Fractal Dropsonde Trajectories and Anomalous Turbulence Exponents in the Vertical

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Abstract

Much of our knowledge about the structure of the atmosphere is obtained by *in situ* measurements: aircraft, radiosondes and more recently, dropsondes. However, turbulence, especially in the wind field, affects these measurement platforms by altering the trajectories of measuring devices; they are no longer along straight horizontal or vertical sections. Indeed, a model of turbulence is required in order to interpret the measurements. For example, if the turbulence is isotropic in three dimensional space, then one expects (at least naïvely) that unique exponents will exist and – at least as far as the scaling exponents are concerned – that the nonrectilinear trajectories are unimportant. Similarly, in 2D isotropic turbulence, the vertical structure is too smooth to lead to biases. However, if the turbulence is anisotropic (neither 3D nor 2D) – and growing evidence shows that it is indeed in-between with $D \approx 2.55$ – then the trajectories can be perturbed over long ranges. Recently Lovejoy, *et al.* (2004) have shown that aircraft can have fractal trajectories and anomalous horizontal scaling exponents over hundreds of kilometers.

We have investigated the corresponding problem for state-of-the-art dropsondes and the implications for the vertical structure of the atmosphere. Dropsondes measure temperature, humidity and pressure as they fall through the troposphere. The wind velocity is estimated by accurately tracking their position using GPS and then applying a correction using a simple dynamical model of how the sonde responds to the wind. When averaged over all altitudes from 0 to 12 km, the scaling is generally excellent but yields a scaling exponent ~30% higher than the Bolgiano-Obukhov value of 3/5. However, when "conditional" statistics are examined, it is found that, at least for altitudes below 6 km, Bolgiano-Obukhov scaling holds as expected. The deviation from Bolgiano-Obukhov statistics is apparently due to strong shear layers above 6 km, which are associated with jet streams. We also show that apparently stable layers consist in reality of a succession of nested, successively smaller, alternating unstable and stable layers in a fractal pattern.

Lovejoy, S., D. Schertzer and A. F. Tuck (2004), Fractal aircraft trajectories and nonclassical turbulent exponents, *Physical Review E*, **70**, 036306-1-5.

Introduction to Scaling

Here we use the quantity H to denote the scaling exponent calculated from a data series $f(t)$ by application of the first order structure function. The q^{th} order structure function of $f(t)$ is defined by

$$S_q(r;f) = \langle |f(t+r) - f(t)|^q \rangle$$

where the lag r is real and positive, the angle brackets denote an average over t and ensemble averaging over f. We denote by $\zeta(q)$ the functional relationship of $\log S_q(r;f)$ to $\log(r)$ and implicitly define the constant H by

$$\zeta(q) = qH - K(q)$$

where $K(q)$ is an intermittency correction. It turns out that for conservative multifractals such as we are dealing with here, $K(1) = 0$, leading to a particularly simple expression for H as $\zeta(1)$. In fact, even for $q = 2$, $K(q)$ is not very large, so it is possible to write $H \approx \zeta(2)/2$.

The quantity H is called a scaling *exponent* because when $\zeta(q)$ is linear it follows that

$$\langle |\Delta f(\Delta t)|^q \rangle \approx (\Delta t)^{\zeta(q)}.$$

Similarly the spectral exponent β indicates that an energy spectrum is in power law relationship with its wave numbers:

$$E(\omega) \approx \omega^{-\beta}$$

ζ, β and H are related by

$$\beta = 1 + \zeta(2) \approx 1+2H.$$

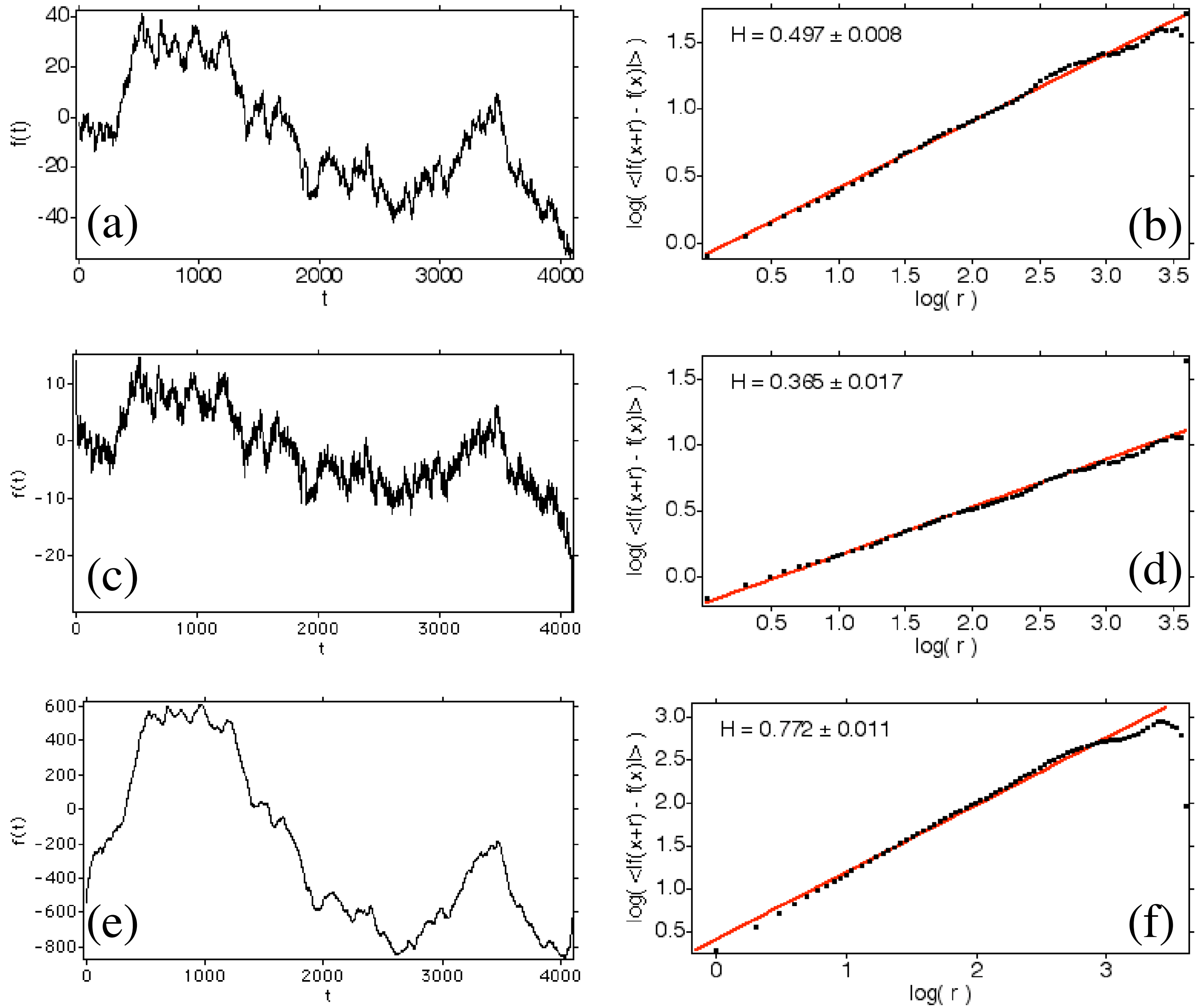
Bolgiano-Obukhov theory posits that the atmosphere evinces $H = 3/5$ ($\beta = 11/5$) in the vertical.

Examples

The usual way to generate a synthetic signal having $H = 0.5$ is to integrate a Gaussian noise, which has spectral exponent $\beta = 0$. Integrating always adds 2 to β, which then yields $H = (\beta - 1)/2 = 0.5$. Such a signal and its variogram are indicated in figures (a) and (b) to the right.

By spectral filtering to reduce β to a value below 2, we get a signal with $H < 0.5$, as illustrated in figures (c) and (d). Notice that the signal has become more rough. Whereas in figure (a) neighboring points are uncorrelated, in figure (b), they generally have negative correlation. That is, there is a tendency for the signal to change in a manner that opposes the current trend. This is called *antipersistence*.

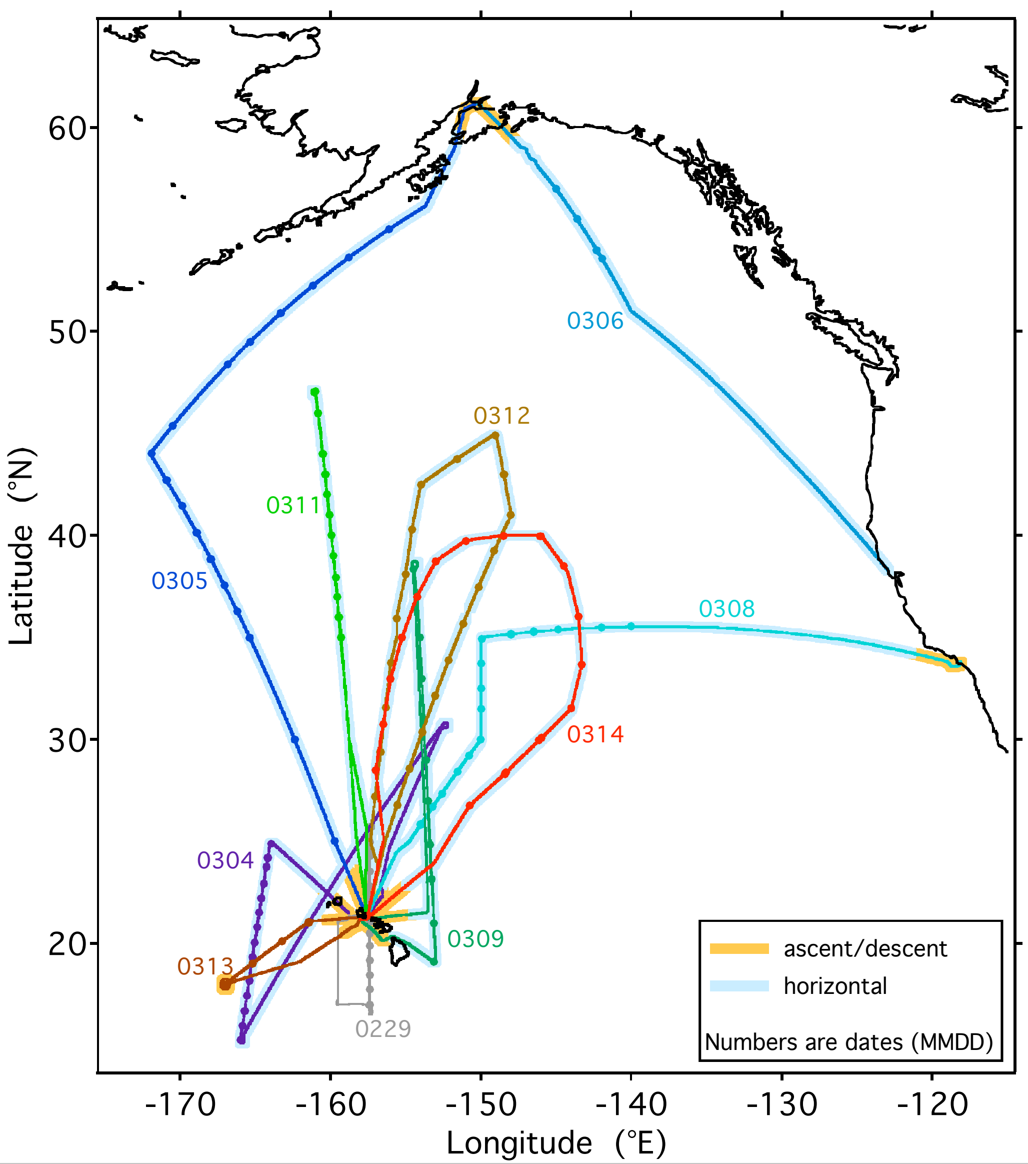
Similarly, by spectral filtering to increase β to a value above 2, we get a signal with $H > 0.5$, as illustrated in figures (e) and (f). Notice how smooth the signal has become. This is indicative of positive neighbor-to-neighbor correlation, also called *persistence*.



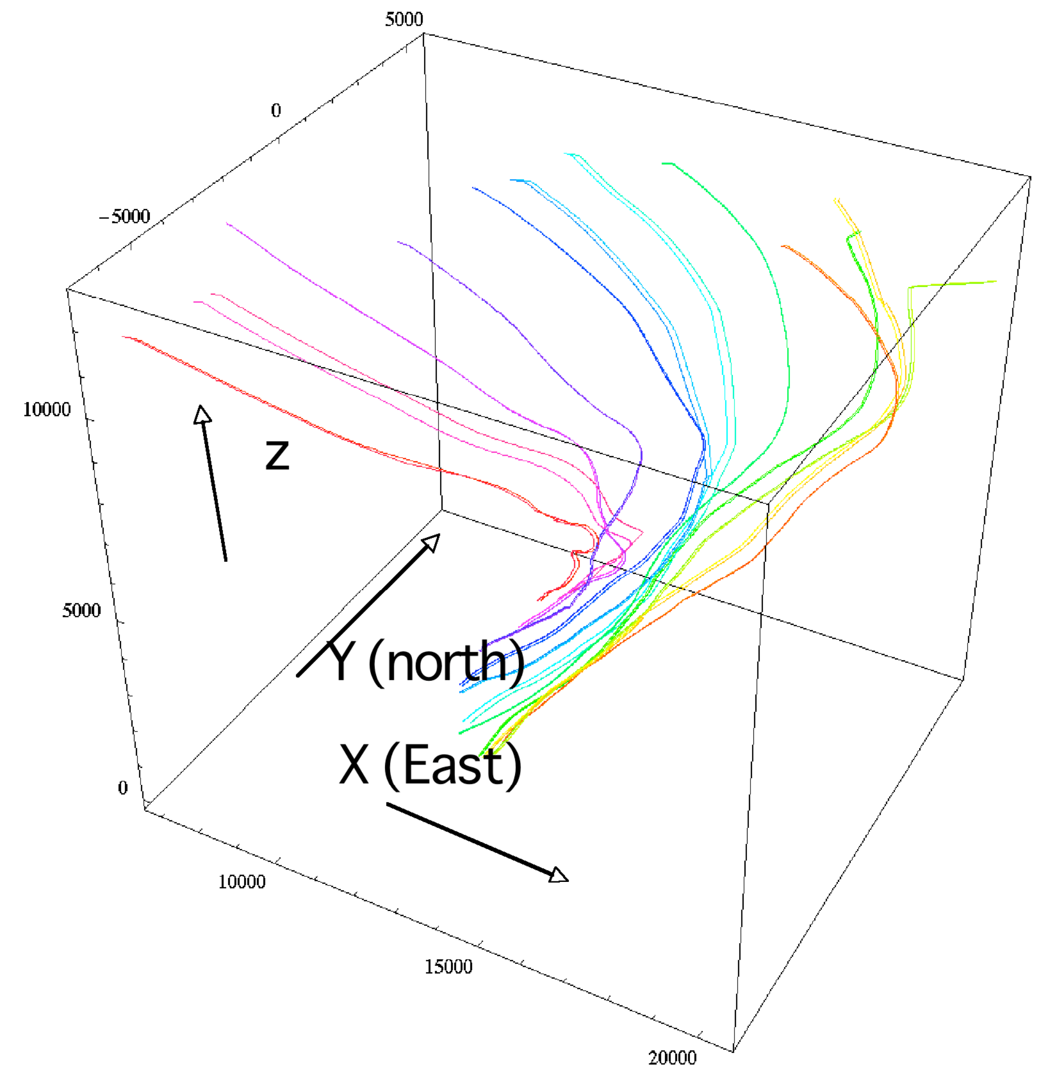
It should be noted that the *persistent* and *antipersistent* classifications are reserved for Gaussian processes and are not used to describe Lévy or multifractal processes.

Data

The data are from the 2004 Winter Storms mission, during which the NOAA Gulfstream 4SP released 261 dropsondes as indicated in the following map.



The analysis presented here was performed on data from the 24 dropsondes released on 29 February 2004. The plot below shows the trajectories. Note that often the dropsondes were released in pairs with as little as a 3-seconds separation, so that there are not a full 24 distinctly visible trajectories.



Initial Results

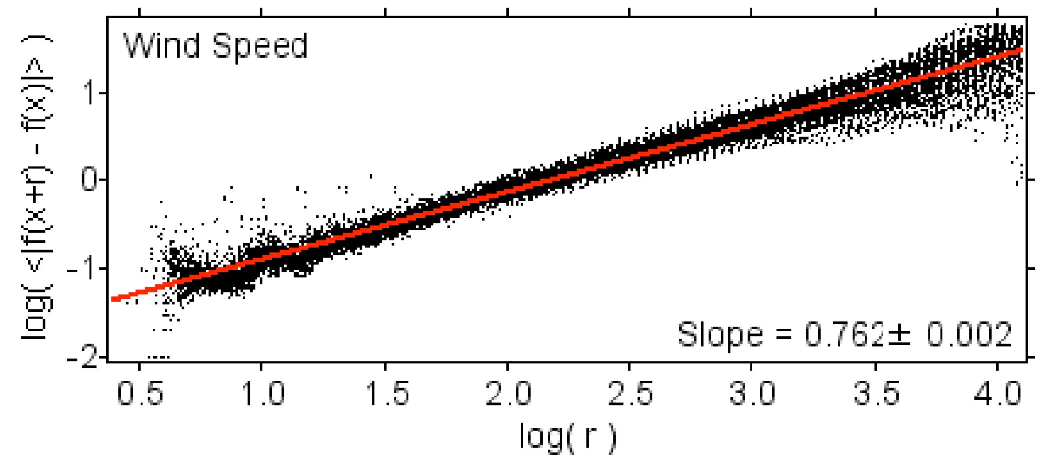
The figures to the right are *composite variograms*, created by overlaying the individual variograms computed for each dropsonde and then fitting a line to the aggregate.

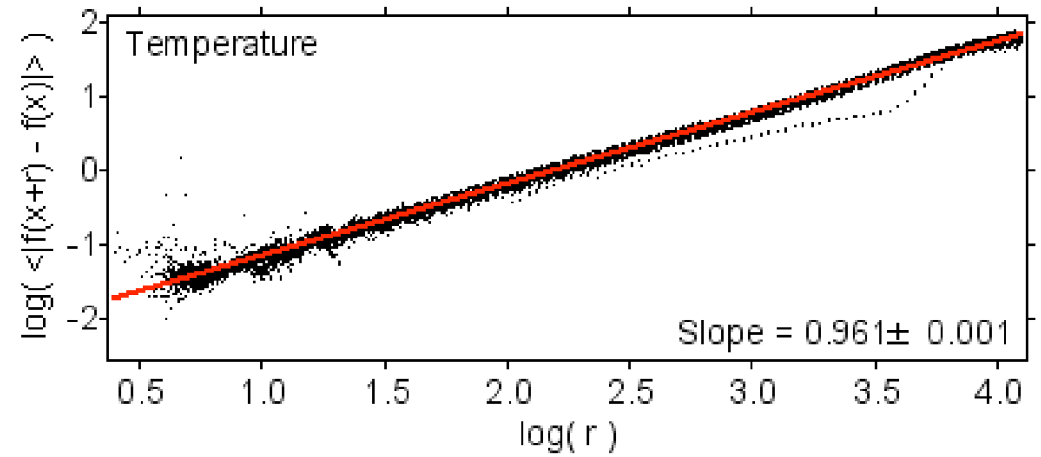
While variograms typically involve variance, we use the first order structure function in order to minimize intermittency corrections and to facilitate comparison with theoretical (dimensional analysis) exponents.

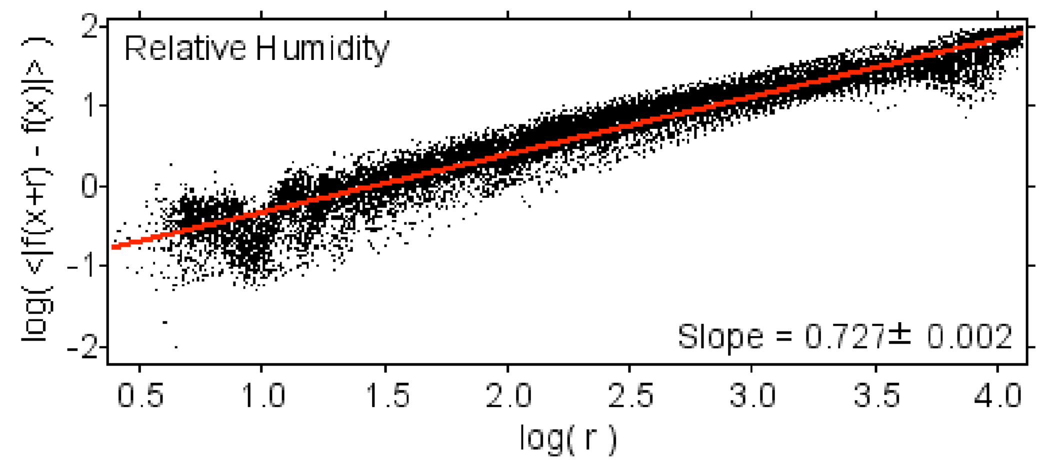
Each individual variogram contained about 100 points, and there were 235 drops that successfully measured wind speed, and 246 that measured temperature and relative humidity. Therefore the lines to the right are each fitting roughly 24,000 points. The errors are 95% confidence intervals.

The surprise is that the slope (i.e. H) for horizontal wind speed, came out appreciably higher than the Bolgiano-Obukhov theoretical value of 0.6. This indicates smoother than expected horizontal wind speed profiles. It is clear also that temperature behaves differently in the vertical than the other variables.

Subsequent spectral analysis has shown that the near-unity value of H for temperature is an artifact of the structure function method, which does not produce a good estimate of H when $H > 1$ or $H < 0$. For the data of 20040229, the spectral method yielded $H \approx 5/4$, again a value unique to temperature.

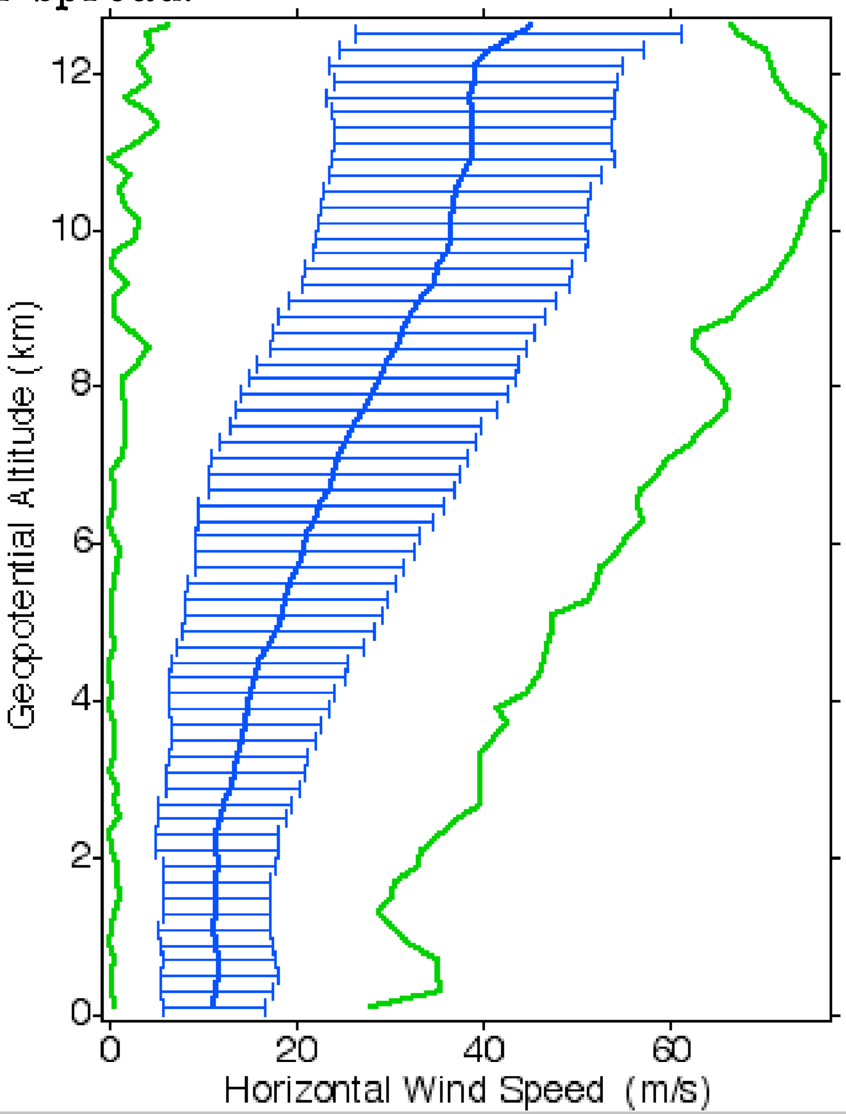


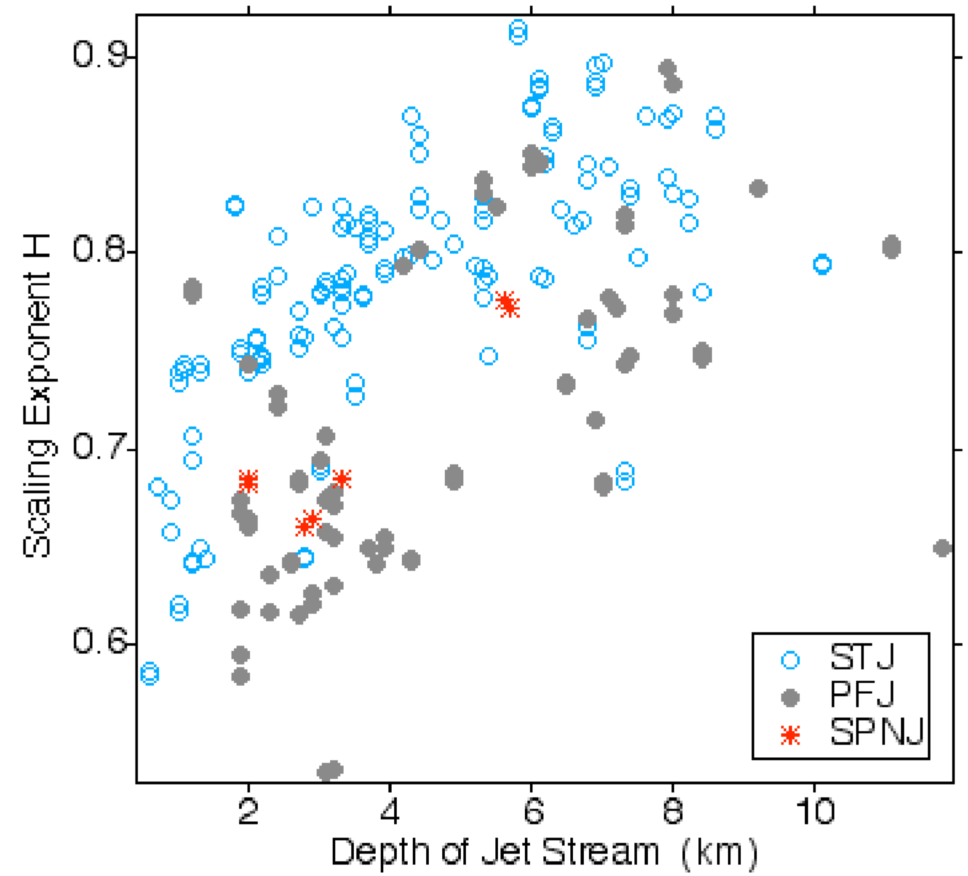




Horizontal Wind Speed vs Altitude

Below is an average (blue) of all of the 235 horizontal wind speed profiles. The error bars are standard deviations. The green traces indicate minima and maxima. Note that at high altitudes the wind speed manifests both a greater mean and a greater spread.

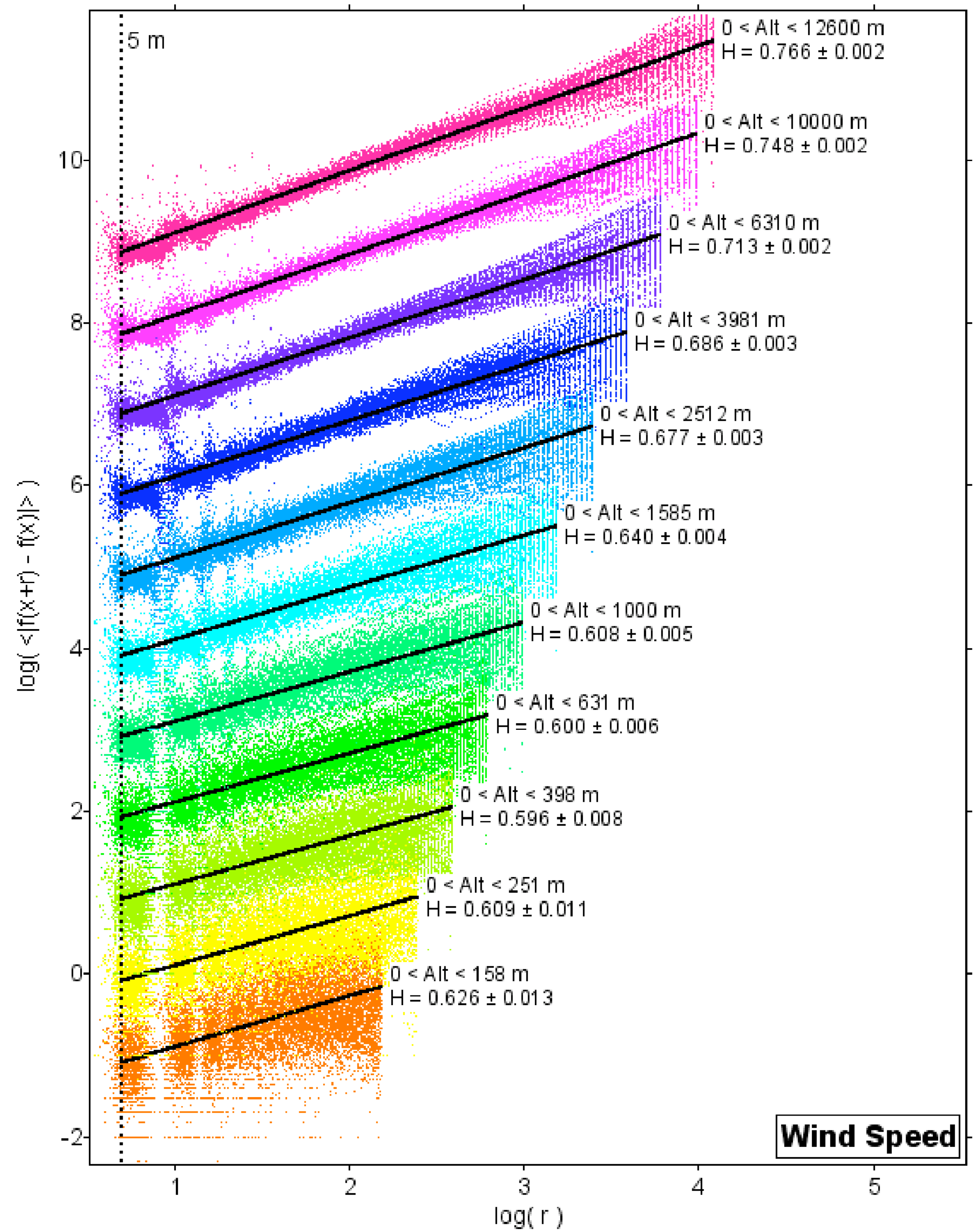




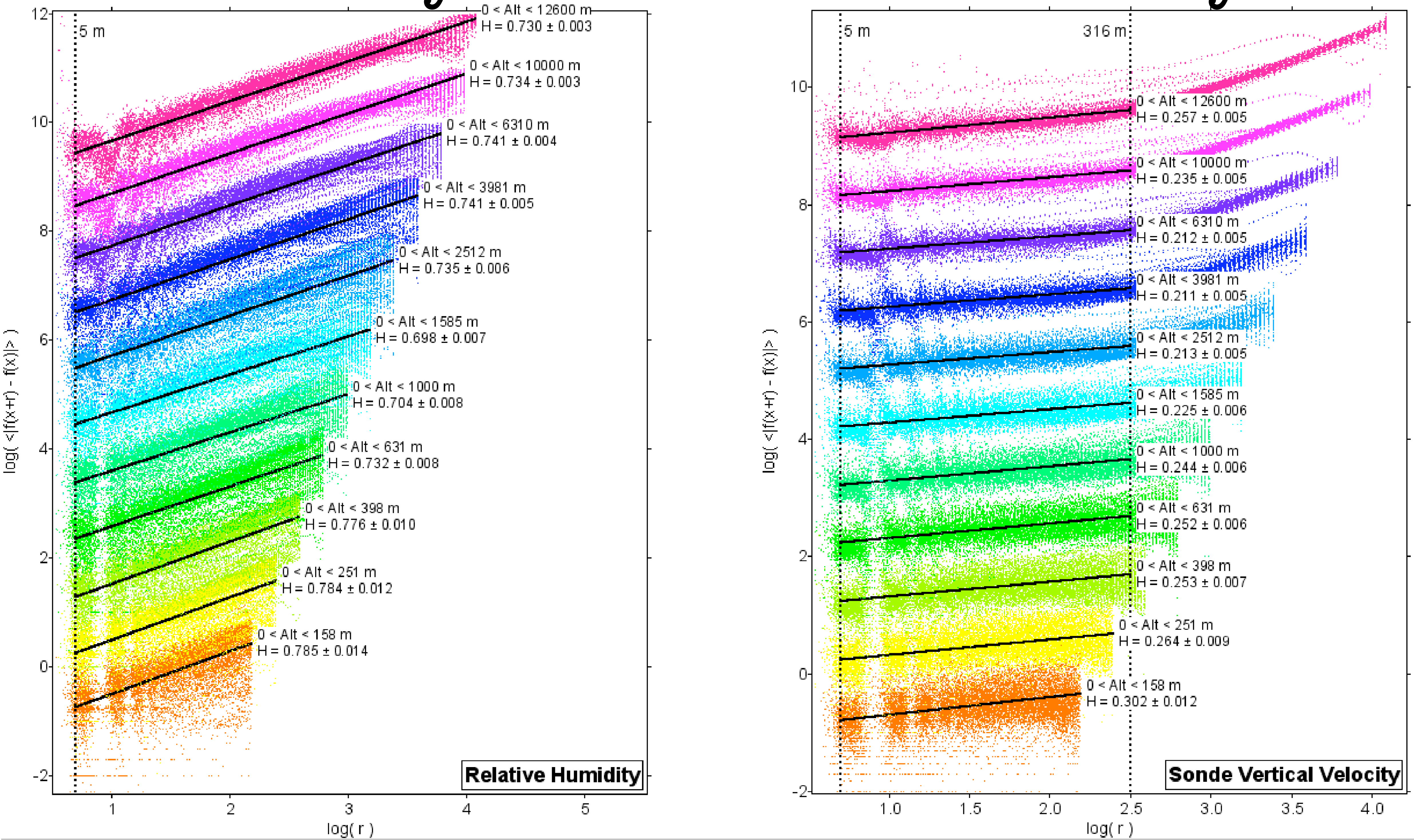
Above is a scatter plot of H for horizontal wind speed as a function of jet stream vertical depth for the subtropical jet (STJ), the polar front jet (PFJ), and the stratospheric polar night jet (SPNJ). Each value of H was calculated by computing the structure function for the entire drop. Here, the Bolgiano-Obukhov theoretical value of $H = 0.6$ is associated with a jet stream thickness of roughly 2 km, but the values for the whole drop are spread between 0.6 and 0.9 and display a positive correlation with jet stream depth.

"Conditional" *H* for Wind Speed

We computed structure functions conditional on the data coming from layers below a given altitude. Each color to the right corresponds to a different maximum height. The data (from all drops indicated in frame 4) were filtered so that the structure function would be applied only to those points lying below that altitude threshold. For example, the lowest (orange) points correspond to a maximum altitude of 158 m; the middle (cyan) trace corresponds to a maximum altitude of 1585 m. The traces were offset in the vertical by 1.0 for clarity. The points to the left of the vertical dashed black line ($\Delta z < 5$ m) are dominated by altitude measurement noise. Including these points in the determination of H decreases *H* by about 0.002. The line fits to each colored set of points are shown in black, with slopes (i.e. *H*) printed to the right of each fit. Note that Bolgiano-Obukhov 0.6 scaling theory holds up to about 1000 m, beyond which there is a steady increase in *H* so that the scaling up through 12600 m is nearly 0.8.

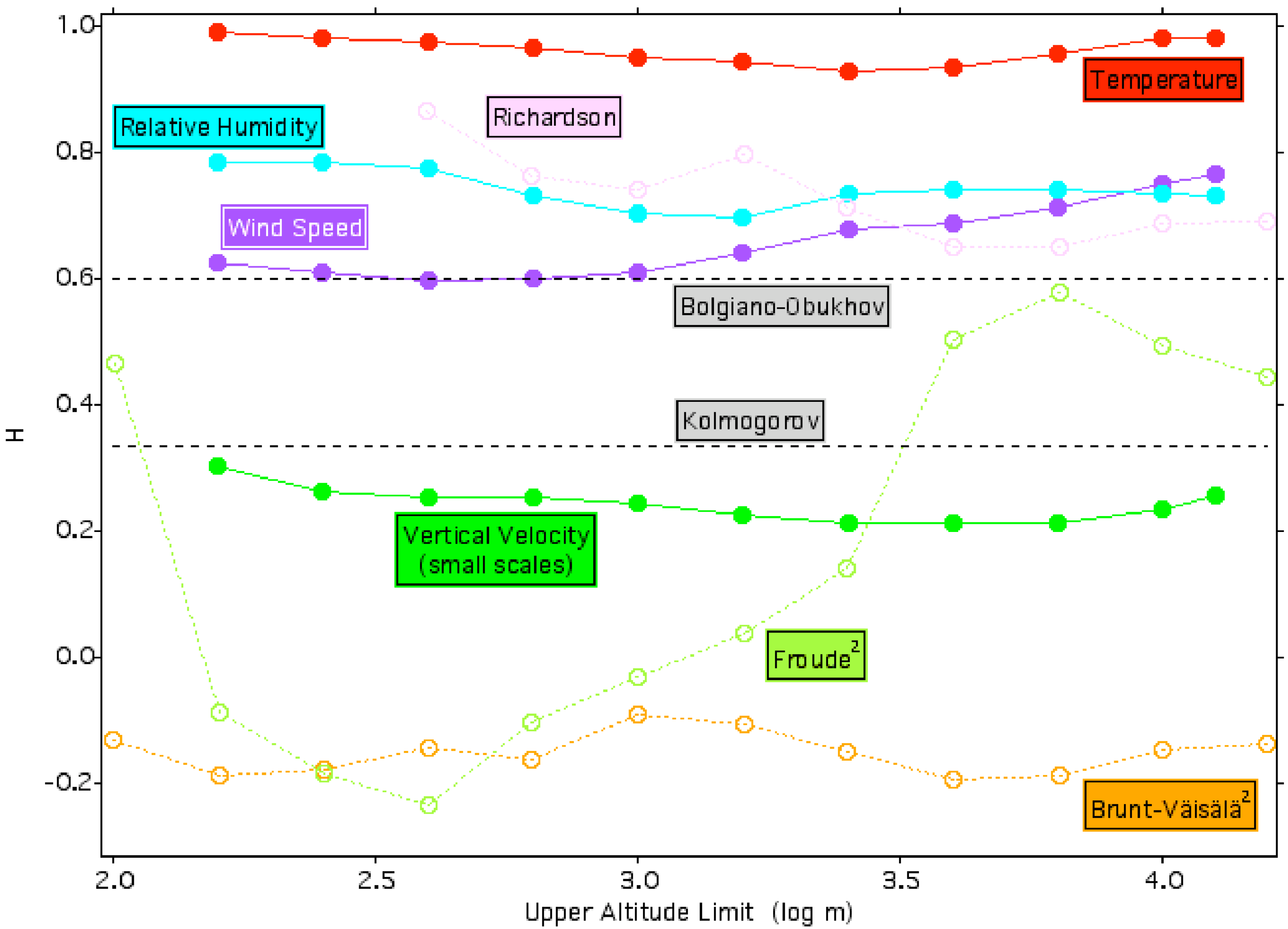


Humidity and Sonde Vertical Velocity

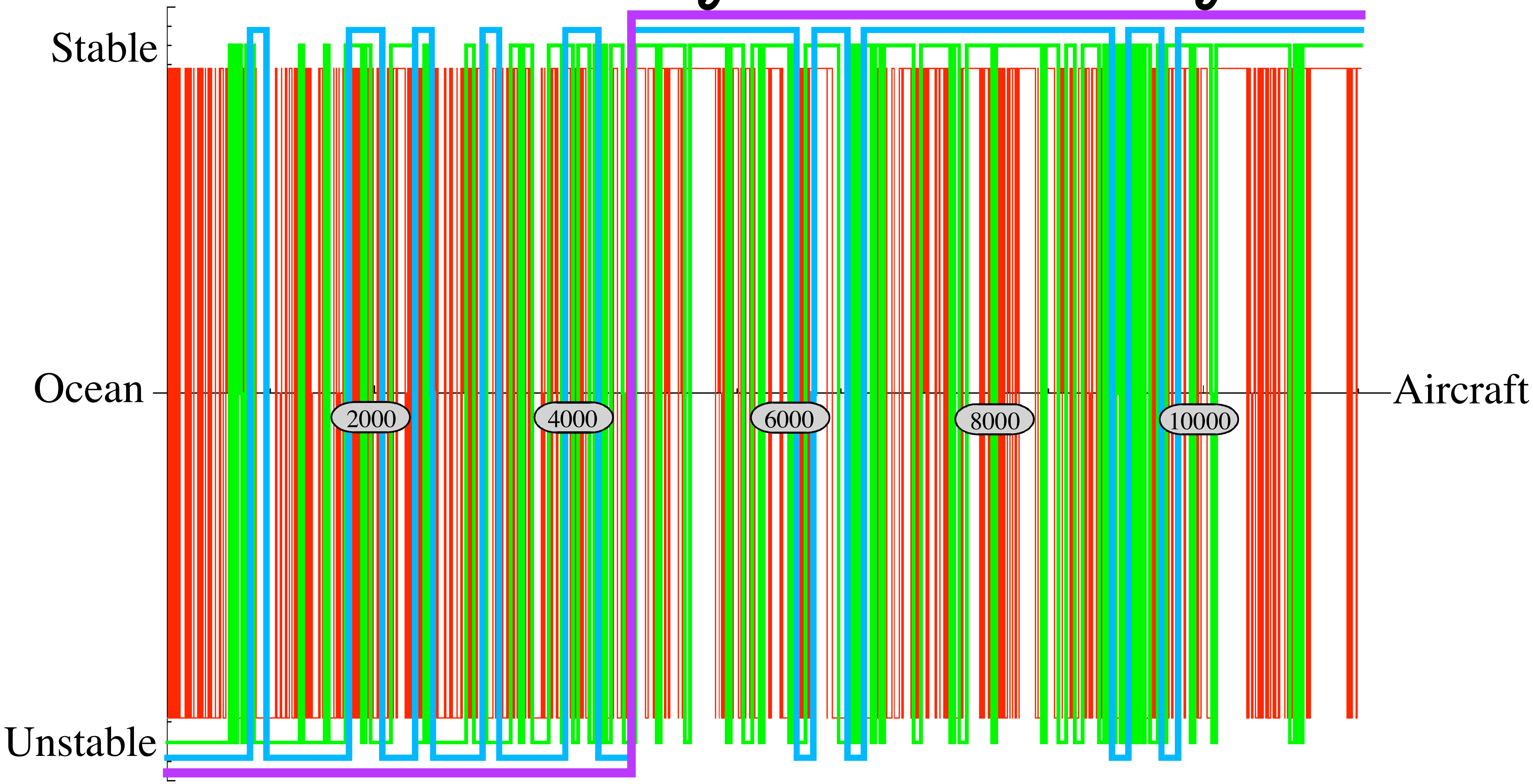


Intercomparison

This figure compares the results obtained from the analysis that produced the preceding two frames. For example, the horizontal wind speed trace (purple) is from frame 7 and the humidity (cyan) and sonde vertical velocity (green) traces are from frame 8. Filled circles indicate the calculation involved all the sonde data from the flights of frame 4. Open circles indicate the calculation was performed only on the data from 20040229. The latter are seen on the traces of several computed quantities: the *Richardson Number*, $Ri = g(\partial \log\theta/\partial z)(\partial v/\partial z)^{-2}$, where v is velocity, g is the acceleration due to gravity, and θ is potential temperature; the *Froude Number*, $Fr = 1/Ri^{1/2}$; and the *Brunt-Väisälä Frequency* (squared), $N^2 = -(g/\rho)(\partial\rho/\partial z)$, where ρ is density.



Richardson Dynamic Stability



This figure shows (for sonde #2 of 20040229), at higher and higher resolution, the Richardson dynamic stability criterion for layers 640 m, 160 m, 40 m, and 5 m thick. It shows that within each stable layer, there are unstable layers, within which is embedded another stable layer, etc. The fractal pattern has fractal correlation codimension 0.09, i.e. a dimension of 1 - 0.09 = 0.91, so that the transitions are sparse but not too sparse. It would seem that the notion of a homogeneous stable layer is quite academic!

Simulation

For scales much larger than the sphero-scale (apparently of the order of 10 cm), for a given spatial displacement, the vertical gradients are much larger than the horizontal gradients, so that the approximation for the horizontal wind speed, $v_x(x(t),z(t),t) \approx v_x(z(t))$, is relatively accurate. It is therefore not necessary to make a 2D or 3D simulation of the horizontal wind; the zonal wind $u(z)$ is adequate.

Fourteen independent universal multifractal simulations with $H = 3/5$ (the Bolgiano-Obukhov value) were used with a resolution of 1 meter over a range of 12 km. The mean wind as a function of altitude was determined using a quadratic fit to the data, as was the mean "spread" about the average wind.

The vertical wind w was taken as an independent realization of a multifractal process with the same multifractal parameters but with 1/10 the amplitude of fluctuations and with zero mean.

Ignoring the second horizontal (y) component, we may express the vertical acceleration of the sonde as

$$\ddot{z} = -g\left[1 + e^{-z/z_0}\left(\dot{z} - w\right)\frac{\Delta v}{v_{0,z}^2}\right]$$

and the horizontal acceleration as

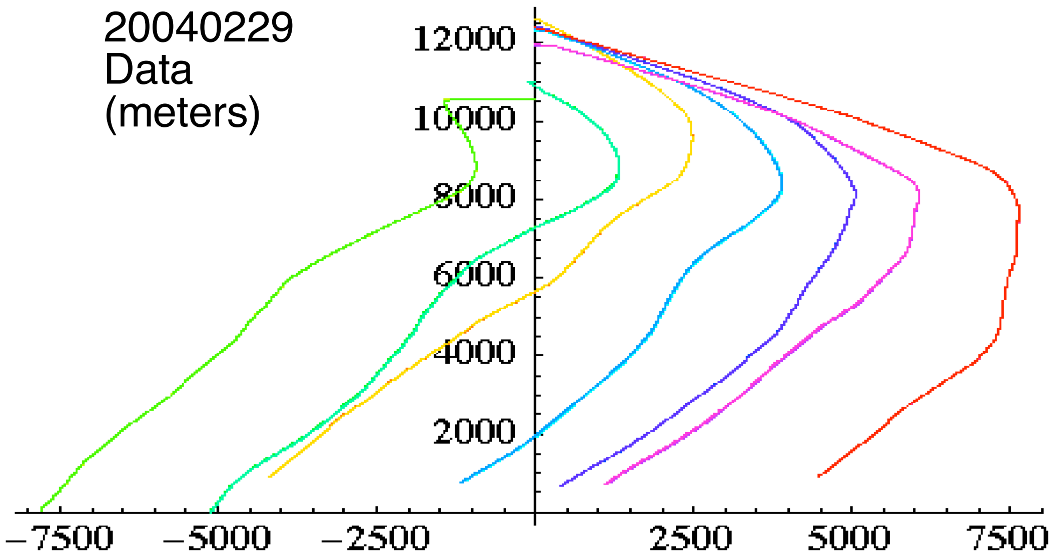
$$\ddot{x} = -ge^{-z/z_0}(\dot{x} - u)\frac{\Delta v}{v_{0,z}^2},$$

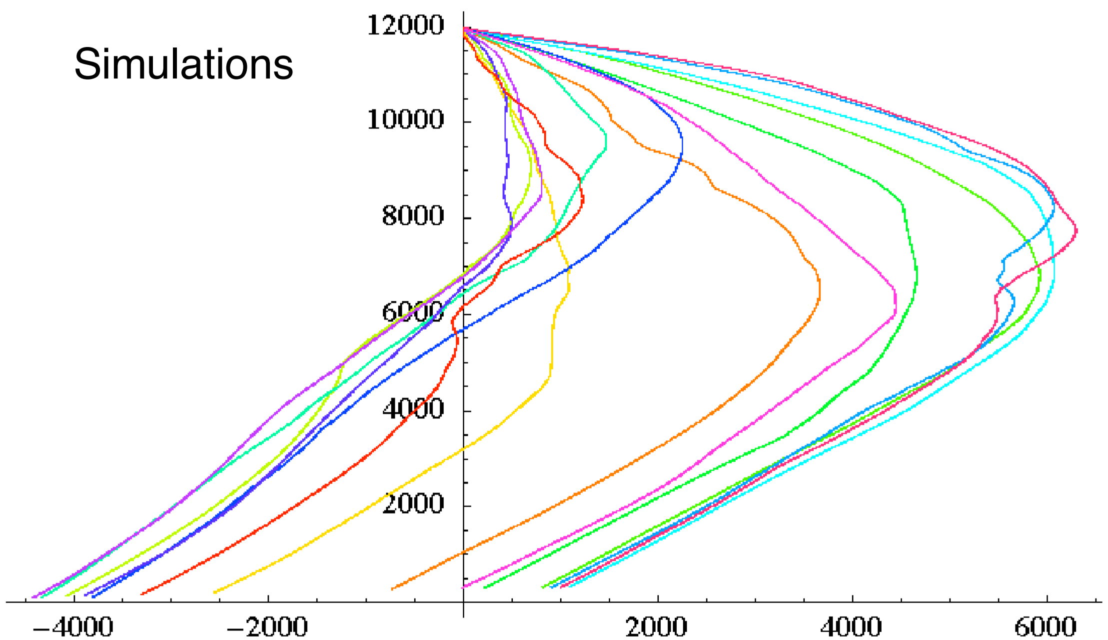
where the dots refer to time derivatives, $\Delta v = \left|\underline{v} - \dot{\underline{r}}\right|$, $\underline{v} = (u, w)$ is wind speed, $\underline{r} = (x, z)$, z_0 is the scale height of the the atmosphere (~8 km), $v_{0,z} = \left(\frac{mg}{b}\right)^{1/2}$ is the mean vertical speed, m is the mass of the sonde, g is the acceleration due to gravity, $b = \frac{C_D A \rho_0}{2}$ is the friction, C_D is the coefficient of drag, A is the cross-sectional area of the sonde, and ρ_0 is the standard density at sea level. The friction evaluates to $b = 0.019\ \mathrm{m}^{-1}$, corresponding to a length scale of about $1/b = 50$ m. The spread in the b values is ±14% from one sonde to the next.

Taking into account the change in density yields an estimate for $v_{0,z}$ of 11 m/s.

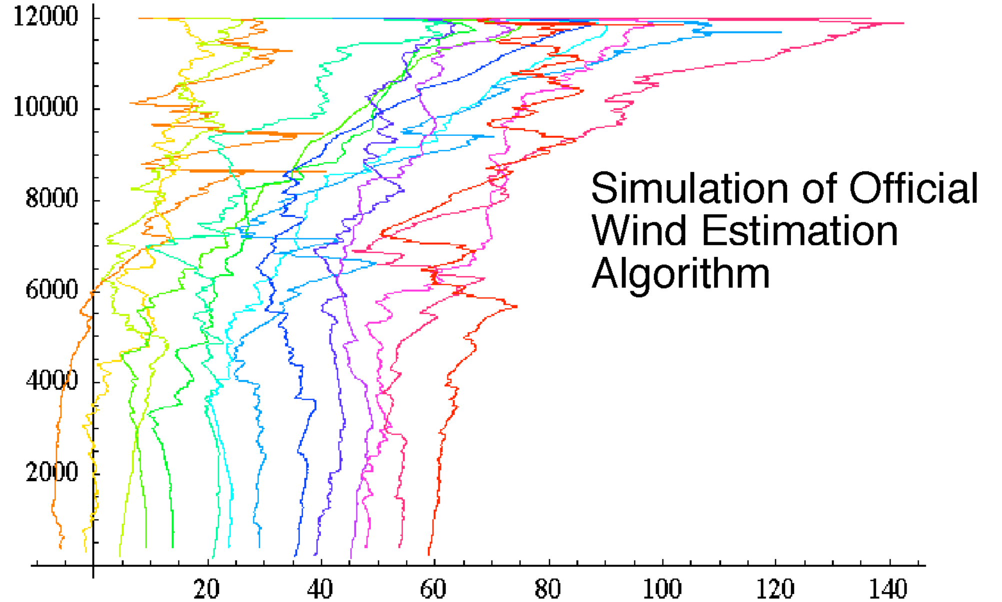
Sample Simulation Results

x-z Cross-sections

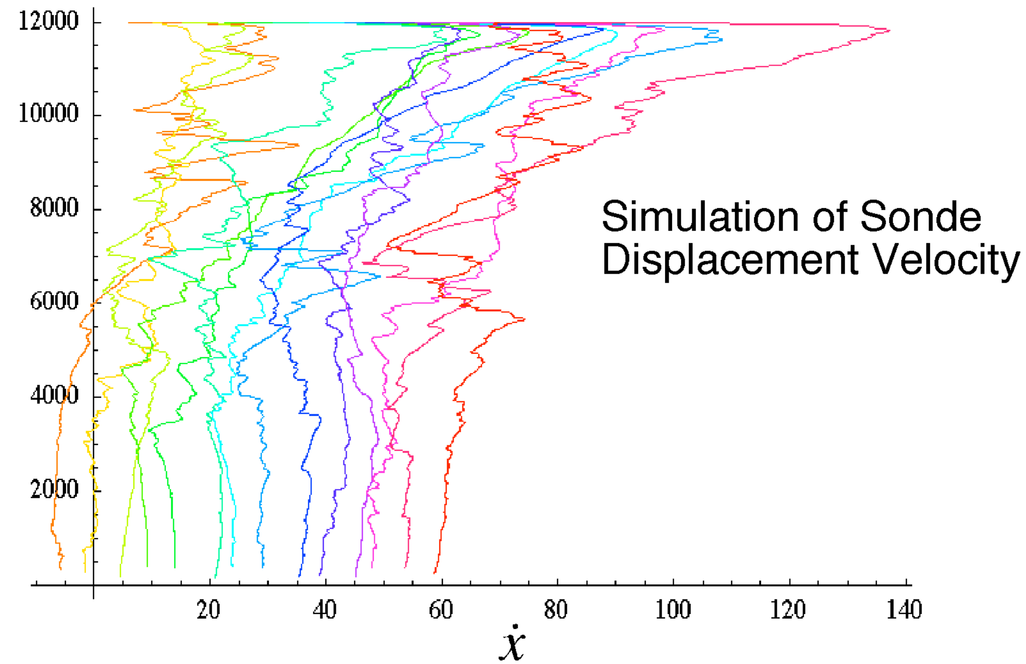




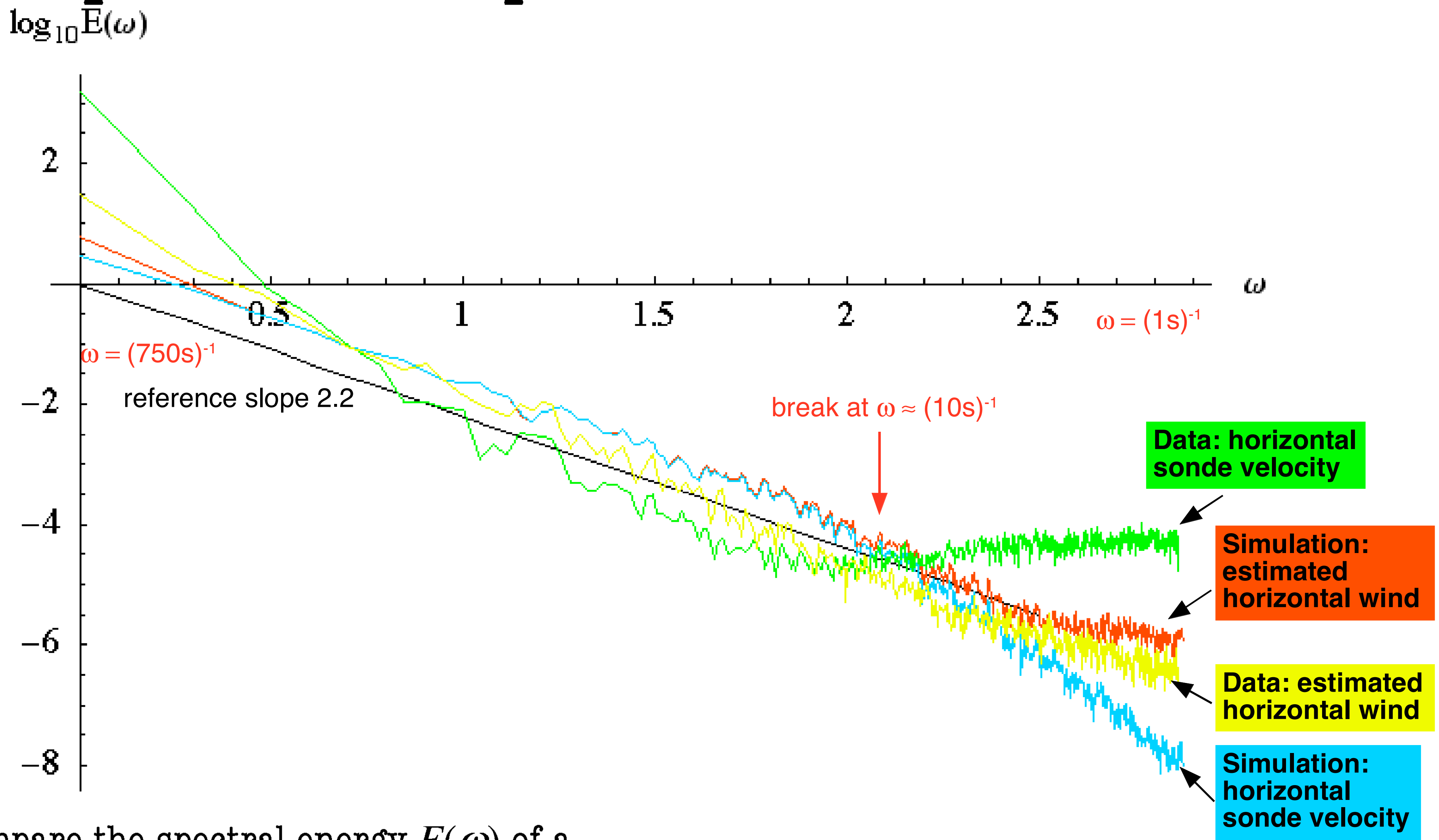
Horizontal Velocities



$$u_{est} = \dot{x} - \frac{\ddot{x}\,\dot{z}}{g}$$

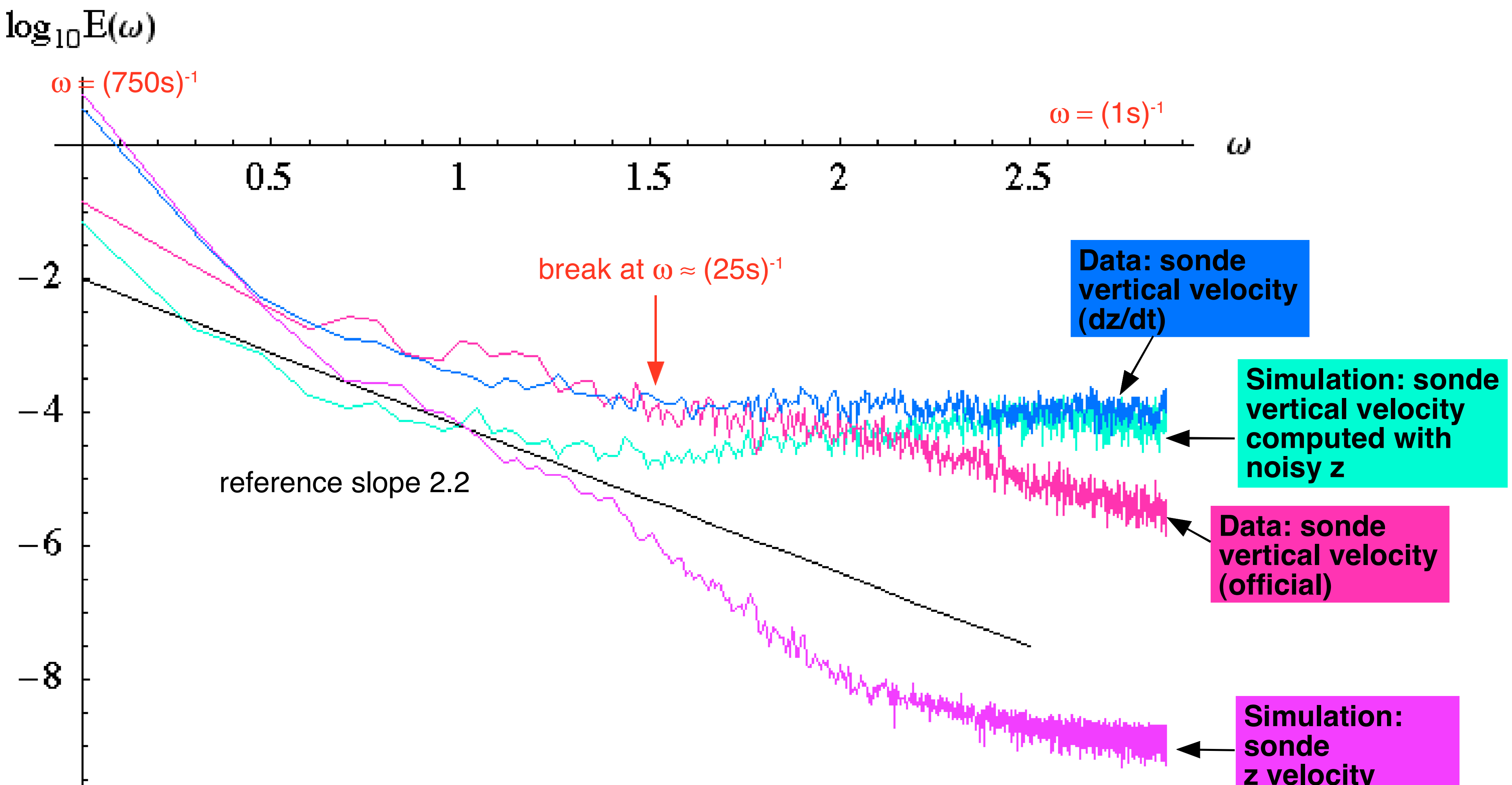


Spectral Comparison - Horizontal



Here we compare the spectral energy $E(\omega)$ of a simulation with that of the actual sonde data. A quadratic fit was used to reproduce the mean vertical shear and the mean spread. $v_{0,z}$ was taken to be 11 m/s. The simulation was computed with 0.5 s time resolution. Note the agreement with actual data for times less than about 10 s. (Length of series is 1448/2 s.) The reference slope of 2.2 corresponds to $H = 3/5$, i.e. the Bolgiano-Obukhov value.

Spectral Comparison - Vertical



The agreement between blue (data) and cyan (simulation) curves shows that the noisy z coordinate explains the spectrum of the sonde vertical velocity quite well. Without it, the spectrum would be very different (bottom magenta). The break at about 25 s is well reproduced; this is an inertial effect which causes smoothing (spectral steepening) in the actual velocity spectrum, but not in the noisy one. The parameters: $v_{0,z} = 11$ m/s , $w_{mean} = 0.1$. (That is, the vertical velocity has fluctuations 0.1 times the amplitude of the horizontal. As long as this is not too big, the results are insensitive to this.)

Summary

Data Analysis

For all of the drops indicated in frame 4, first order structure functions conditional upon all data coming from layers below logarithmically increasing altitude thresholds were determined for the horizontal wind, sonde vertical velocity, temperature and humidity. Also determined for 20040229 were the Richardson number and Brunt-Väisälä frequencies. The effort to interpret the results in the light of Bolgiano-Obukhov theory is ongoing. The climb in wind speed H to a value near 0.8 is probably attributable to jet streams. The vertical scaling of temperature is also unique, probably reflecting the effect of gravity acting through the hydrostatic relation.

One of the most compelling results is that the usual idealization of the atmosphere into homogeneous layers is untenable since we find that within each stable layer there is a hierarchy of unstable sublayers whose fractal dimension we estimate. Elsewhere we examine the consequences for gravity waves and other linearizations of the dynamical equations.

The internal structure revealed here in stable layers will have implications for tropopause folds, both chemically and dynamically. The mixing rate with the free troposphere will be affected, as will the rates of chemical reactions, both internally and at the boundaries of the fold. The ultimate stable layer is the stratosphere itself - will it have this sort of vertical structure?

Simulations

In order to understand the limitations of the data, simulations were made of the sonde trajectories using multifractal horizontal wind fields and Newton's laws with a quadratic friction law.

For both horizontal and vertical displacements, the low frequencies were accurately simulated as well as breaks in the scaling due to inertial effects at about 25 s, 10 s (vertical, horizontal respectively). We were able to well simulate the high frequency vertical statistics by assuming that the dynamical pressure leads to erroneous pressure altitude corrections. The high frequency horizontal statistics imply noisy estimates of the horizontal position (scales < 10 m).