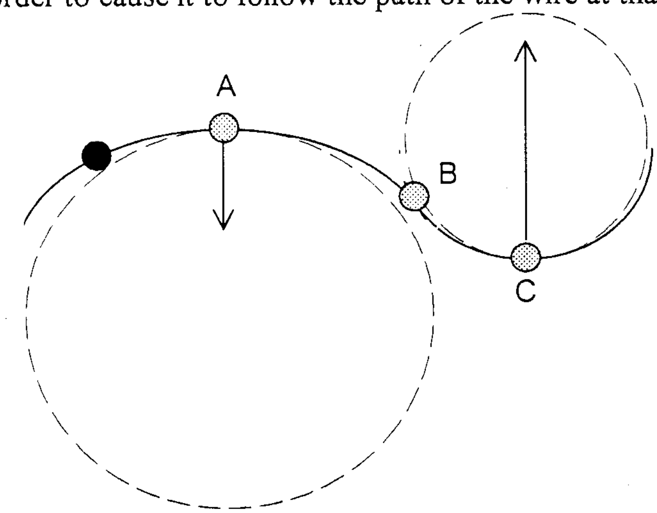
This error is the fictitious force $\vec{F}_f = m\vec{a}_B^A$ . Newton's Laws of motion do not hold in non-inertial frames of reference. If Newton's Second Law is applied in accelerating frames of reference, then the fictitious force must be introduced to ensure $\vec{F}_{net} = m\vec{a}$.

(b) A passenger in a car travelling at 60 km/hour pours a cup of coffee for the tired driver, with it taking the coffee 0.10 seconds to reach the cup. Describe the path of the coffee as it moves from a Thermos bottle into a cup as seen by (i) the passenger, and (ii) someone standing beside the road and looking in the window of the car as it drives past. (iii) What happens if the car accelerates while the coffee is being poured?

**<u>Solution:</u>**
(i) The passenger sees the coffee pouring nearly vertically into the cup, just as if s/he were standing on the ground pouring it.
(ii) The stationary observer sees the coffee moving in a parabolic path with a constant horizontal velocity of 60 km/hour (17 m/s) and a downward acceleration of g. If it takes the coffee 0.10 seconds to reach the cup, then the stationary observer sees the coffee moving 1.7 m horizontally before it hits the cup.
(iii) If the cars slows suddenly, the coffee reaches the place where the cup would have been had there been no change in velocity and continues falling because the cup has not yet reached that location.

If the car rapidly speeds up, then the coffee falls behind the cup. If the car accelerates sideways, then the coffee ends up somewhere other than in the cup.

(c) A bead slides freely along a curved wire at constant speed, as shown in the following overhead view. At each of the points A, B, and C, describe the magnitude and direction of the force that the wire exerts on the bead in order to cause it to follow the path of the wire at that point.




**Solution:**
At point A, the path is along the circumference of the larger circle. Therefore, the wire must be exerting a force on the bead directed towards the centre of the circle, i.e., downwards in the figure. Because the speed is constant, there is no tangential force component. At point B, the path is not curved and so the wire exerts no force on the bead. At point C, the path is again curved and so the wire again exerts a force on the bead. This time, the force is directed towards the centre of the smaller circle, i.e., upwards in the figure. Because the radius of this circle is smaller, the magnitude of the force exerted on the bead at C must be larger than it is at A (using $F = ma_r = mv^2/R$).

(d) A person steps from a boat towards a dock. Unfortunately s/he forgot to tie the boat to the dock, and the boot scoots away as s/he steps from it. Analyze this situation in terms of Newton's Third Law. Would the outcome be the same for a small dog jumping from the boat? Why or why not?

**Solution:**
As the person steps out of the boat, s/he pushes against it with her/his foot, expecting the boat to push back so that s/he accelerates towards the dock. However, because the boat is untied, the force exerted by the foot causes the boat to scoot away from the dock. As a result, the person is not able to exert a very large force on the boat before it move out of reach. Therefore, the boat does not exert a very large force on the person, who thus ends up not being accelerated sufficiently to make it to the dock. Consequently, the person falls into the water instead.

If a small dog were to jump from the untied boat towards the dock, the force exerted by the boat on the dog would probably be enough to ensure the dog's successful landing because of the dog's small mass.

***[Each of the following five questions is worth 16 marks.]***

2. A basketball is dropped from rest 3.0 m above the floor. After bouncing from the floor, the ball reaches a height of 1.5 m.
   (a) What is the speed of the ball the instant before hitting the floor?
   (b) What is the speed of the ball just after leaving the floor?
   (c) The ratio of the speed after the bounce to the speed before the bounce is called the coefficient of restitution ε of the ball. Find ε for this basketball.
   (d) If the ball is in contact with the floor for 0.025 s, what is the magnitude of the average acceleration of the ball during this time interval?

**Solution:**

a) Choose a coordinate system with $\hat{\imath}$ pointing up and origin on the floor. Then $x_0 = 3.0 \text{ m}$, $v_{x0} = 0 \text{ m/s}$, and $a_x = -9.81 \text{ m/s}^2$. The equation for the velocity component is:

$$v_x(t) = v_{x0} + a_x t = 0 \text{ m/s} - (9.81 \text{ m/s}^2)t = -(9.81 \text{ m/s}^2)t.$$

The equation for its position is

$$x(t) = x_0 + v_{x0}t + a_x\frac{t^2}{2} = 3.0 \text{ m} - 9.81 \text{ m/s}^2\,\frac{t^2}{2}.$$

When the ball hits the floor, the position is 0 m, so

$$0 \text{ m} = 3.0 \text{ m} - (9.81 \text{ m/s}^2)\frac{t^2}{2}.$$

Solve for $t$:

$$t = 0.78 \text{ s}.$$

Then the velocity component on impact is $v_x = (-9.81 \text{ m/s}^2)(0.78 \text{ s}) = -7.7 \text{ m/s}$. The *speed* of the ball on impact is the magnitude of the velocity:

$$v = 7.7 \text{ m/s}.$$

b) After impact, we have a new problem. Using the same coordinate system, we now have $x_0 = 0 \text{ m}$ and $a_x = -9.81 \text{ m/s}^2$. The equation for the velocity component is

$$v_x(t) = v_{x0} + a_x t = v_{x0} - (9.81 \text{ m/s}^2)t.$$

When the ball is at maximum height, its velocity component is zero, so

(1) $$0 \text{ m/s} = v_{x0} - (9.81 \text{ m/s}^2)t.$$

The equation for its position is

$$x(t) = x_0 + v_{x0}t + a_x\frac{t^2}{2} = v_{x0}t - (9.81 \text{ m/s}^2)\frac{t^2}{2}.$$

When the ball reaches its maximum height of 1.5 m, we have

(2) $$1.5 \text{ m} = v_{x0}t - (9.81 \text{ m/s}^2)\frac{t^2}{2}.$$

Solve equation (1) for $t$ and substitute into equation (2).

$$1.5 \text{ m} = v_{x0}\left(\frac{v_{x0}}{9.81 \text{ m/s}^2}\right) - (9.81 \text{ m/s}^2)\frac{\left(\frac{v_{x0}}{9.81 \text{ m/s}^2}\right)^2}{2}.$$

Now solve this equation for $v_{x0}$.

$$v_{x0} = 5.4 \text{ m/s}.$$

The *speed* of the ball just after the bounce is the magnitude of its the initial velocity, so $v = 5.4$ m/s.

c) The *coefficient of restitution* is the ratio of the speed just after the bounce to the speed just before the bounce:

$$\epsilon = \frac{5.4 \text{ m/s}}{7.7 \text{ m/s}} = 0.70.$$

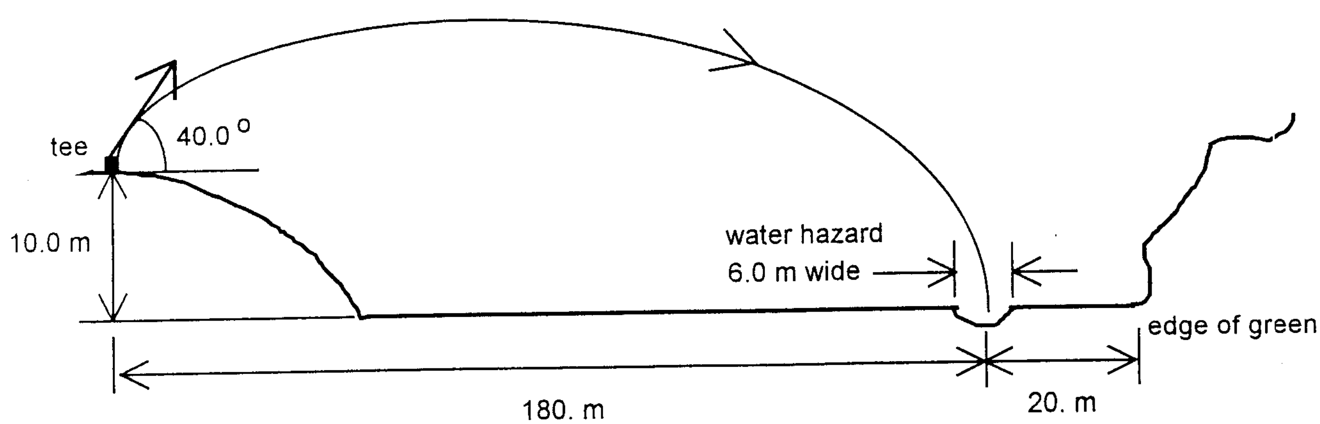
Note that if the ball returned to its original height, the coefficient of restitution would be one, and if the ball didn't bounce at all the coefficient of restitution would be zero. When $\epsilon = 1$, we say that the collision is "totally elastic," when $\epsilon = 0$ we say it is "totally inelastic." Thus, when the ball is dropped on a hard surface, $\epsilon$ is a measure of the elasticity of the ball.

d) The average acceleration of the ball during the short interval it is in contact with the floor is

$$\begin{aligned} \vec{a}_{ave} &= \frac{\Delta \vec{v}}{\Delta t} \\ &= \frac{\vec{v}_f - \vec{v}_i}{\Delta t} \\ &= \frac{(5.4 \text{ m/s})\hat{i} - (-7.7 \text{ m/s})\hat{i}}{0.025 \text{ s}} \\ &= (5.2 \times 10^2 \text{ m/s}^2)\hat{i}. \end{aligned}$$

The magnitude of the average acceleration is, therefore, $5.2 \times 10^2$ m/s$^2$.

3. The 18th green lies 200. m from a golf tee as indicated in the figure below. Golf pro Sandy Trappe clubs the ball heroically, and it leaves the tee making an angle of 40.0° with the horizontal. Unfortunately, the ball lands in a water hazard 180. m from the tee.
   (a) What was the speed of the golf ball as it left the tee?
   (b) If the ball left the tee at the same speed, but at an angle of 45.0° to the horizontal, could the ball have landed on the green?
   (c) Does the 45.0° launch angle ensure that the ball will travel the maximum horizontal distance? Explain your answer.



## Solution:

a) Choose the origin 10.0 m below the tee, with $\hat{\imath}$ pointing to the right and $\hat{\jmath}$ pointing up. Then

$$\begin{aligned} y_0 &= 10.0 \text{ m} & x_0 &= 0 \text{ m} \\ v_{y0} &= v_0 \sin 40.0^\circ & v_{x0} &= v_0 \cos 40.0^\circ \\ a_y &= -g & a_x &= 0 \text{ m/s}^2 . \end{aligned}$$

So, the equations for the velocity and position components are

$$\begin{aligned} v_y(t) &= v_0 \sin 40.0^\circ - gt & v_x(t) &= v_0 \cos 40.0^\circ \\ y(t) &= 10.0 \text{ m} + (v_0 \sin 40.0^\circ)t - g\frac{t^2}{2} & x(t) &= (v_0 \cos 40.0^\circ)t \end{aligned}$$

The $x$-coordinate of the impact point is 180 m. Use this information in the equation for $x$.

$$180 \text{ m} = (v_0 \cos 40.0^\circ)t \implies t = \frac{180 \text{ m}}{v_0 \cos 40.0^\circ} = \frac{235 \text{ m}}{v_0}$$

The $y$-coordinate of the impact is 0 m. Hence at impact time the equation for $y$ is

$$\begin{aligned} 0 \text{ m} = 10.0 \text{ m} + (v_0 \sin 40.0^\circ)t - g\frac{t^2}{2} &= 10.0 \text{ m} + (v_0 \sin 40.0^\circ)\frac{235 \text{ m}}{v_0} - g\frac{\left(\frac{235 \text{ m}}{v_0}\right)^2}{2} \\ &= 10.0 \text{ m} + 151 \text{ m} - g\frac{(235 \text{ m})^2}{2v_0^2} = 161 \text{ m} - g\frac{(235 \text{ m})^2}{2v_0^2} \end{aligned}$$

When we solve $0 \text{ m} = 161 \text{ m} - g\frac{(235 \text{ m})^2}{2v_0^2}$ for $v_0$, we find $v_0 = \sqrt{\frac{(9.81 \text{ m/s}^2)(235 \text{ m})^2}{2(161 \text{ m})}} = 41.0 \text{ m/s}.$

b) Since the initial speed is 41.0 m/s and the launch angle is 45.0°, then

$$\begin{aligned} y_0 &= 10.0 \text{ m} & x_0 &= 0 \text{ m} \\ v_{y0} &= (41.0 \text{ m/s}) \sin 45.0^\circ = 29.0 \text{ m/s} & v_{x0} &= (41.0 \text{ m/s}) \cos 45.0^\circ = 29.0 \text{ m/s} \\ a_y &= -g & a_x &= 0 \text{ m/s}^2 . \end{aligned}$$

So, the equations for the velocity and position components are

$$\begin{aligned} v_y(t) &= 29.0 \text{ m/s} - gt & v_x(t) &= 29.0 \text{ m/s} \\ y(t) &= 10.0 \text{ m} + (29.0 \text{ m/s})t - g\frac{t^2}{2} & x(t) &= (29.0 \text{ m/s})t \end{aligned}$$

Since impact occurs where $y = 0$ m, the equation for $y$ becomes

$$0 \text{ m} = 10.0 \text{ m} + (29.0 \text{ m/s})t - g\frac{t^2}{2}$$

Use the quadratic formula to solve this for $t$. Since impact occurs after $t = 0$ s, choose the positive root.

$$t = 6.24 \text{ s}.$$

The $x$-coordinate of the impact point is

$$x = (29.0 \text{ m/s})(6.24 \text{ s}) = 181 \text{ m}.$$

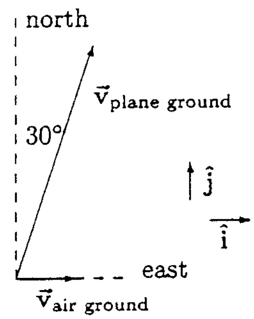
The ball travels only 1 m farther, and so it still falls prey to the water hazard and misses the green.

c) The 45.0° launch angle does not ensure maximum range. The 45.0° angle only yields maximum range if the ball lands at the same elevation as its launch.

4. You are flying an airplane in a strong wind. An air traffic controller reports your velocity to be 700. km/hour 30.0° east of north. The wind is reported to be 120. km/hour due east.
   (a) Sketch the situation and introduce an appropriate coordinate system.
   (b) What is the velocity of the air with respect to the airplane?
   (c) What is the speed of the air with respect to the airplane?

**Solution:**

a) The air traffic controller measures the speed of the plane with respect to the ground. The wind is blowing at 120 km/h towards east. (It is blowing from the west). Here's the picture.




We've chosen the coordinate system so that $\hat{\imath}$ points east and $\hat{\jmath}$ points north.

b) The velocity of the air with respect to the ground is

$$\vec{v}_{\text{air ground}} = (120 \text{ km/h})\hat{\imath}.$$

The velocity of the plane with respect to the ground is

$$\vec{v}_{\text{plane ground}} = (700 \text{ km/h})(\sin 30.0°)\hat{\imath} + (700 \text{ km/h})(\cos 30.0°)\hat{\jmath} = (350 \text{ km/h})\hat{\imath} + (606 \text{ km/h})\hat{\jmath}.$$

The velocity of the air with respect to the plane is found from the relative velocity addition equation.

$$\vec{v}_{\text{air plane}} = \vec{v}_{\text{air ground}} + \vec{v}_{\text{ground plane}}$$

But

$$\vec{v}_{\text{ground plane}} = -\vec{v}_{\text{plane ground}}.$$

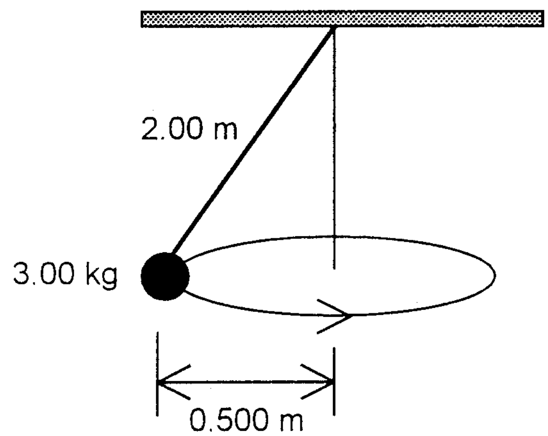
Hence the relative velocity addition equation becomes

$$\vec{v}_{\text{air plane}} = (120 \text{ km/h})\hat{\imath} - \left((350 \text{ km/h})\hat{\imath} + (606 \text{ km/h})\hat{\jmath}\right) = (-230 \text{ km/h})\hat{\imath} - (606 \text{ km/h})\hat{\jmath}.$$

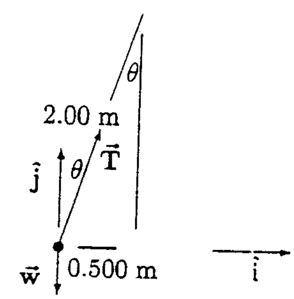
c) The speed of the air with respect to the plane is the magnitude of $\vec{v}_{\text{air plane}}$.

$$v_{\text{air plane}} = \sqrt{(-230 \text{ km/h})^2 + (-606 \text{ km/h})^2} = 648 \text{ km/h}.$$

5. A 3.00 kg mass attached to a string is swung around at constant speed in a circle of radius 0.500 m by means of a massless string of length 2.00 m, as indicated in the figure below.
(a) Draw the force diagram, indicating the forces on the mass at some instant.
(b) Is the total force on the mass zero? Explain.
(c) Find the magnitude of the tension in the string.
(d) What is the speed of the mass?
(e) What is the time needed for the mass to complete one circular trajectory?




**Solution:**




a) The forces acting on the system are:

1. its weight $\vec{w}$, equal in magnitude to $mg$, directed downward; and
2. the force $\vec{T}$ of the string on the system, directed along the string away from the mass.

Here's the second law force diagram, and a coordinate system.

b) Since the mass is in circular motion, there is a nonzero centripetal acceleration. From Newton's second law, there is a nonzero total force on the system.

c) Use the coordinate system and picture shown in part a). Apply Newton's second law in each direction. There is zero acceleration along the $\hat{j}$ direction, so

$$F_{y\ \text{total}} = ma_y \implies T\cos\theta - mg = 0\ \text{N} \implies T = \frac{mg}{\cos\theta}.$$

From the picture

$$\sin\theta = \frac{0.500\ \text{m}}{2.00\ \text{m}} = 0.250 \implies \cos\theta = \sqrt{1-(0.250)^2} \implies$$

$$T = \frac{mg}{\sqrt{1-(0.250)^2}} = \frac{(3.00\ \text{kg})(9.81\ \text{m/s}^2)}{\sqrt{1-(0.250)^2}} = 30.4\ \text{N}.$$

d) To find the speed of the mass, consider the force in the $\hat{i}$ direction. The acceleration in the $\hat{i}$ direction is the centripetal acceleration of magnitude $\frac{v^2}{r}$. Hence

$$F_{x\ \text{total}} = ma_x \implies T\sin\theta = m\frac{v^2}{r} \implies v = \sqrt{\frac{rT\sin\theta}{m}} = \sqrt{\frac{(0.500\ \text{m})(30.4\ \text{N})(0.250)}{3.00\ \text{kg}}} = 1.13\ \text{m/s}.$$

e) The time required to complete one circumference is

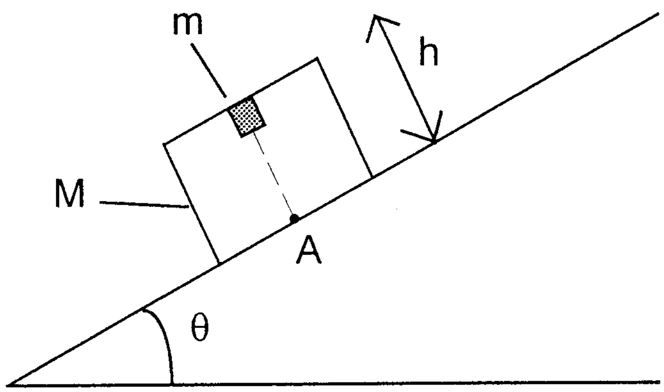
$$\frac{2\pi r}{v} = \frac{2\pi(0.500\ \text{m})}{1.13\ \text{m/s}} = 2.78\ \text{s}.$$

6. A block of mass M and height h slides down a slope as shown below. The slope is inclined at angle θ from the horizontal. Connected to the top of the box is an object of mass m. The object is directly above point A on the floor of the box, as shown.

(a) Draw free-body diagrams for mass m and for mass M and identify all forces acting on each at the point when mass m has just been released from the top of the box (i.e., ignore any normal force between the two masses).

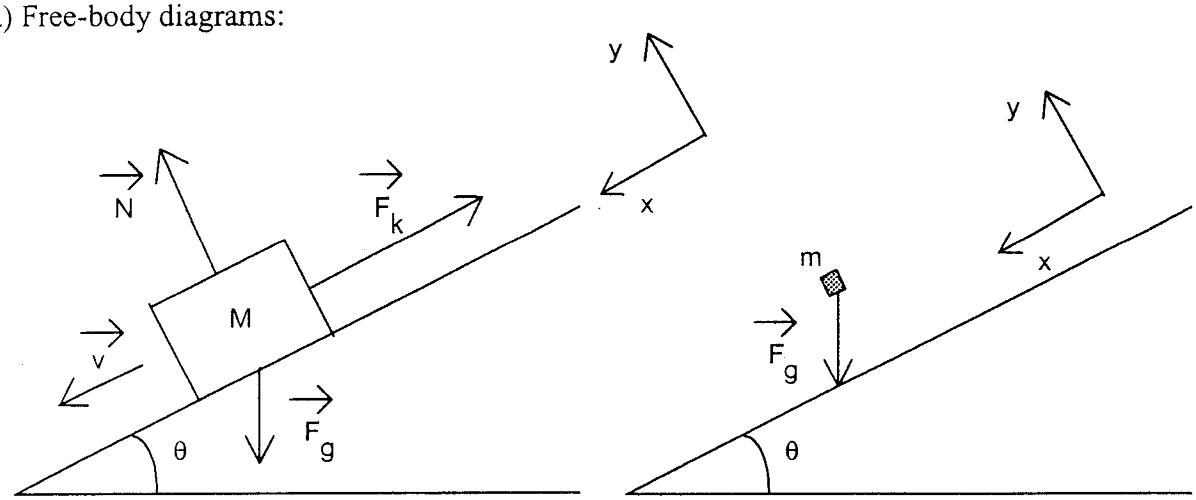
(b) If mass m drops from the top of the box, where will it fall relative to A when it reaches the floor, and how long will it take to fall, assuming that the slope is frictionless.

(c) Repeat part (b) for the case in which the coefficient of kinetic friction between the box and the slope is $\mu_k$.




**Solution:**

(a) Free-body diagrams:



(b) Apply Newton's Second Law to each mass, ignoring friction: $\vec{F}_{net} = \vec{F}_g + \vec{N} = m\vec{a}$

mass M, x direction:
$$Mg\sin\theta = Ma_x$$
$$a_x = g\sin\theta$$

mass M, y direction:
$$N - Mg\cos\theta = Ma_y = 0$$
$$N = Mg\cos\theta$$

mass m, x direction:
$$mg\sin\theta = ma_x$$
$$a_x = g\sin\theta$$

mass m, y direction:
$$-Mg\cos\theta = Ma_y = 0$$
$$a_y = -g\cos\theta$$

Since the acceleration in the x direction is the same for both masses, mass m will fall on point A when it reaches the floor.

To find the time that mass m takes to fall, start with the acceleration in the y direction.

$$a_y = -g\cos\theta$$
$$v_y(t) = v_{oy} + a_y t = v_{oy} - gt\cos\theta$$
$$y(t) = y_o + v_{oy}t + \tfrac{1}{2}a_y t^2 = y_o + v_{oy}t - \tfrac{1}{2}gt^2\cos\theta$$

$$y(t) - y_o = v_{oy}t - \tfrac{1}{2}gt^2\cos\theta$$

Thus: $-h = (0)t - \tfrac{1}{2}gt^2\cos\theta$

$$t = \sqrt{\frac{2h}{g\cos\theta}}$$

(c) Now reapply Newton's Second Law to each mass, including kinetic friction:

$$\vec{F}_{net} = \vec{F}_g + \vec{N} + \vec{F}_k = m\vec{a}$$

mass M, y direction:
$$N - Mg\cos\theta = Ma_y = 0$$
$$N = Mg\cos\theta$$
(same as in part (b))

mass M, x direction:
$$Mg\sin\theta - \mu_k N = Ma_x$$
$$Mg\sin\theta - \mu_k Mg\cos\theta = Ma_x$$
$$a_x^M = g\sin\theta - \mu_k g\cos\theta$$

mass m, x direction:

$$mg \sin\theta = ma_x$$
$$a_x^m = g \sin\theta$$

(same as in part (b))

mass m, y direction:

$$-Mg \cos\theta = Ma_y = 0$$
$$a_y = -g\cos\theta$$

(same as in part (b))

Since the acceleration in the y direction is the same as in part (b), the time taken for mass m to fall to the floor will be the same: $t = \sqrt{\dfrac{2h}{g\cos\theta}}$

The acceleration in the x direction is no longer the same for both masses, mass m will not fall on point A when it reaches the floor. Mass m will acclerate more quickly and so it will fall some distance $\Delta x$ beyond point A.

Need to consider the difference in accleration between the two masses, i.e., their relative acceleration. Note that $x_o$ and $v_{ox}$ are the same for both masses.

$$\Delta a_x = a_x^m - a_x^M = g\sin\theta - (g\sin\theta - \mu_k g\cos\theta) = \mu_k g\cos\theta$$
$$\Delta v_x(t) = \Delta v_{ox} + \Delta a_x t = \mu_k gt\cos\theta$$
$$\Delta x(t) = \Delta x_o + \Delta v_{ox} t + \tfrac{1}{2}\Delta a_x t^2 = \tfrac{1}{2}\mu_k g t^2 \cos\theta$$
$$= \tfrac{1}{2}\mu_k g\left(\sqrt{\frac{2h}{g\cos\theta}}\right)^2 \cos\theta$$
$$= \mu_k h$$

So mass m will fall a distance $\mu_k h$ downslope of point A when friction is present.

**END**