$E = mc^2$ on the Taipei 101 building

Albert Einstein Memorial, Washington, DC

Sculptor: Robert Berks

$E = mc^2$ on the Walk of Ideas in Berlin

Images from www.wikipedia.com

$E = mc^2$: Energy equals milk chocolate squared

Current Assignments ...

For today

- Read Sections 10.8

For Lecture 17

- Read Sections 11.1 - 11.7

Office hours:
2-3 Tuesday &
3-4 Thursday

Homework #3 (new due date)

- Handed out February 14, due **FRIDAY, MARCH 8**

Homework #4

- Handed out today. Due 11:00 AM, Friday, March 22

Writing Assignment #2

- Posted Feb. 28. Due 11:00 AM, Thursday, April 4

Suggested Conceptual Exercises

- Chapter 10: 21,25,27,31,35,37,39,41,43,47,49,51

Tutorial #7

Review of Lecture 15

Textbook, Sections 10.6 - 10.7

- Time travel
- The relativity of space - length contraction
- The relativity of mass

Plan for Lecture 16

Textbook, Sections 10.8

- Space-time diagrams and worldlines
- $E = mc^2$

"If a body releases the energy L in the form of radiation, its mass is decreased by L / V^2."

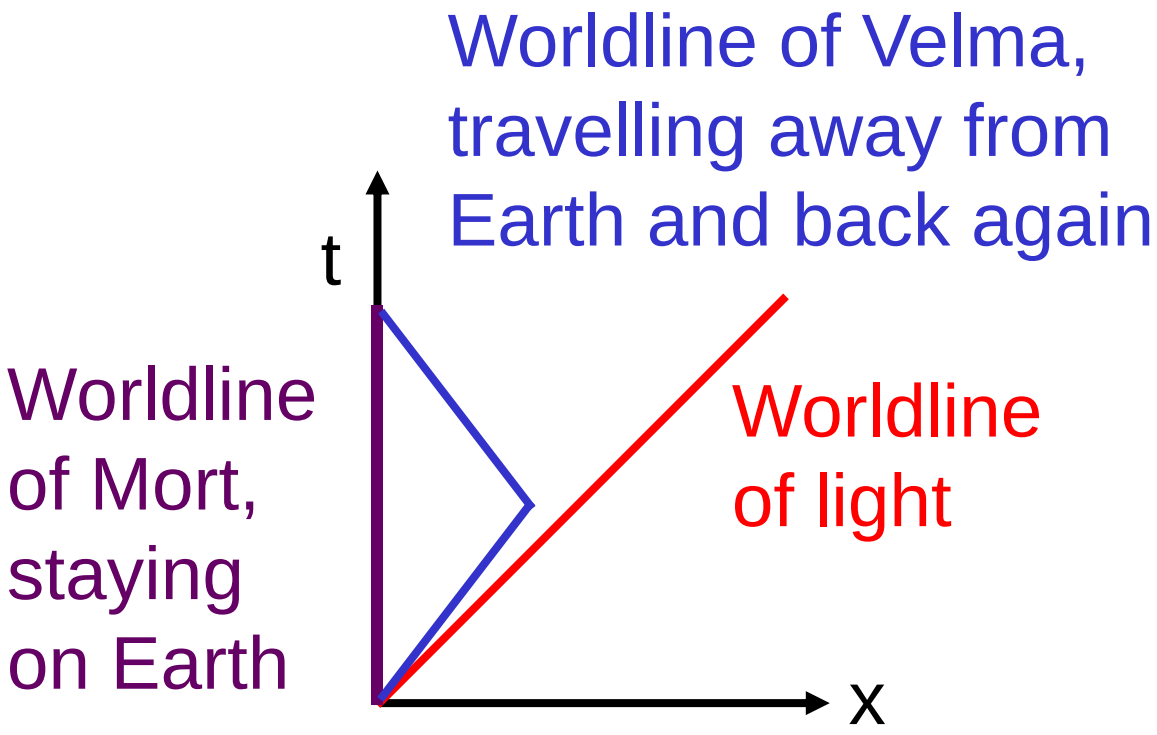
Albert Einstein (1879-1955)

from *Annalen der Physik*, 18, 639-641, 1905.

L = energy
V = lightspeed

Space-Time Diagrams

- Provide a visual illustration of the path of an object through space and time - its worldline
- Plot distance on x-axis and time on y-axis

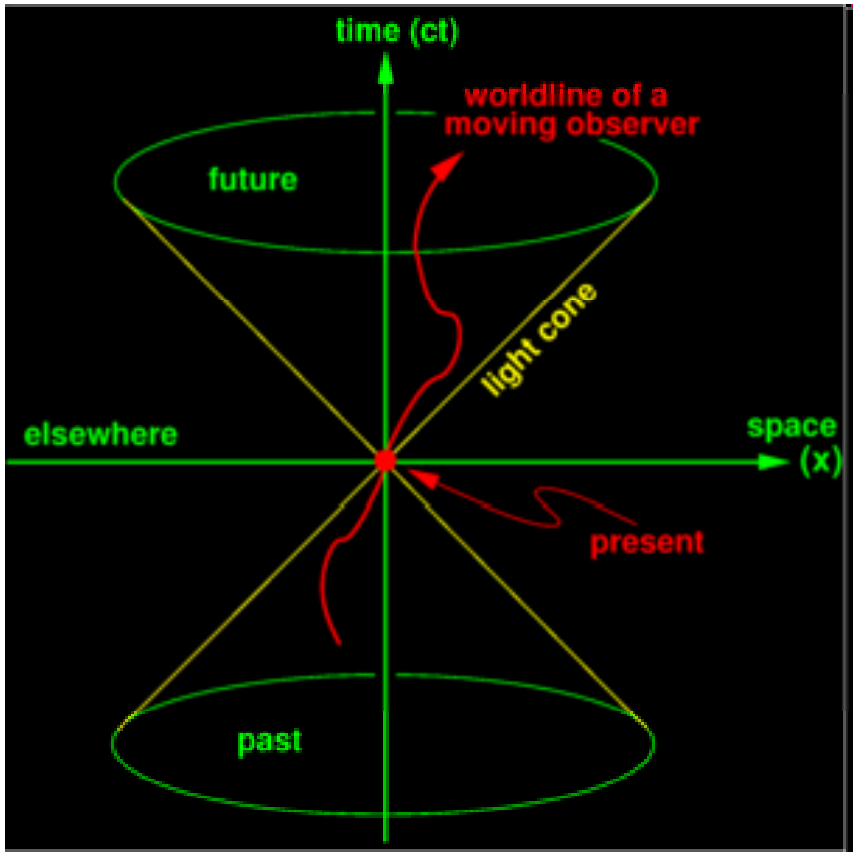


Space-time diagram for the twin "paradox"

More on Space-Time Diagrams: Light Cones

- This light cone diagram shows a 2D representation of 4D spacetime.
  - →The red line is the worldline of an observer moving in 2D.

http://einstein.stanford.edu/SPACETIME/spacetime2.html



What about two approaching spacecraft?

What about two approaching spacecraft?

- Is their relative speed 1.6c?
- Galilean relativity says yes, and would be wrong! Special relativity says no.
- Both time and space are transformed at high speeds - this results in the "Lorentz velocity transformation".

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{v_B - v_A}{1 - v_B v_A/c^2} = \frac{-0.85c - 0.75c}{1 - (-0.85c)(0.75c)/c^2} = -0.98c$$

where u' = speed of B with respect to A

Mass and Energy

- By combining Special Relativity and Conservation of Energy, Einstein found that any increase in the energy of an object results in an increase to its mass.
- The change in mass is equal to the change in energy, divided by the square of the speed of light: $\Delta m = \Delta E / c^2$
- The speed of light is very large, so the change in mass is undetectable in ordinary situations.

Nuclear Reactions

- This change in mass due to changes in energy becomes large in nuclear reactions.
- The energies are extremely large when nuclei are involved.
- The fission (splitting) of 1 kg of uranium decreases its mass by 1 gram, which is easily measured.
- This contradicts the conservation of matter – mass has actually been converted to energy.

$E = mc^2$

- Thus rest mass (= matter) is not conserved.
- Einstein extended the energy-mass relationship to all masses.
- In words: The total mass of a system is equal to the total energy of that system, divided by the square of the speed of light.

$$m = E/c^2 \quad \text{or} \quad \mathbf{E = mc^2}$$

- This gives the energy content of any mass.
- Rest mass alone is not conserved, but energy – including mass energy – is.

The Principle of Mass-Energy Equivalence

Energy has mass; that is, energy has inertia.

And mass has energy; that is, mass has the ability to do work.

The quantitative relation between the energy of any system and the mass of that system is $E = mc^2$.

$E = mc^2$ Again

- More strictly, all objects have an intrinsic, or rest, energy $\mathbf{E_o = m_o c^2}$
  where m_o is the rest mass of the object.
- This is true even if the object is stationary.
- If the object is moving at speed v, then the total energy = rest energy + kinetic energy:

$$\mathbf{E = mc^2 = \gamma m_o c^2 = m_o c^2 + K}.$$

- "$E=mc^2$" either means $E=m_oc^2$ for an object at rest, or $E=\gamma m_o c^2$ when the object is moving.

$E = mc^2$: In which of the following processes does the system's total mass change?

(A) An automobile speeds up

(B) A rubber ball is squeezed

(C) The total mass changes in both of the above processes

(D) The total mass changed in neither of the above processes.

An increase in energy causes an increase in total mass ($m = E/c^2$). Both A and B increase the total energy (kinetic and elastic) of the system, and so increase the mass.

$E = mc^2$: In which of the following processes does the system's rest mass change?

(A) An automobile speeds up

(B) A rubber ball is squeezed

(C) The total mass changes in both of the above processes

(D) The total mass changed in neither of the above processes.

The rest mass of the automobile is unchanged. The rubber ball is at rest, but its energy increases (work is done), so its rest mass increases.

The Ultimate Speed is c.

- Objects cannot be accelerated to velocities equal to or greater than the speed of light.
- This has been verified by experiments on electrons in particle accelerators.
  - → The graph of speed vs. kinetic energy of the electrons approaches, but does not reach, c.
- According to $E = \gamma m_o c^2$, as v approaches c, γ, and therefore E, approach infinity.
- It would take an infinite amount of energy for an object to reach a speed of c.

The Ultimate Speed is c.

Or is it?

Can Neutrinos Move Faster Than Light?

- September 2011: The OPERA experiment announced detection of the **faster-than-light neutrino anomaly**.
- They created muon neutrinos (subatomic particles) at the CERN lab and detected them in Italy, using GPS (the global positioning system) to measure the locations and times of their creation and detection.
- They found that the neutrinos appeared to travel faster than light, arriving 60.7 ns sooner than light would have, thus violating Special Relativity.
- The initial press release stated that further scrutiny and independent tests were necessary to confirm or disprove the results.

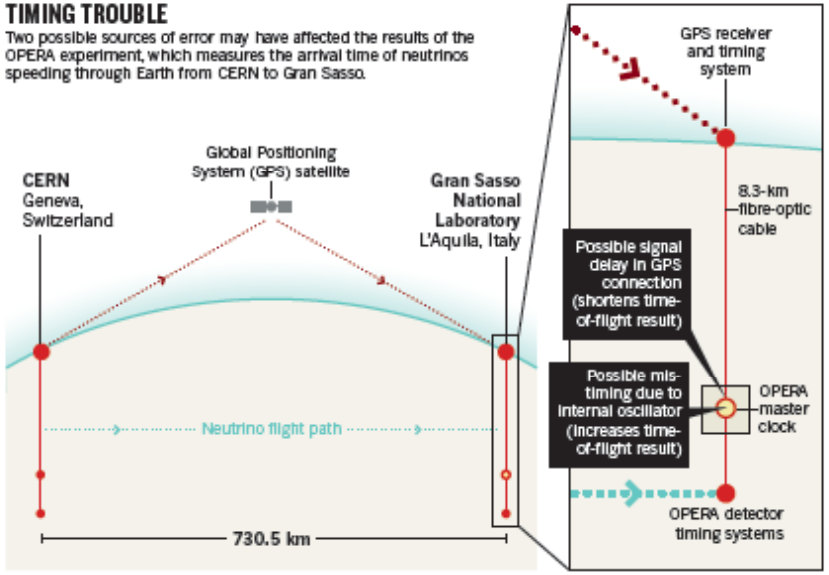
Nature, 1 March 2012

"Is it an epic blunder or a textbook demonstration of how science should work?"

The OPERA team confirmed two possible errors:

(1) a faulty connection from a fibre-optic cable bringing the GPS signal into the master clock, causing this clock to run slow and the travel time to seem shorter than it was;

(2) an oscillator in the master clock was running fast; this would cause the travel time seem longer.



PHYSICS

Timing glitches dog neutrino claim

Team admits to possible errors in faster-than-light finding.

BY EUGENIE SAMUEL REICH

Is it an epic blunder or a textbook demonstration of how science should work? To some physicists, the OPERA (Oscillation Project with Emulsion-tracking Apparatus) collaboration deserves credit for disclosing possible errors in its paradigm-challenging measurement of neutrinos travelling faster than light. "I think we did the right thing to continue to investigate," says Dario Autiero of the Institute of Nuclear Physics of Lyons in France, who presented the original results and notes that the collaboration had spent six months checking its result before its announcement last September.

To others, the revelation shows that the OPERA team went public too soon with its claim that neutrinos from CERN, the European particle-physics laboratory near Geneva in Switzerland, were flouting Albert Einstein's absolute limit on the speed of light as they travelled the 730 kilometres to the OPERA detector at the underground Gran Sasso National Laboratory near L'Aquila, Italy. "I find it embarrassing," says Luca Stanco of the National Institute of Nuclear Physics in Padova, Italy, an OPERA member who initially refused to sign a paper about the result. "Maybe we should have been more cautious and done more checks."

On 23 February, OPERA team members reported two possible sources of error in the experiment. The initial result suggested that the neutrinos were reaching the detector 60 nanoseconds faster than the speed of light would allow. Both potential errors would affect the neutrinos' arrival time, as measured by OPERA's master clock (see 'Timing trouble'). The first is a faulty connection at the point at which the light from a fibre-optic cable brings a synchronizing Global Positioning System (GPS) signal into the master clock. The fault could have delayed the GPS signal, causing the master clock to run slow and thus causing the neutrinos' travel time to appear shorter than it actually was.

"It's a subtle effect," says Autiero, and one that was evident only when the team examined many measurements of signals passing through the connection. Tests of the timing system turned up a second, opposing effect: an oscillator within the master clock that keeps time between the arrivals of synchronization signals was running fast. That would have made the neutrinos' travel time seem longer.

The collaboration says that it has not yet worked out the magnitude of these effects. Autiero says that because of the high profile of the result and the possibility of rumours and leaks, the collaboration wanted to disclose the potential errors promptly. The OPERA team plans to correct the faults and repeat the experiment after CERN's neutrino beam is switched on again in March, following a winter break.

Two independent checks of the measurement are also being considered. One, at Japan's Tokai to Kamioka (T2K) neutrino experiment, would still be valuable despite the doubt cast on the OPERA data, but may now prove harder to fund, says international co-spokesman Chang Kee Jung, a physicist at Stony Brook University in New York. But another, the Main Injector Neutrino Oscillation Search (MINOS) experiment, which fires neutrinos from Fermilab in Batavia, Illinois, to an underground detector in northern Minnesota, will proceed, at a cost of about US$500,000. "It's never a bad idea to have multiple measurements," says MINOS co-spokesman Rob Plunkett.

Jorge Páramos, a physicist at the Higher Technical Institute in Lisbon, says that the admissions by OPERA point to an honest mistake, albeit one that should have been avoided. "The putative origin of the systematic error reflects the innards of the experiment — something that should have been checked exhaustively before any public announcement," he says. ■