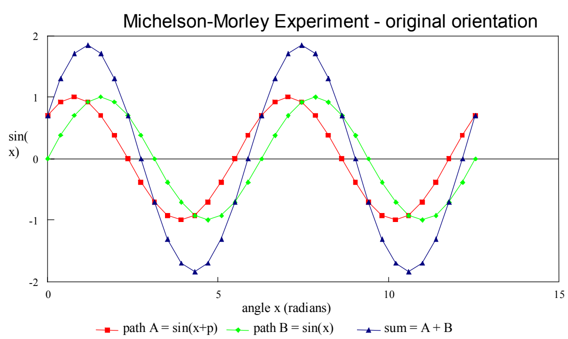
LECTURE #28 – SUMMARY

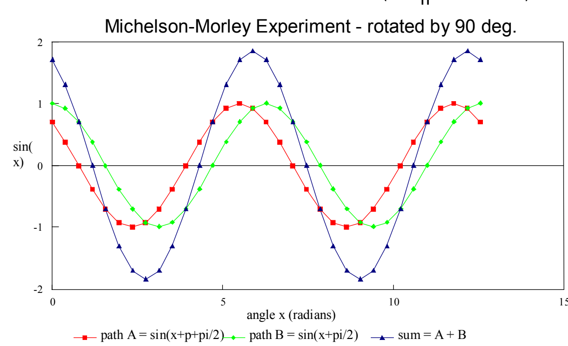
Michelson-Morley Experiment *…continued*

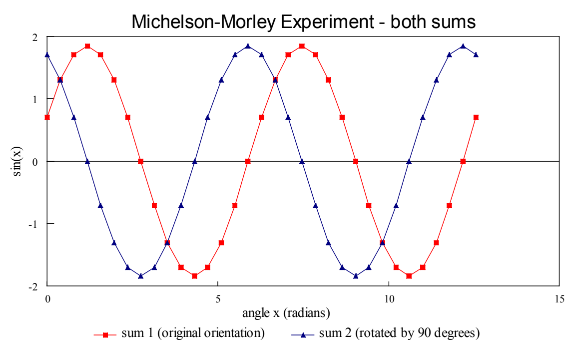
Now, we don't know which way the ether is directed. Also, we can only observe the combination of the two light beams (waves) - we can't distinguish them individually. The combination generates interference fringes of dark and bright bands. Michelson and Morley rotated their interferometer by 90°, interchanging the role of the two light paths. They observed the combination of the two beams in the original orientation and their combination after rotation by 90°.

Rotation should shift the interference fringes. Total path difference is $\Delta d = 2\frac{Lv^2}{c^2}$.

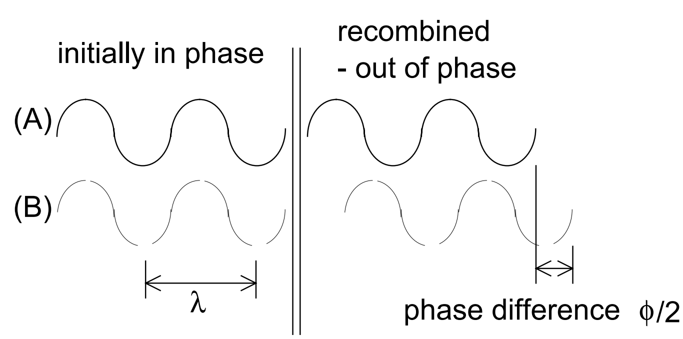
The *change in the time difference* between the orientations is $2(\Delta t_{\|} - \Delta t_{\perp})$.







The two beams are waves - let's say that they start in phase. A change in pathlength of one λ corresponds to a shift of one fringe. Let φ = the phase shift between the two combined beams:



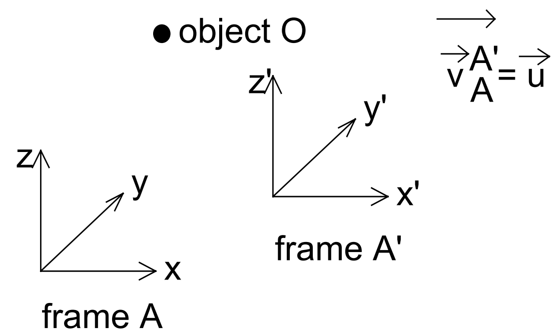
$$\phi = \frac{\Delta d}{\lambda} = 2\frac{Lv^2}{\lambda c^2}$$

Michelson and Morley predicted φ = 0.4 using their interferometer, with L = 11 m, $\lambda = 5.90 \times 10^{-7}$ m (590 nm), and v = orbital speed of Earth, so that $v/c = 10^{-4}$. They were capable of observing a shift of 0.01. However, they detected no phase shift! Thus, there was no experimental evidence for the ether wind. The results are consistent with the later STR.

Note: This experiment does not prove STR. Einstein said that no number of experiments could prove STR correct, but a single experiment could prove STR wrong. Such an experiment has yet to be found.

Constancy of the speed of light:
The speed of light in a vacuum has the same value in all inertial frames of reference (regardless of the velocity of the observer or the velocity of the source emitting the light).

V.3 Lorentz Transformations
The Galilean Transformations



Define two inertial reference frames A and A'.

$\vec{v}_A^O = \vec{v}_{A'}^O + \vec{v}_A^{A'} = \vec{v}_{A'}^O + \vec{u}$

$\vec{v}_A = \vec{v}_{A'} + \vec{u}$

$\vec{v} \equiv \vec{v}_A$ and $\vec{v}' \equiv \vec{v}_{A'}$ $\boxed{\therefore \vec{v} = \vec{v}' + \vec{u}}$

With "Newton's relativity", we have
(for A' moving in +x direction w.r.t. A, and for A moving in –x direction w.r.t. A'):

(1) absolutes: $t = t'$, $m = m'$, $a = a'$, $F = F'$, $L = L'$

(2) transformations:
$\vec{r} = \vec{r}' + \vec{u}t \quad \vec{r}' = \vec{r} - \vec{u}t$

$\vec{v} = \vec{v}' + \vec{u} \quad \vec{v}' = \vec{v} - \vec{u}$

$\vec{a} = \vec{a}' \quad \vec{a}' = \vec{a}$

$\vec{F} = \vec{F}' \quad \vec{F}' = \vec{F}$

There is an implicit assumption here: $\boxed{t = t'}$

$\vec{r} = \vec{r}' + \vec{u}t$

e.g., $\frac{d\vec{r}}{dt} = \frac{d\vec{r}'}{dt} + \vec{u}$ and $\vec{v}' \equiv \frac{d\vec{r}'}{dt'}$ so implicitly assumed $t=t'$ to get $\vec{v} = \vec{v}' + \vec{u}$

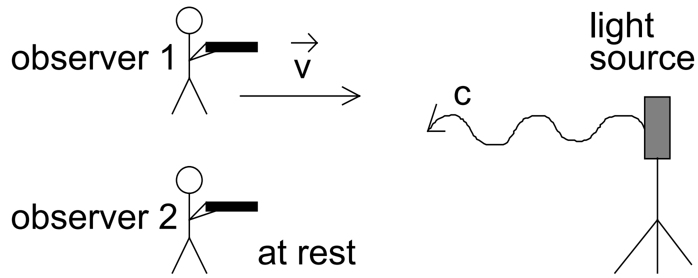
$\vec{v} = \frac{d\vec{r}'}{dt} + \vec{u}$

Implications for Space and Time

Rephrase the "speed of light" assertion: "In every observation of the passage of light from one point to another through empty space, the time taken is simply the relative separation of the points divided by a universal velocity, c."
This statement explicitly links space and time.

If c is constant for all observers, then our notions of space and time must depend on the observer!

Consider the following example…



Let's say that two observers, having metre sticks of length L=1 m, as measured by both of them, are looking at a light source.

In reference frame 2, the time taken for light to traverse the metre stick (which is at rest in frame 2), as measured by observer 2 is $\Delta t_2 = L/c$.
In reference frame 1, the time taken for light to traverse the metre stick (which is at rest in frame 1), as measured by observer 1 is $\Delta t_1 = L/c$.
We still use c because c is the same in all inertial reference frames.

"Common sense" tells us that time on a moving person's stop watch (1) will be shorter than time on a stationary person's stop watch (2), because the person is moving (actually, we might have thought that $\Delta t_1 = L/(c+v)$ but we know from STR that this is not true).

What is wrong? We have been assuming that the concept of space and time is the same in both reference frames. This is incorrect.
Time and space are altered in a way which permits all the laws of physics to be the same in all inertial reference frames. Length and time are not absolutes.

Observer 2 does not measure observer 1's stick to be 1 m.
∴ the time taken for light to traverse the metre stick in frame 1 is not the same, as measured in frame 1 (L/c) as it is when measured from frame 2.

If time and space are coupled, then the Galilean transformations (which assume t=t', L=L') are no longer valid.

Lorentz Transformations
We need a new set of transformations which are appropriate to the postulates of special relativity. → Lorentz transformations

$$\boxed{\begin{aligned} x' &= \gamma(x - vt) \\ x &= \gamma(x' + vt') \end{aligned}} \quad \text{and} \quad \boxed{\begin{aligned} t' &= \gamma\left(t - \frac{vx}{c^2}\right) \\ t &= \gamma\left(t' + \frac{vx'}{c^2}\right) \end{aligned}} \quad \text{with} \quad \boxed{\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}}$$

where
t, L are measured in reference frame A, at rest
t', L' are measured in reference frame A', moving at velocity v with respect to