LECTURE #18 – SUMMARY

Non-Constant Forces - Springs

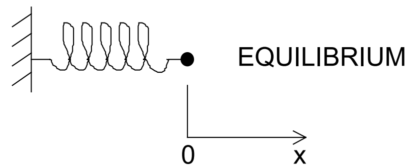
When stretched, a spring exerts tension forces to oppose the stretching.
When compressed, a spring exerts compression forces to oppose compression.
The force exerted by a spring is proportional to distance stretched or compressed.

Hooke's Law (for an ideal spring): $\boxed{F_o^s = -kx}$

where F_o^s = force exerted by spring (on object), k = spring constant (units of N/m), x = distance the spring is stretched or compressed from equilibrium, and "-" sign indicates that the spring force is opposite the stretching or compression.

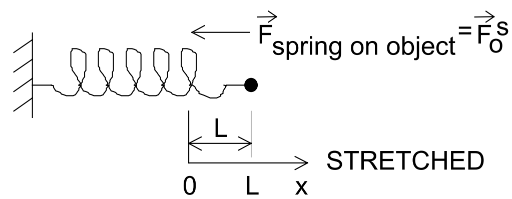
By Newton's Third Law, we can calculate the force applied to the spring.

$$F_s^o = +kx \qquad \text{(from } F_o^s + F_s^o = 0\text{)}$$



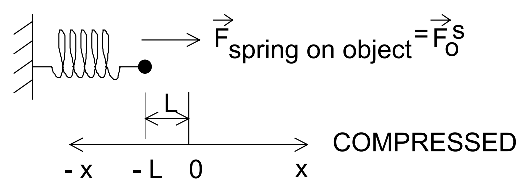
Work done in stretching spring from x=0 to x=L:

$$W = \int_{x_i}^{x_f} F_x dx = \int_0^L F_s^o dx = \int_0^L kx dx = \tfrac{1}{2}kx^2\Big|_0^L = \tfrac{1}{2}kL^2$$



Work done in compressing spring from x=0 to x=-L:

$$W = \int_0^{-L} F_s^o dx = \int_0^{-L} kx dx = \tfrac{1}{2}kx^2\Big|_0^{-L} = \tfrac{1}{2}kL^2 \text{ (same)}$$

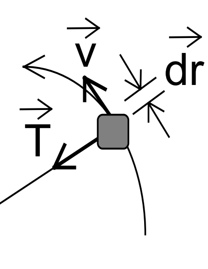


In order to apply the Work-Energy Theorem, we need to include the force that the wall exerts on the spring. By Newton's Second Law, $|F_s^w| = |F_s^o|$, so the net force is zero. This is consistent with $K_f = K_i$.

Non-Constant Forces - Uniform Circular Motion

Consider an object being swung on a string, with no gravity.

$$W_{net} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}_{net} \bullet d\vec{r} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{T} \bullet d\vec{r}$$



$\vec{T} \perp d\vec{r} \qquad \therefore \vec{T} \bullet d\vec{r} = 0$, so $W_{net} = 0$ for UCM

Alternatively, apply the Work-Energy Theorem:

$K = \frac{1}{2}mv^2$ = constant $\quad \therefore K_f = K_i$ and again $W_{net} = 0$

What about non-uniform circular motion?

- v is no longer constant and $\therefore W_{net} \neq 0$
- $\vec{a}_{||} \neq 0$ (changes speed) and $\therefore W_{net} \neq 0$