PHY 140Y – FOUNDATIONS OF PHYSICS
2001-2002
Problem Set #2

HANDED OUT: Friday, October 5, 2001 (in class).

DUE: <u>**5:00 PM, Thursday, October 18, 2001**</u> in the appropriate box, labeled by tutorial group, in the basement at the bottom of the stairs leading down from MP202.

LATE PENALTY: 5 marks/day (which also applies to weekend days!) until 1:00 PM, Monday, October 22, after which it will not be accepted as solutions will then be available in tutorials and on the WWW.

NOTES: Answer all questions. A selected subset (3-4) will be marked out of 100%.
Marks will be given for workings and units, as well as for final answers.

QUESTIONS:

1. A space station 120 m in diameter is set rotating in order to give its occupants "artificial gravity". Over a period of 5.0 minutes, small rockets bring the station steadily to its final rotation rate of 1 revolution every 20. seconds. What are the radial and tangential accelerations of a point on the rim of the station 2.0 minutes after the rockets start firing?

2. You may have noticed that as a cassette tape is played, the radius of the tape left on the supply spool decreases slowly with time as the tape is drawn through the machine at a constant speed v_o. Because the radius of the tape remaining changes slowly compared with the speed of the tape through the machine, to a good approximation we can write:

$$v_o = r\frac{d\theta}{dt} = r\omega > 0$$

where ω is the angular speed of the tape reel. Notice that the tape speed v_o is constant, but radius r changes slowly with time. Therefore the angular speed of the reel also changes slowly with time. Assume that r represents the average radius of each coiled layer (since the tape is actually wound in a spiral). Let r_o be the initial radius of the tape when t = 0 s and $\theta_o = 0$ rad, where θ is the total angle of tape unwound from the spool. Represent the thickness of the tape by b. When the tape has unwound through a total angle θ, the radius of tape remaining is

$$r = r_o - b\frac{\theta}{2\pi}.$$

(a) Show that $\frac{dr}{dt} = -\frac{b}{2\pi}\frac{d\theta}{dt} = -\frac{b}{2\pi}\frac{v_o}{r}$.

(b) Integrate the result of part (a) to show that $\pi(r_o^2 - r^2) = bv_o t$.

What is the geometrical interpretation of the left-hand side of this equation? What is the physical interpretation of the right-hand side of this equation? Illustrate each with a sketch.

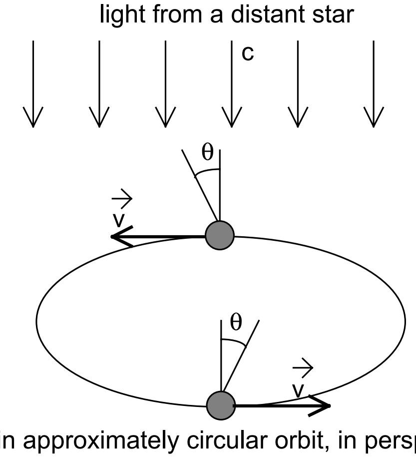
(c) Solve the result of part (b) for r and substitute this for r in Equation (1). Integrate the resulting expression to show that the total angle θ through which the tape has turned at time t is

$$\theta = \frac{2\pi}{b}\left\{ r_o - \sqrt{r_o^2 - \frac{b v_o t}{\pi}} \right\}.$$

Hint: The following integration formula may be useful: $\int \frac{-a}{\sqrt{b - at}}\,dt = 2\sqrt{b - at} + C$.

3. A kayaker attempts to cross a river in which the current has a uniform speed of 5.0 km/hr across the 50.0 m width of the river. The kayaker is paddling at the frenetic speed of 10.0 km/hr with respect to the water and wants to travel straight across the river to avoid a huge waterfall just downstream.
   (a) What is the resulting speed of the kayak with respect to the ground?
   (b) At what angle to the stream flow should the kayaker paddle so that the transit is made without moving upstream or downstream?
   (c) How long does the trip take?

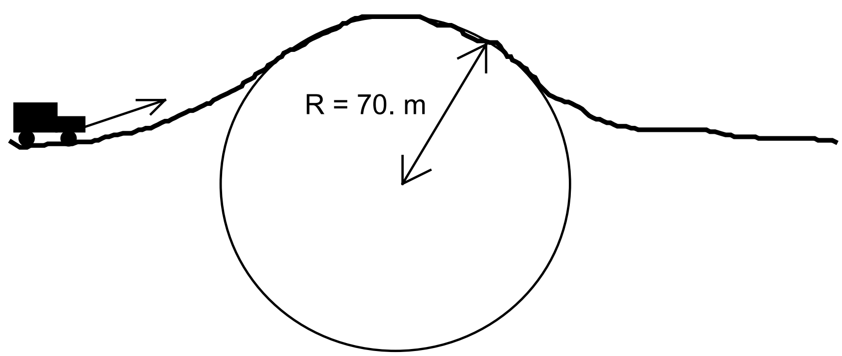
4. Imagine snow falling vertically downward at constant speed v_s. A car is driving horizontally in a straight line through the snow at constant speed v.
   (a) Show that according to the driver, the snow appears to fall along a direction that makes an angle θ with the vertical direction, where $\tan\theta = v / v_s$. Include a sketch of the velocity vectors. If you tilt an umbrella at this angle while walking at speed v in snow (or rain) falling vertically at speed v_s, you keep driest.
   (b) This problem also has implications in astronomy. Imagine the snow to be starlight moving at constant speed c towards the solar system, as shown in the figure below. The Earth, in its orbital motion, travels at speed v. Observers on the Earth see the light coming from a direction θ that is not in the direction of the true position of the star. The angle is found from $\tan\theta = v / c$. Thus a telescope must be tilted in the direction of the orbital velocity of the Earth by an angle θ to see the star. Over the course of a year, the star executes a small circle of angular radius θ about its true position perpendicular to the plane of the orbit of the Earth. This effect is known as *stellar aberration*. The orbital speed of the Earth is about 30 km/s. Evaluate the angle θ. If the star is in the plane of the orbit, then the aberrational path of the star is a line centered on the true position of the star. If the star is between the plane of the orbit and the perpendicular to the orbit, then the aberrational path of the star is an ellipse.



Earth in approximately circular orbit, in perspective but not to scale

5. When the starting gun fires, a 60.0 kg sprinter accelerates with a constant acceleration for the first 10.0 m of the track and then runs with constant speed for the duration of the race, completing the 100. m sprint in 10.0 s. What is the magnitude of the total force causing the acceleration of the sprinter?

6. A car travelling at constant speed reaches the crest of a huge hill that is a cylindrical cross section of radius 70. m as shown in the figure below.
   (a) At the instant when the car is at the crest of the hill, make a Second Law force diagram indicating schematically all forces acting on the car.
   (b) What is the maximum speed that the car can travel without becoming airborne at the crest of the hill? Express your result in km/hr.

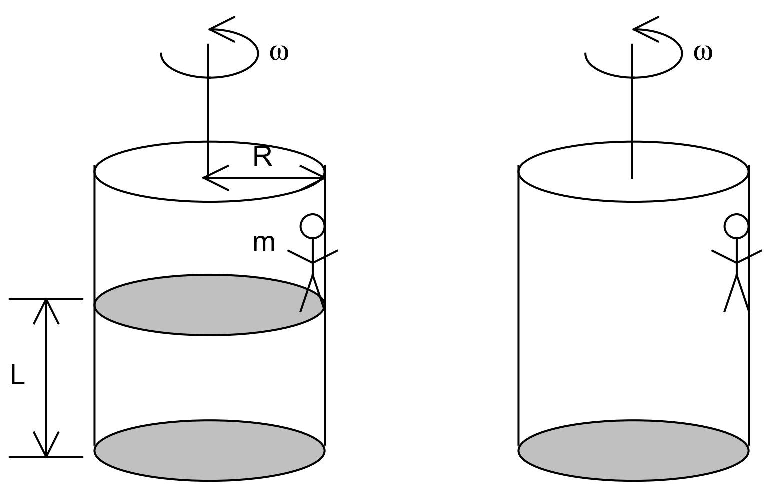


7. A large Ferris wheel of radius 15.0 m is rotating wildly out of control at 4.00 rev/min. A student of mass 80.0 kg is becoming increasingly uncomfortable with the ride. On the other hand, you (of mass 50.0 kg) are rather enjoying the wild ride. To calm your companion's panic, you decide to explain soothingly the physics of the effects that you are both experiencing on the Ferris wheel. You first point out that during the entire ride, there are only two forces acting on each of you: (1) your weight, and (2) the normal force of the bench seat on your backside.
   (a) When the chair is at the top of the circle, which of the two forces has the greater magnitude? What is the magnitude of the difference between your weight and the normal force of the seat on your backside?
   (b) When the chair is at the bottom of the circle, which of the two forces has the greater magnitude? What is the difference in the magnitudes of the forces on you at this location?
   (c) When the chair is not at the top or bottom of the circle, the two forces are no longer antiparallel to each other, since the chair swivels. At what two places in the motion is the deviation of the direction of the normal force from the local vertical a maximum? At these locations, what is the angle θ between the direction of this force and the local vertical direction?
   (d) At what angular speed must the wheel rotate so that you are both apparently "weightless" at the top of the spin? Give the result in rev/min. Are you truly weightless at the top of the spin? Explain why or why not.

Hints: You may need to use the Law of Cosines. Also, an extremum of function f(x) can be found using df(x)/dx=0.

8. A string made from braided dental floss has a maximum permissible tension of 500. N and is used by a 60.0 kg inmate to escape prison (apparently this actually happened!). The inmate starts at rest and slides vertically down the string for 8.0 m.
(a) Draw a force diagram indicating all forces acting on the inmate in the descent.
(b) Determine the magnitude of the acceleration of the inmate so that the string barely does not break. Is this a minimum or a maximum value? Explain.
(c) What is the maximum speed of the former inmate at the end of the 8.0 m descent if the string does not break?

9. Amusement parks occasionally have a large cylindrical tube mounted to spin about the vertical axis of the cylinder as shown in the figure below. Patrons stand against the wall of the cylinder and it is set into rotation. At a certain critical angular velocity, the floor on which the patrons are standing is lowered and the patrons are left hanging on the wall (but on the verge of slipping) due to the force of static friction on them. Let R be the radius of the cylinder, m the mass of one patron, and μ_s the coefficient of static friction for this patron on the wall.
(a) Draw the force diagram for the patron, indicating all forces acting on the patron.
(b) Is there an acceleration of the patron in the vertical direction? What does this imply about the relationship between the static frictionless force on the patron and the weight of the patron? Are these forces a Newton's Third Law pair?
(c) Is there an acceleration of the patron in the horizontal direction? How is this acceleration related to the speed of the patron?
(d) How is the angular velocity ω of the cylinder related to the tangential speed of the patron? If the patron is ready to slip, find the angular velocity ω of the cylinder in terms of R, μ_s, and g. Express your answer in rev/s.
(e) If the floor of the cylinder drops a distance L, how long does it take for the patron to slide down the distance L? Consider the coefficient of kinetic friction to be μ_k.



10. Two masses, m_1 and m_2, are connected by an ideal cord as shown in the figure below. The cord passes over an ideal pulley. A student determines that when the surface is inclined at an angle θ to the horizontal, m_1 is just on the verge of slipping to the right up the incline.

(a) Draw a force diagram showing all forces acting on mass m_2.

(b) Draw a force diagram showing all forces acting on mass m_1.

(c) Introduce appropriate coordinate system(s) and show that the coefficient of static frictions for m_1 on the inclined plane is

$$\mu_s = \frac{|m_2 - m_1 \sin\theta|}{m_1 \cos\theta}.$$

Now the inclination angle of the inclined plane is increased to angle ϕ so that the mass m_1 is just on the verge of slipping to the left down the incline.

(d) Draw a force diagram showing all forces acting on mass m_2.

(e) Draw a force diagram showing all forces acting on mass m_1.

(f) Show that the coefficient of static frictions for m_1 on the inclined plane is now

$$\mu_s = \frac{|m_1 \sin\phi - m_2|}{m_1 \cos\phi}.$$

Notice that if the suspended mass $m_2 = 0$ kg, then $\phi = 0$ and this equation reduces to

$$\mu_s = \tan\phi = \tan\theta.$$

(g) Is this the same coefficient of friction as that determined in part (c)? Why or why not?

(h) Show that the ratio of the two masses is given by the rather elegant expression

$$\frac{m_2}{m_1} = \frac{\tan\theta + \tan\phi}{\sec\theta + \sec\phi}.$$