New 3D-Var Balance Constraints at Environment Canada

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September 4, 2007

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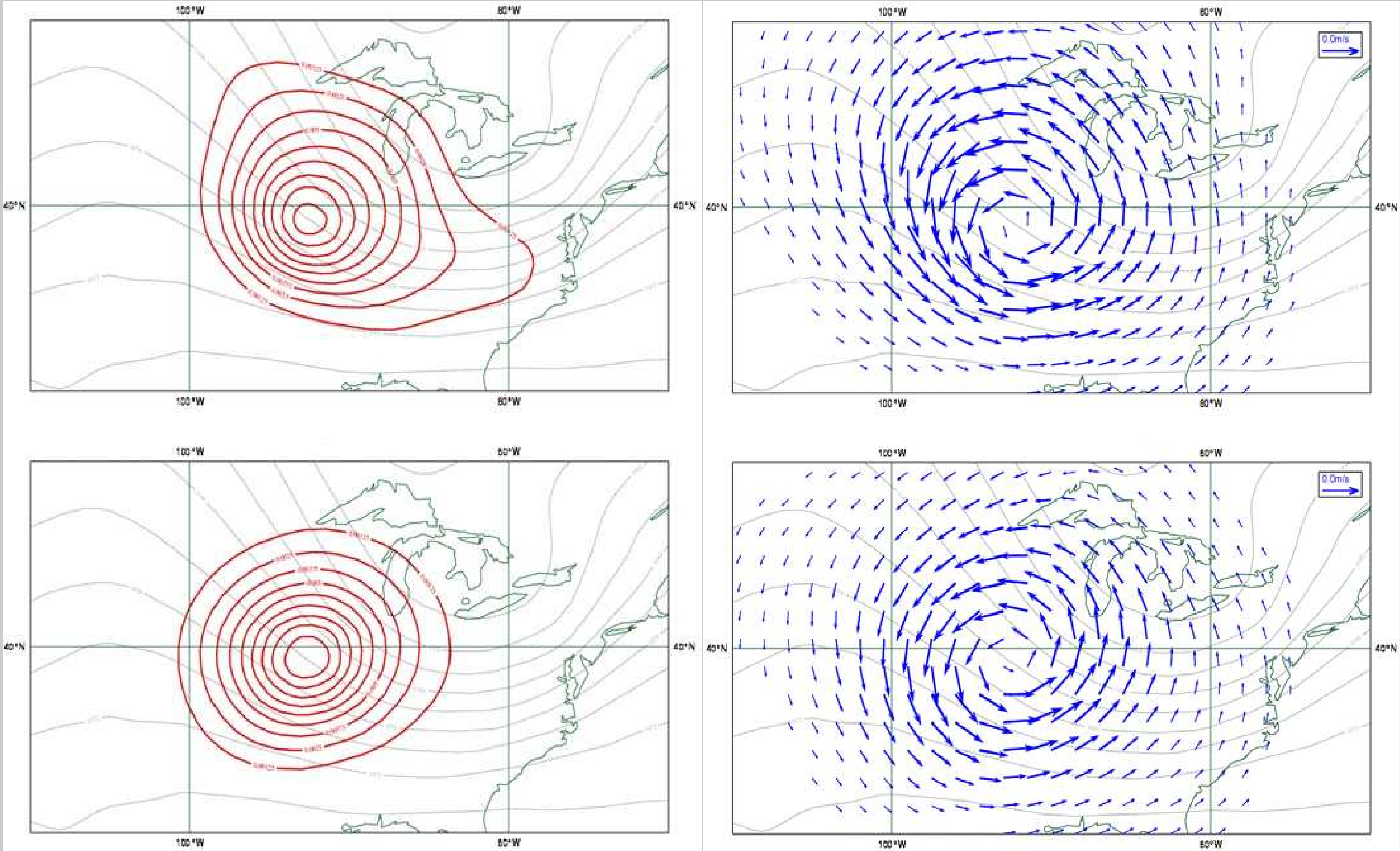
Outline

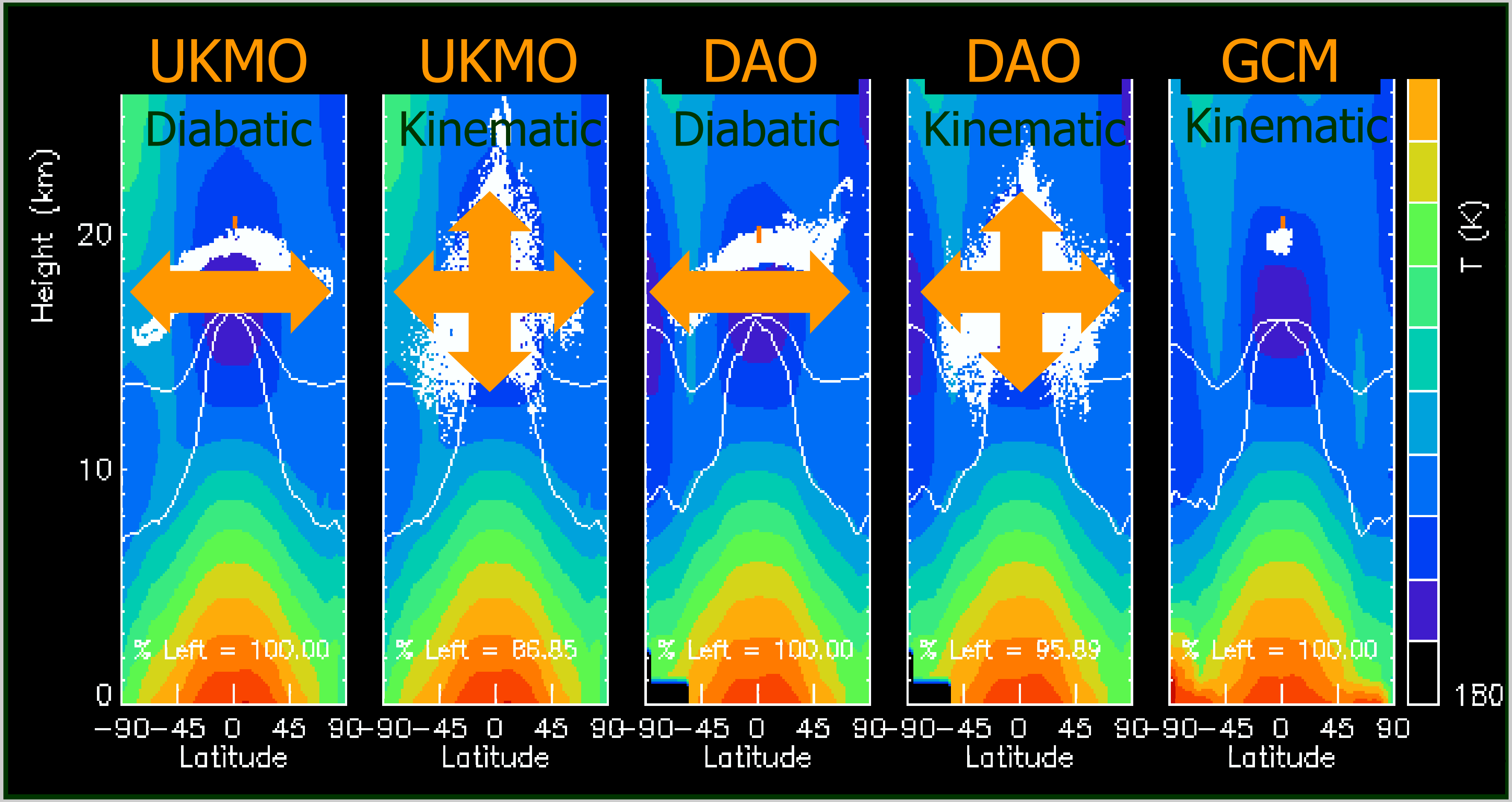
- Motivation
- Introduction
    - Linear and Charney nonlinear balance
    - Quasigeostrophic (QG) omega equation
    - Implementation in 3D-Var
- Online 3D-Var tests
    - Comparisons of increments
    - Scores
    - Efficiency
- Ongoing work

Motivation: Flow Dependence



Temperature and wind increments at 228 hPa for an observation at 300 hPa with (top) Charney and QG omega constraints and (bottom) statistical mass-wind balance (Fisher 2003, ECMWF tech. note).

Motivation: Mixing



3D trajectory calculations at 50 days. Excessive mixing in analyses contributes to incorrect meridional overturning, species dispersal and “age of air” characteristics. Free model shows little dispersion. Trajectory calculations based on (Kinematic) 3D velocity or (Diabatic) heating rates (Schoeberl et al, JGR 2003).

Quick review of 3D-Var

- **To find analysis X, minimize**

$$J(X) = \frac{1}{2}(X - X_{\mathsf{b}})^T \mathbf{B}^{-1}(X - X_{\mathsf{b}}) + \frac{1}{2}(\mathbf{H}(X) - Y)^T \mathbf{O}^{-1}(\mathbf{H}(X) - Y)$$

where $Y =$ **observation vector**

$X_b =$ **background field (from model)**

$\mathbf{H} =$ **forward interpolation**

$\mathbf{O} =$ **observation error covariance**

$\mathrm{B} =$ **background error covariance**

- **Balances introduced by transforming $X \longrightarrow X_u$**

$$X = \begin{bmatrix} \Psi \\ \chi \\ T \\ \ln q \\ \ln p_s \end{bmatrix} = \begin{bmatrix} \mathbf{I} & 0 & 0 & 0 & 0 \\ \mathbf{E} & \mathbf{I} & 0 & 0 & 0 \\ \mathbf{G} & 0 & \mathbf{I} & 0 & 0 \\ 0 & 0 & 0 & \mathbf{I} & 0 \\ \mathbf{D} & 0 & 0 & 0 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \Psi \\ \chi_u \\ T_u \\ \ln q \\ \ln p_{s,u} \end{bmatrix} = \mathcal{M} X_u$$

where G, D = mass-wind balance, E = divergence constraint

- **Operationally**
  - $\mathrm{G}(\phi, p)$: **time-averaged operator (statistics)**
  - E: **near-surface "Ekman" balance**

- **Want to implement**
  - **Charney balance (and hydrostatic balance)**

$$\nabla^2 \Phi_B = \nabla \cdot (f \nabla \Psi) + \frac{2}{a^2} J \left(\frac{1}{\cos\phi} \frac{\partial \Psi}{\partial \lambda}, \frac{\partial \Psi}{\partial \phi} \right)$$

$$T_B = -\frac{p}{R} \frac{\partial \Phi_B}{\partial p}$$

  - **QG omega equation (and continuity equation)**

$$\left(\nabla^2 + \frac{f^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \omega_B = \frac{f}{\sigma} \frac{\partial}{\partial p} [\mathbf{u}_\psi \cdot \nabla (f + \nabla^2 \Psi)] + \frac{R}{\sigma p} \nabla^2 \left(\mathbf{u}_\psi \cdot \nabla T_B \right)$$

$$\nabla^2 \chi_B = -\frac{\partial \omega_B}{\partial p}$$

- **Origin of Charney balance**
  - **Neglect "small" terms in divergence equation**

$$\Longrightarrow \nabla^2 \Phi_B = \nabla\cdot(f\nabla\Psi) + \frac{2}{a^2} J\left(\frac{1}{\cos\phi}\frac{\partial\Psi}{\partial\lambda}, \frac{\partial\Psi}{\partial\phi}\right)$$

- **Origin of QG omega equation**

$$\frac{\partial\zeta}{\partial t} + \mathbf{u}_\psi\cdot\nabla(f+\zeta) - f_0\frac{\partial\omega}{\partial p} = 0 \qquad \textbf{(vorticity eq.)}$$

$$\frac{\partial T}{\partial t} + \mathbf{u}_\psi\cdot\nabla T - \left(\frac{\sigma p}{R}\right)\omega = \frac{J}{c_p} \qquad \textbf{(thermodynamic eq.)}$$

  - **Use Geostrophy** $\zeta = \frac{1}{f_0}\nabla^2\Phi$**, hydrostatic relation** $\frac{\partial\Phi}{\partial p} = -\frac{RT}{p}$

$$\Longrightarrow \left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega_B = \frac{f_0}{\sigma}\frac{\partial}{\partial p}[\mathbf{u}_\psi\cdot\nabla(f+\nabla^2\Psi)] + \frac{R}{\sigma p}\nabla^2\left(\mathbf{u}_\psi\cdot\nabla T_B\right)$$

- **Simplistic extension to sphere** $\nabla \rightarrow \left(\frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}, \frac{1}{a}\frac{\partial}{\partial\phi} \right)$, **etc...**
- **Formulate tangent linear equations, e.g.**

$$\delta\Phi_B = \nabla_\eta^{-2} \left\{ \nabla_\eta \cdot (f \nabla_\eta \delta\Psi) + 2(\widehat{\mathbf{z}} \times \nabla_\eta u_r) \cdot \nabla_\eta \left(\frac{1}{a\cos\phi}\frac{\partial\delta\Psi}{\partial\lambda} \right) \right.$$
$$\left. + 2\nabla_\eta v_r \cdot \left[\widehat{\mathbf{z}} \times \nabla_\eta \left(-\frac{1}{a}\frac{\partial\delta\Psi}{\partial\phi} \right) \right] \right\}$$

- **Transformations for** $p = p(\eta)$

$$\frac{\partial F}{\partial p} \rightarrow \frac{\partial\eta}{\partial p}\frac{\partial F}{\partial\eta} \qquad\qquad \nabla_p F \rightarrow \nabla_\eta F - \frac{\partial\eta}{\partial p}\frac{\partial F}{\partial\eta}\nabla_\eta p$$

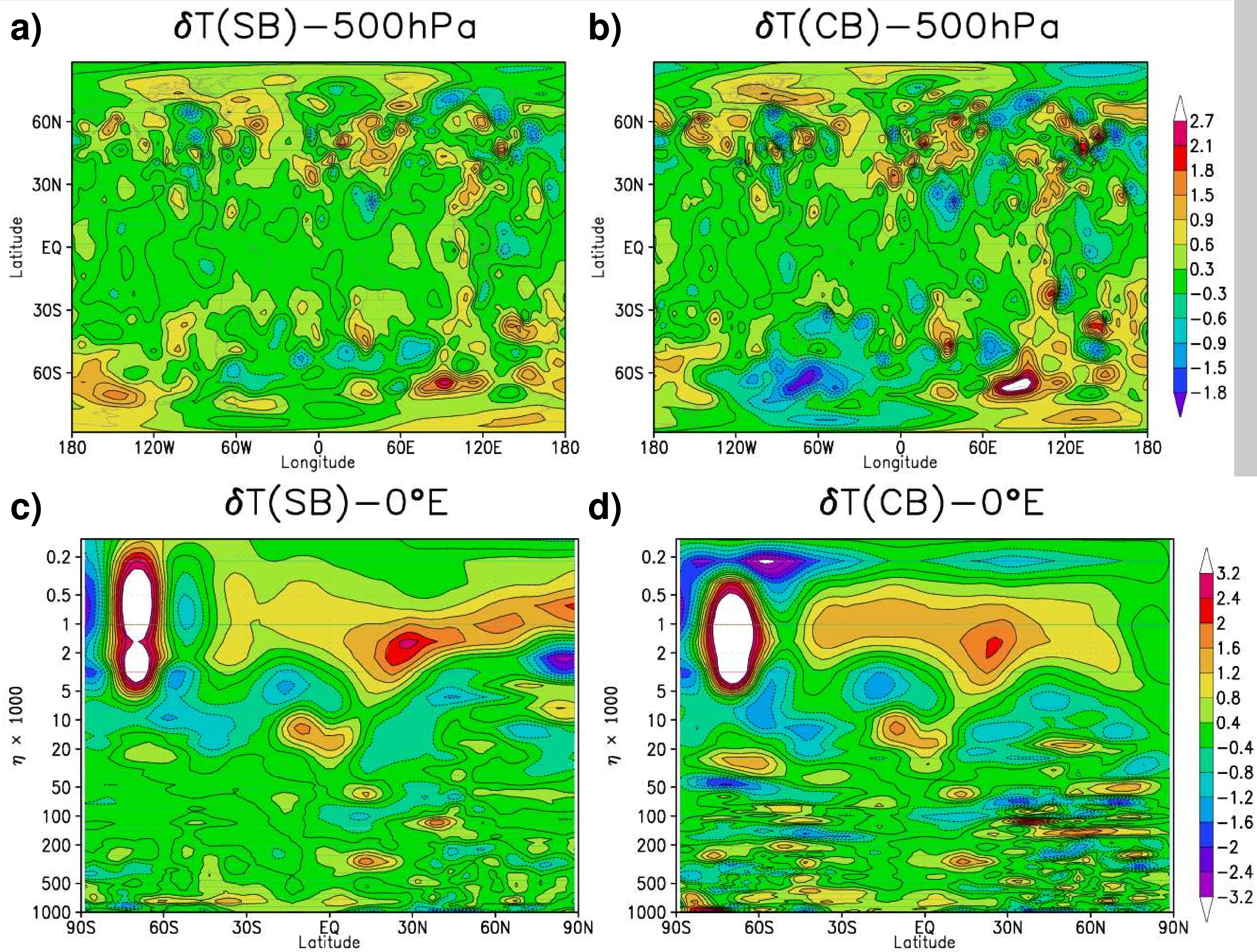
- **Simplifying assumptions**
  - **for LHS and** ∇_p^2 **let** $p = p_0\eta$
  - **static stability** $\sigma =$ **const**
- **Formulate and test adjoint model**

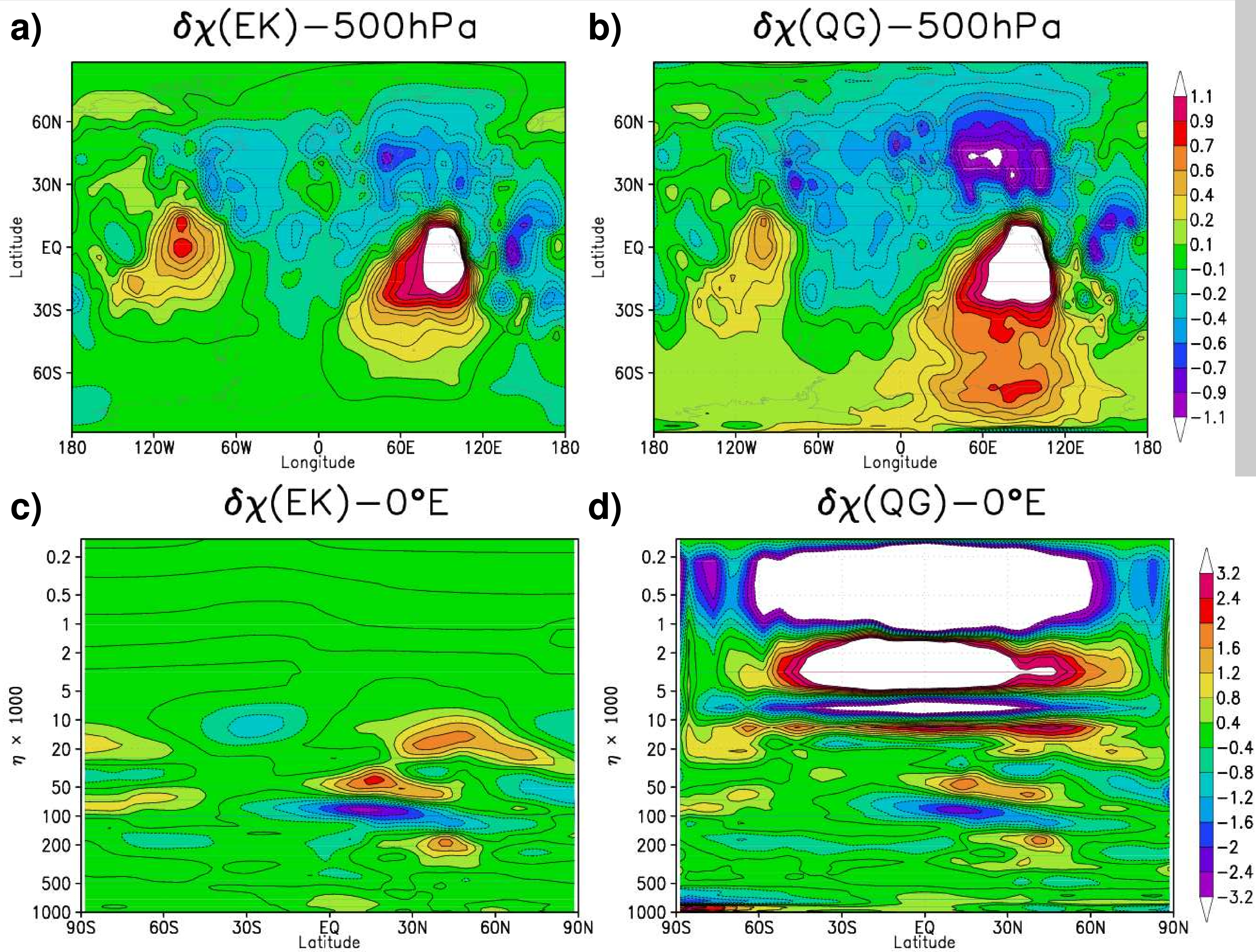
Computational setup

- GCM: GEM-Strato V3.2.2, lid at 0.1 hPa
  - full physics, GW schemes, chemistry
- Assimilation: CMC 3D-Var V10.0.0
  - Note, only meteorology assimilated

Experiments (2 weeks)

- Control
  - (SB) statistical mass-wind balance
  - (EK) Ekman surface balance
- Experiment 1
  - (CB) Charney balance
  - (EK) Ekman surface balance
- Experiment 2
  - (SB) statistical mass-wind balance
  - (QG) omega balance
- Experiment 3
  - (LB) Linear balance
  - (QG) omega balance


a)
δT(SB)−500hPa
b)
δT(CB)−500hPa
c)
δT(SB)−0°E
d)
δT(CB)−0°E
Latitude
Longitude
η × 1000
60N
30N
EQ
30S
60S
180
120W
60W
0
60E
120E
90S
90N
0.2
0.5
1
2
5
10
20
50
100
200
500
1000
2.7
2.1
1.8
1.5
0.9
0.6
0.3
−0.3
−0.6
−0.9
−1.5
−1.8
3.2
2.4
2
1.6
1.2
0.8
0.4
−0.4
−0.8
−1.2
−1.6
−2
−2.4
−3.2



a)
δχ(EK)−500hPa
b)
δχ(QG)−500hPa
c)
δχ(EK)−0°E
d)
δχ(QG)−0°E
Latitude
Longitude
η × 1000
60N
30N
EQ
30S
60S
180
120W
60W
0
60E
120E
180
1.1
0.9
0.7
0.6
0.4
0.2
0.1
−0.1
−0.2
−0.4
−0.6
−0.7
−0.9
−1.1
0.2
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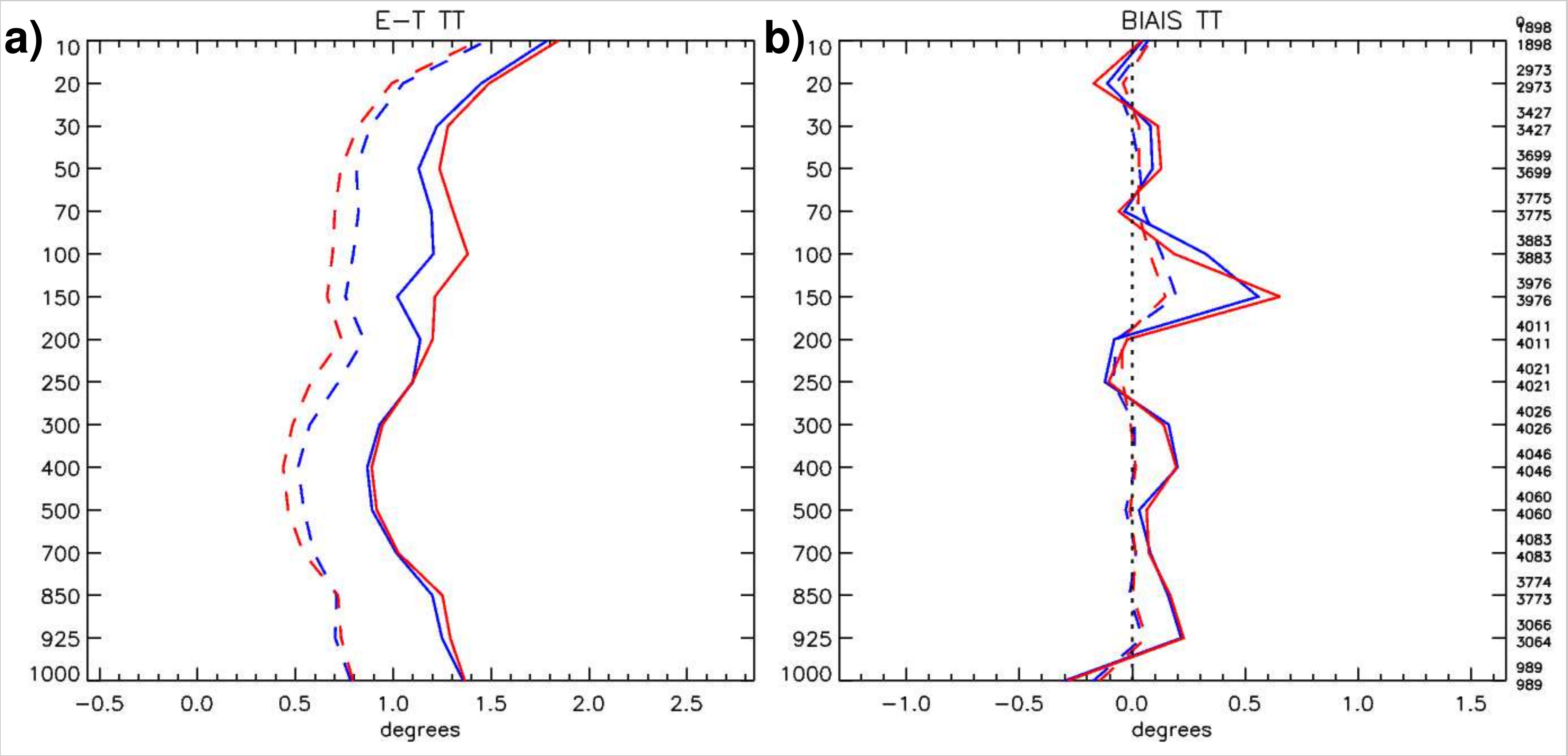


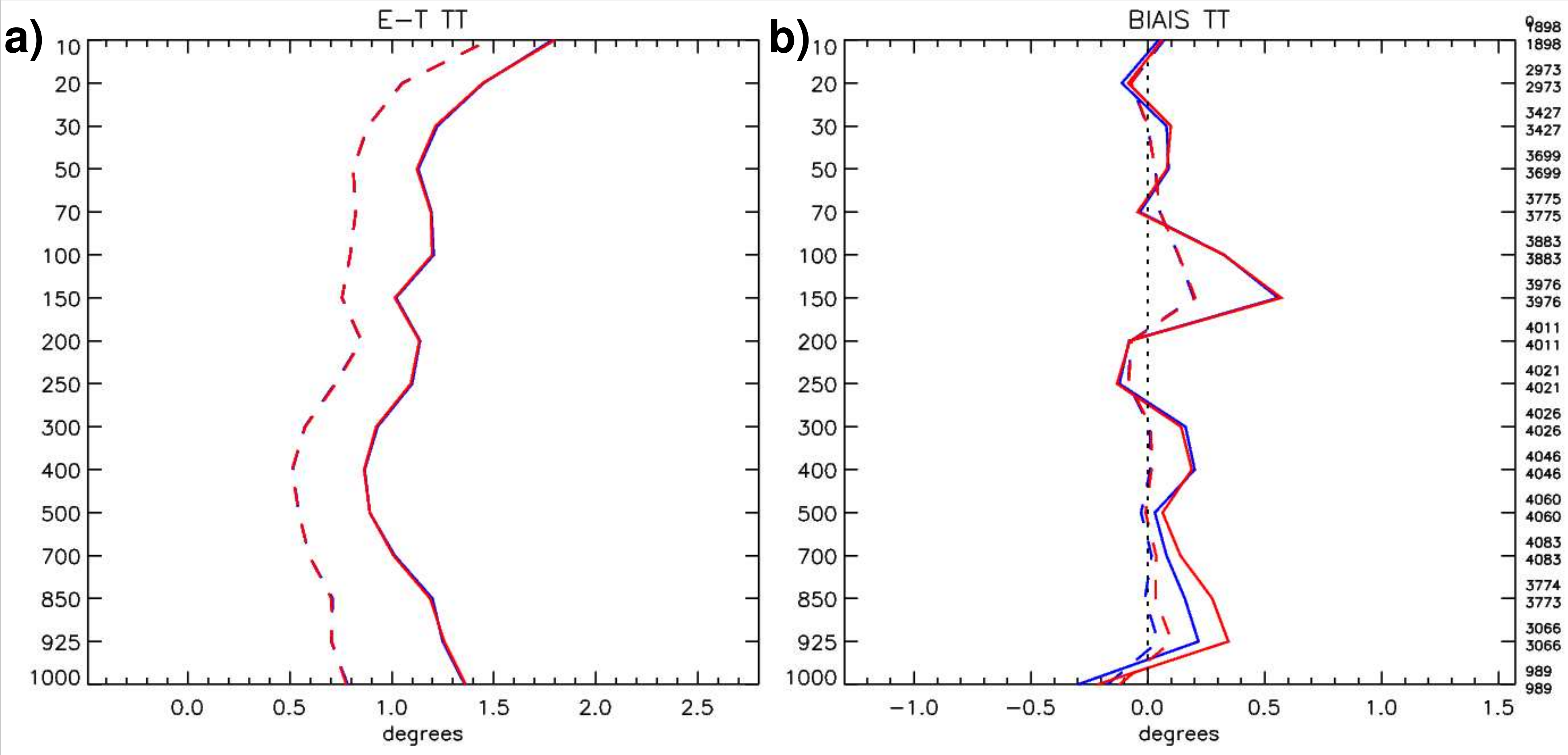
Scores: a) Standard deviation, b) Bias

Dashed blue: Control O-A

Dashed red : Exp. 1 O-A

Continuous blue: Control O-P (6 hour)

Continuous red: Exp. 1 O-P (6 hour)



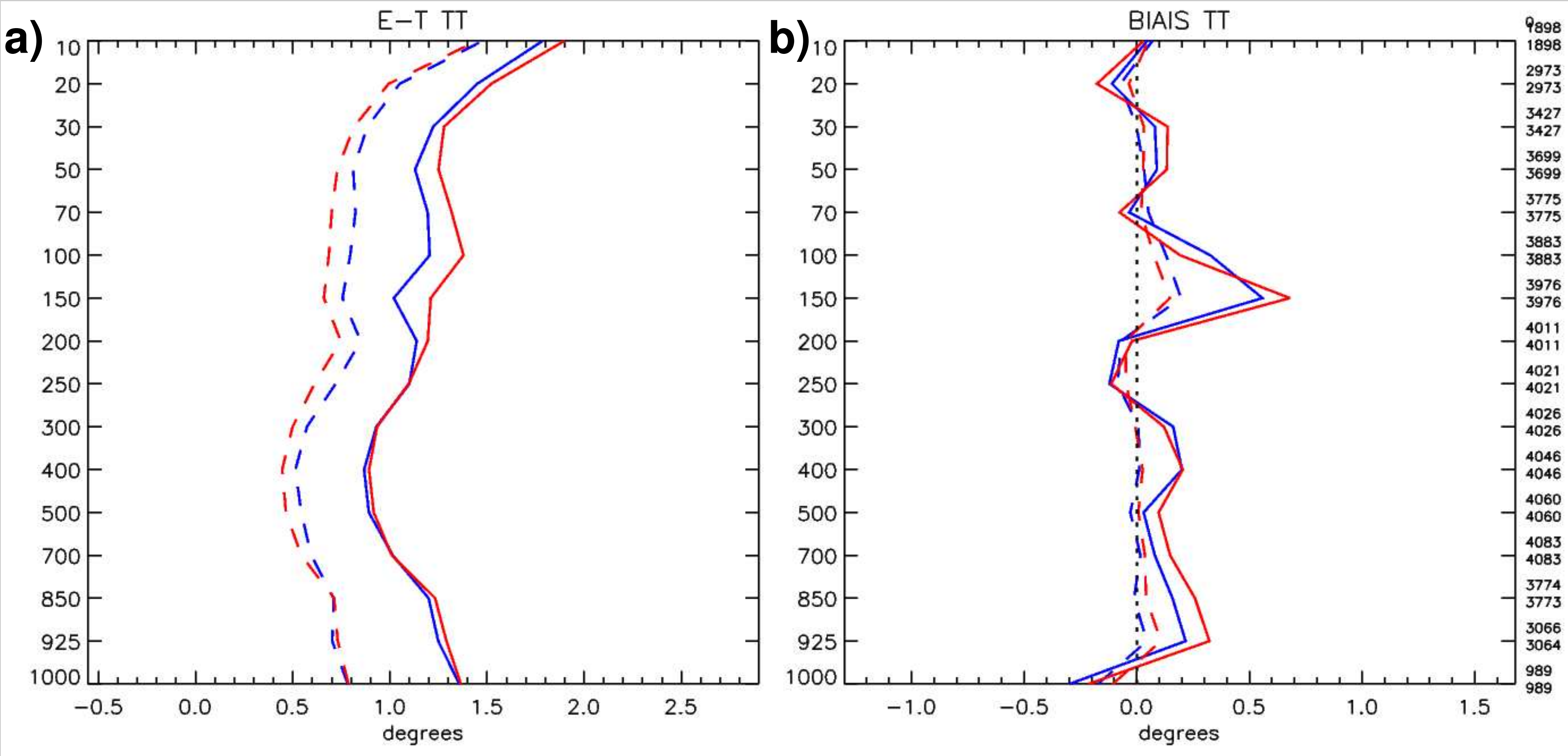
Scores: a) Standard deviation, b) Bias

Dashed blue: Control O-A

Dashed red : Exp. 2 O-A

Continuous blue: Control O-P (6 hour)

Continuous red: Exp. 2 O-P (6 hour)



Scores: a) Standard deviation, b) Bias

Dashed blue: Control O-A

Dashed red : Exp. 3 O-A

Continuous blue: Control O-P (6 hour)

Continuous red: Exp. 3 O-P (6 hour)

Question:

Is the minimization forced to work overtime in experiments 1, 2, 3, i.e. does it take longer to converge than the control?

Answer:

Check the efficiency of the minimization scheme.

Efficiency

- **Values for one analysis time (averaged over 2-week period)**

	Control	Exp. 1	Exp. 2	Exp. 3
Number of iterations	101	98	100	94
Number of simulations	108	102	108	98
3D-Var duration (minutes)	15	33	61	57

- **Note, duration of analysis step is about 1/3 of total time**

Ongoing work

- Test physical impact of new constraints
- Improve scores (e.g. relax assumptions)
- Fix noisy temperature increments
- Introduce diabatic heating effects
- Combine Ekman and QG omega balances?
- Improve efficiency of code

Acknowledgments

Many thanks to:

Stephen Beagley (EC, York), Mark Buehner (EC), Cecilien Charette (EC), Martin Charron (EC), Jean de Grandpré (EC), Pierre Gauthier (EC), Sylvie Gravel (EC), Peter Houtekamer (EC), Herschell Mitchell (EC), Josée Morneau (EC), Yulia Nezlin (U of T), Shuzhan Ren (U of T), Alain Robichaud (EC), Yves Rochon (EC), Kirill Semeniuk (York), Ted Shepherd (U of T), Monique Tanguay (EC), Yan Yang (EC), and others...