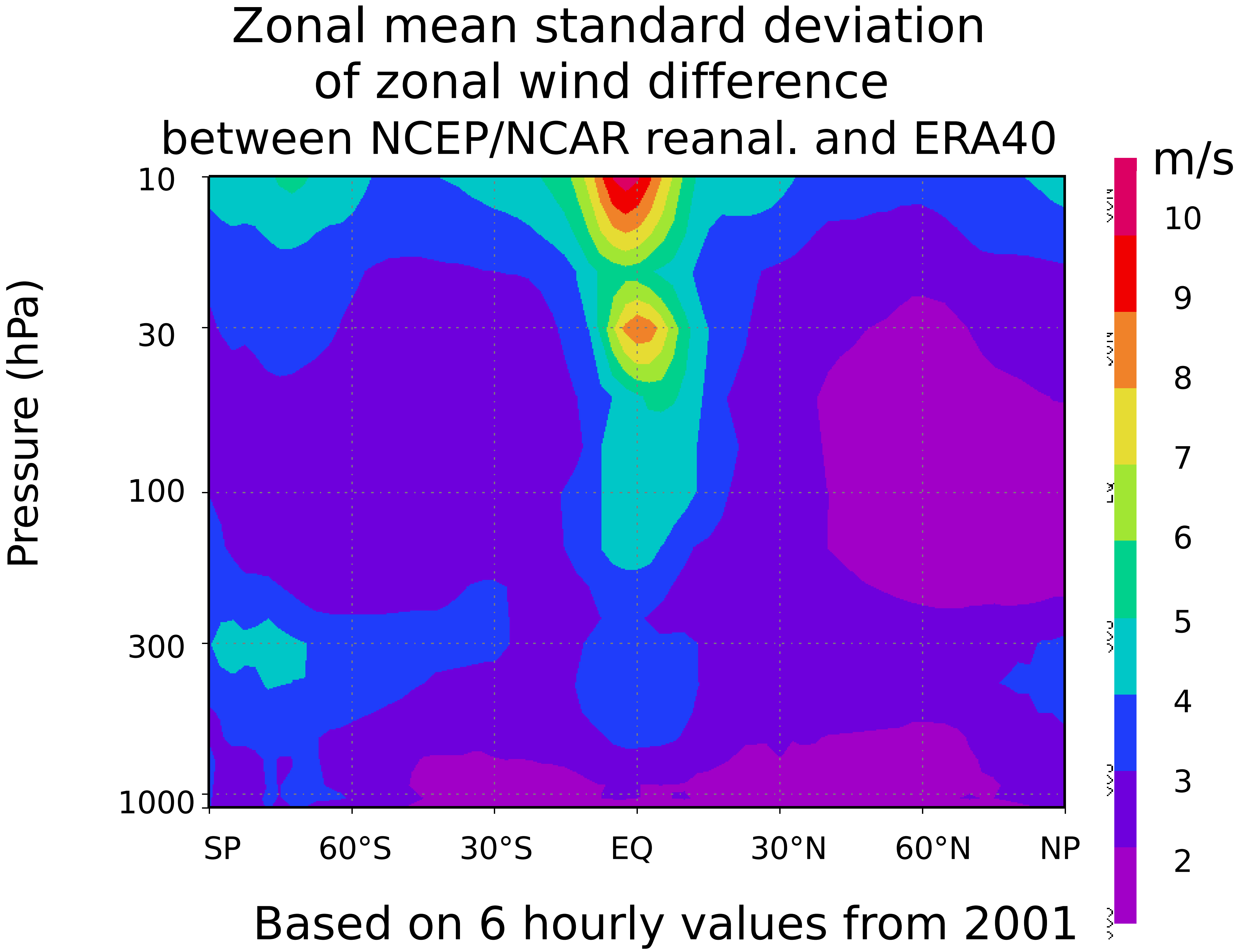
Equatorial waves as a balance relationship in global data assimilation

Heiner Körnich
MISU, Meteorological Institute at Stockholm University

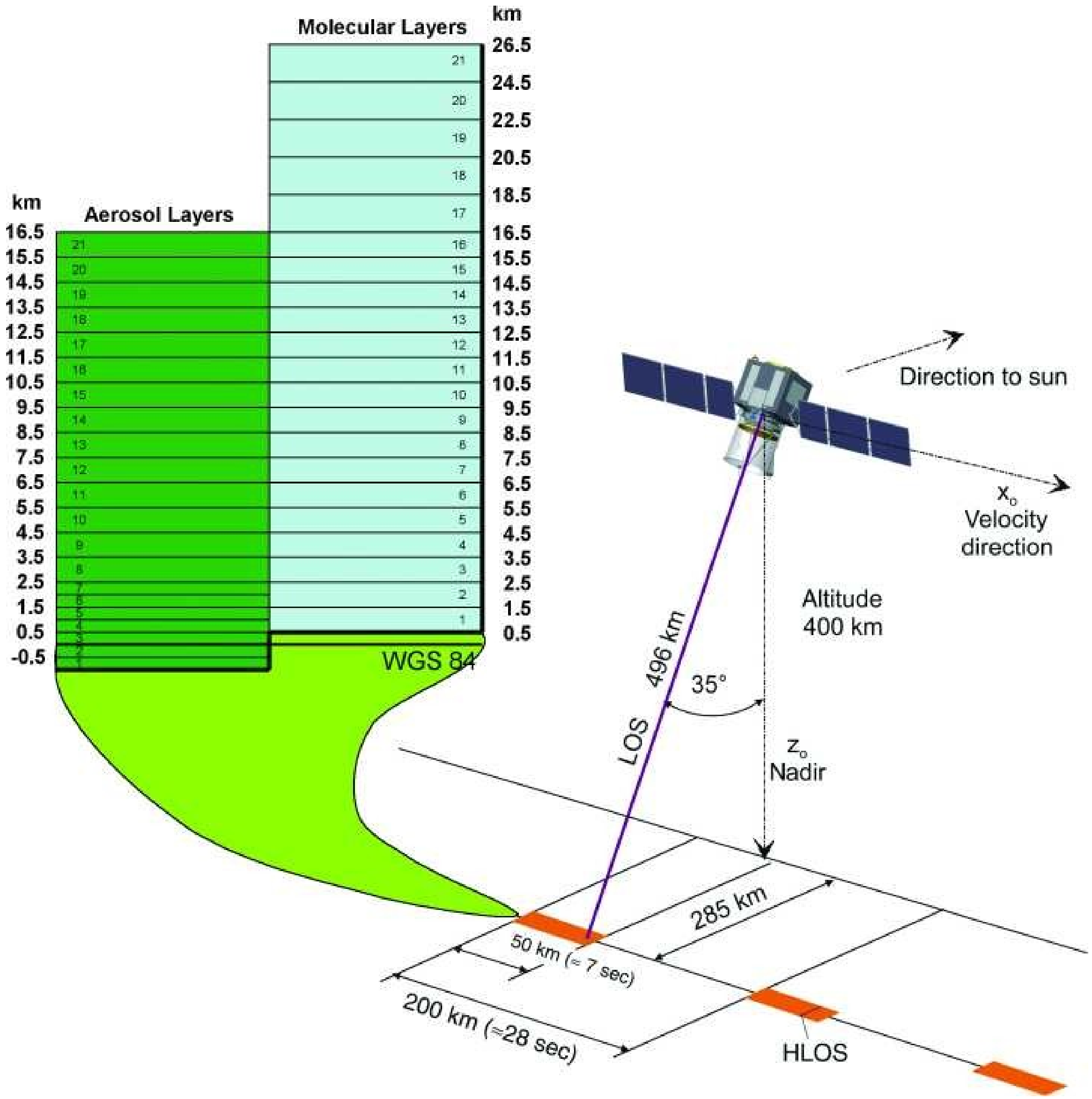
Overview

- Motivation: Analysis are problematic in the Tropics.
- Method: Equatorial waves as a balance relationship in data assimilation.
- Characterization of waves
  - in free model runs and analysis
  - impact of QBO and tides
- Data assimilation with equatorial waves
- Conclusions



Zonal mean standard deviation
of zonal wind difference
between NCEP/NCAR reanal. and ERA40
m/s
10
9
8
7
6
5
4
3
2
Pressure (hPa)
10
30
100
300
1000
SP
60°S
30°S
EQ
30°N
60°N
NP
Based on 6 hourly values from 2001

The Earth Explorer Atmospheric Dynamics Mission (ADM-Aeolus)



- Doppler Wind Lidar yields vertical profiles of the horizontal wind
- Flexible vertical resolution
- Accuracy: 2-3 m/s
- Secondary products: aerosol and cloud properties

Variational data assimilation

$$J(\delta x) = \underbrace{\delta \boldsymbol{x}^{\boldsymbol{T}} \boldsymbol{B}^{\boldsymbol{-1}} \delta \boldsymbol{x}}_{\text{Background term}} + \underbrace{\left[\boldsymbol{y} - H(\boldsymbol{x_b} + \delta \boldsymbol{x})\right]^T \boldsymbol{R}^{\boldsymbol{-1}} \left[\boldsymbol{y} - H(\boldsymbol{x_b} + \delta \boldsymbol{x})\right]}_{\text{Observation term}}$$

Analysis: $\boldsymbol{x_a} = \underbrace{\boldsymbol{x_b}}_{\text{background}} + \underbrace{\delta \boldsymbol{x}}_{\text{increment}}$

Observations: $\boldsymbol{y}$

Background and observation error covariance: $\boldsymbol{B}, \ \boldsymbol{R}$

Forward model: H

Variational data assimilation

$$J(\delta x) = \underbrace{\delta \boldsymbol{x}^{T} \boldsymbol{B}^{-1} \delta \boldsymbol{x}}_{\text{Background term}} + \underbrace{\left[\boldsymbol{y} - H(\boldsymbol{x_b} + \delta \boldsymbol{x})\right]^{T} \boldsymbol{R}^{-1} \left[\boldsymbol{y} - H(\boldsymbol{x_b} + \delta \boldsymbol{x})\right]}_{\text{Observation term}}$$

Analysis: $\boldsymbol{x_a} = \boldsymbol{x_b} + \delta \boldsymbol{x}$ (background, increment)

Observations: $\boldsymbol{y}$

Background and observation error covariance: $\boldsymbol{B}, \boldsymbol{R}$

Forward model: H

$$B^{1/2} = B^{1/2}_{\text{multivariate}} B^{1/2}_{\text{space}}$$

Variational data assimilation

$$J(\delta x) = \delta \boldsymbol{x}^T \boldsymbol{B}^{-1} \delta \boldsymbol{x} + [\boldsymbol{y} - H(\boldsymbol{x_b} + \delta \boldsymbol{x})]^T \boldsymbol{R}^{-1} [\boldsymbol{y} - H(\boldsymbol{x_b} + \delta \boldsymbol{x})]$$

Background term   Observation term

Analysis: $\boldsymbol{x_a} = \boldsymbol{x_b} + \delta \boldsymbol{x}$

background   increment

Observations: $\boldsymbol{y}$

Background and observation error covariance: $\boldsymbol{B}, \boldsymbol{R}$

Forward model: H

$$\boldsymbol{B}^{1/2} = \boldsymbol{B}^{1/2}_{\text{multivariate}} \boldsymbol{B}^{1/2}_{\text{space}}$$

Use mass/wind balance relationships in $\boldsymbol{B}$

Variational data assimilation

$$J(\delta x) = \underbrace{\delta \boldsymbol{x}^T \boldsymbol{B}^{-1} \delta \boldsymbol{x}}_{\text{Background term}} + \underbrace{\left[\boldsymbol{y} - H(\boldsymbol{x_b} + \delta \boldsymbol{x})\right]^T \boldsymbol{R}^{-1} \left[\boldsymbol{y} - H(\boldsymbol{x_b} + \delta \boldsymbol{x})\right]}_{\text{Observation term}}$$

Analysis: $\boldsymbol{x_a} = \underbrace{\boldsymbol{x_b}}_{\text{background}} + \underbrace{\delta \boldsymbol{x}}_{\text{increment}}$

Observations: $\boldsymbol{y}$

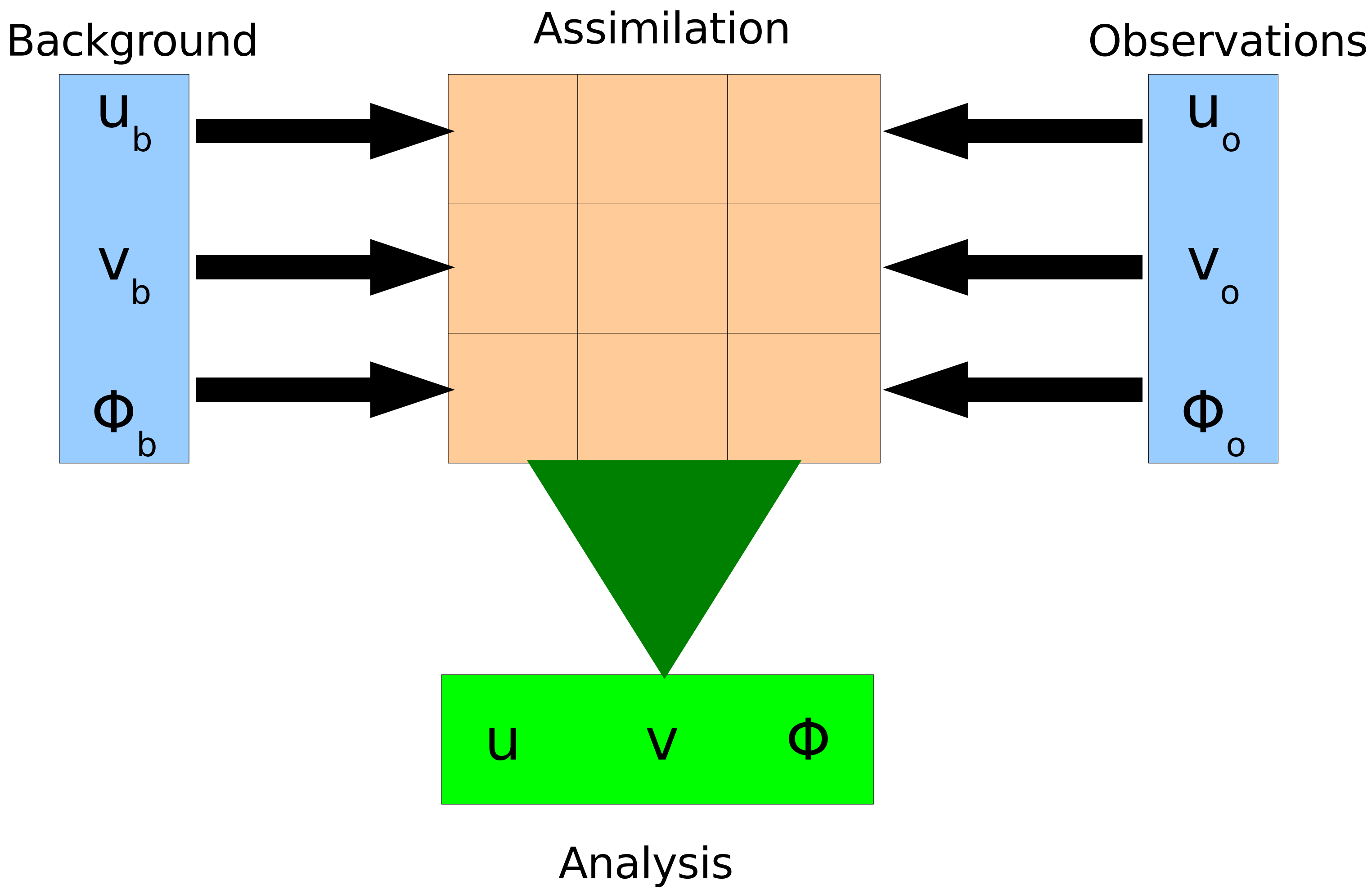
Background and observation error covariance: $\boldsymbol{B}, \ \boldsymbol{R}$

Forward model: H

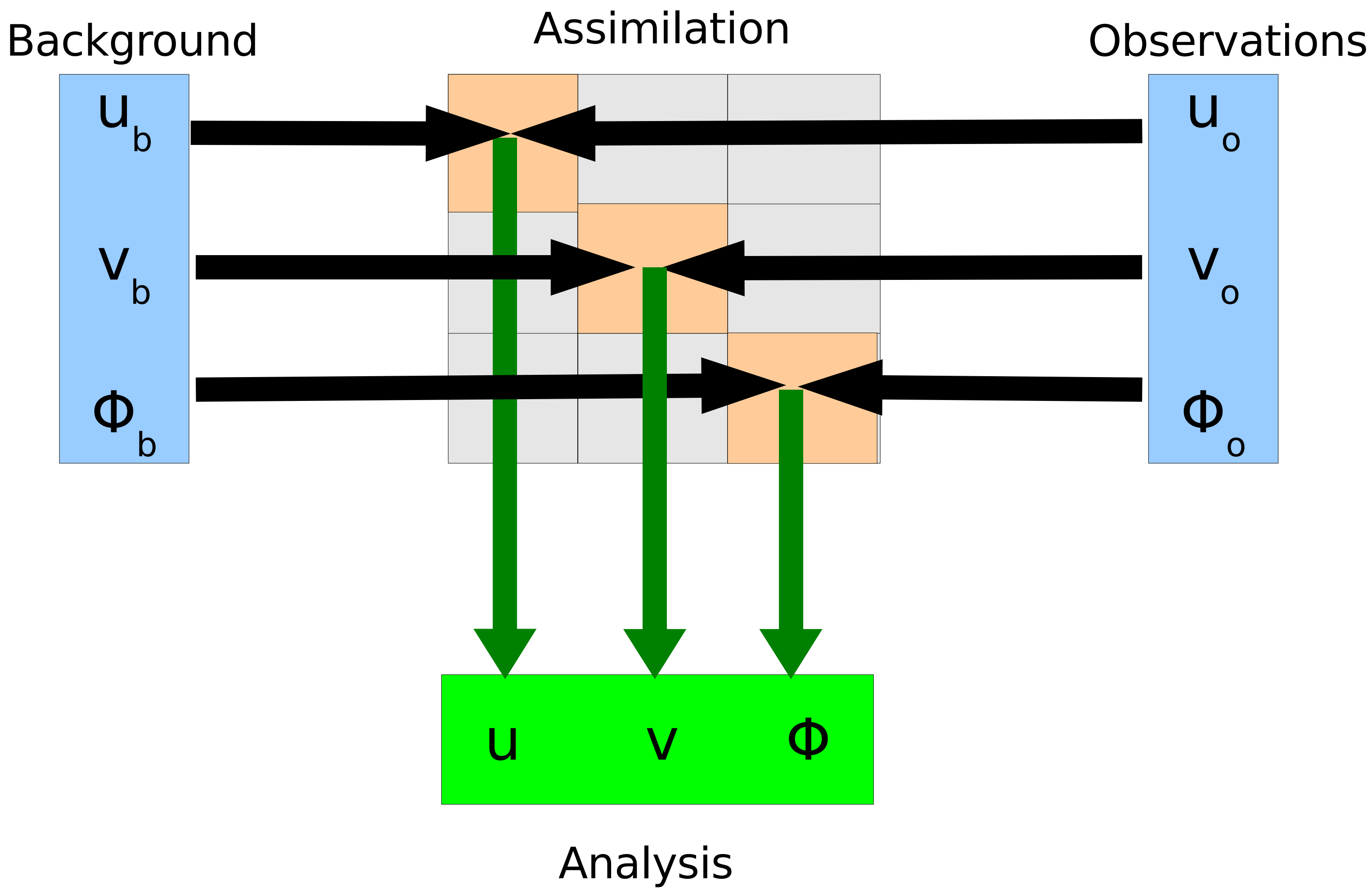
$$\boldsymbol{B}^{1/2} = \boldsymbol{B}^{1/2}_{\text{multivariate}} \boldsymbol{B}^{1/2}_{\text{space}}$$

Use mass/wind balance relationships in $\boldsymbol{B}$

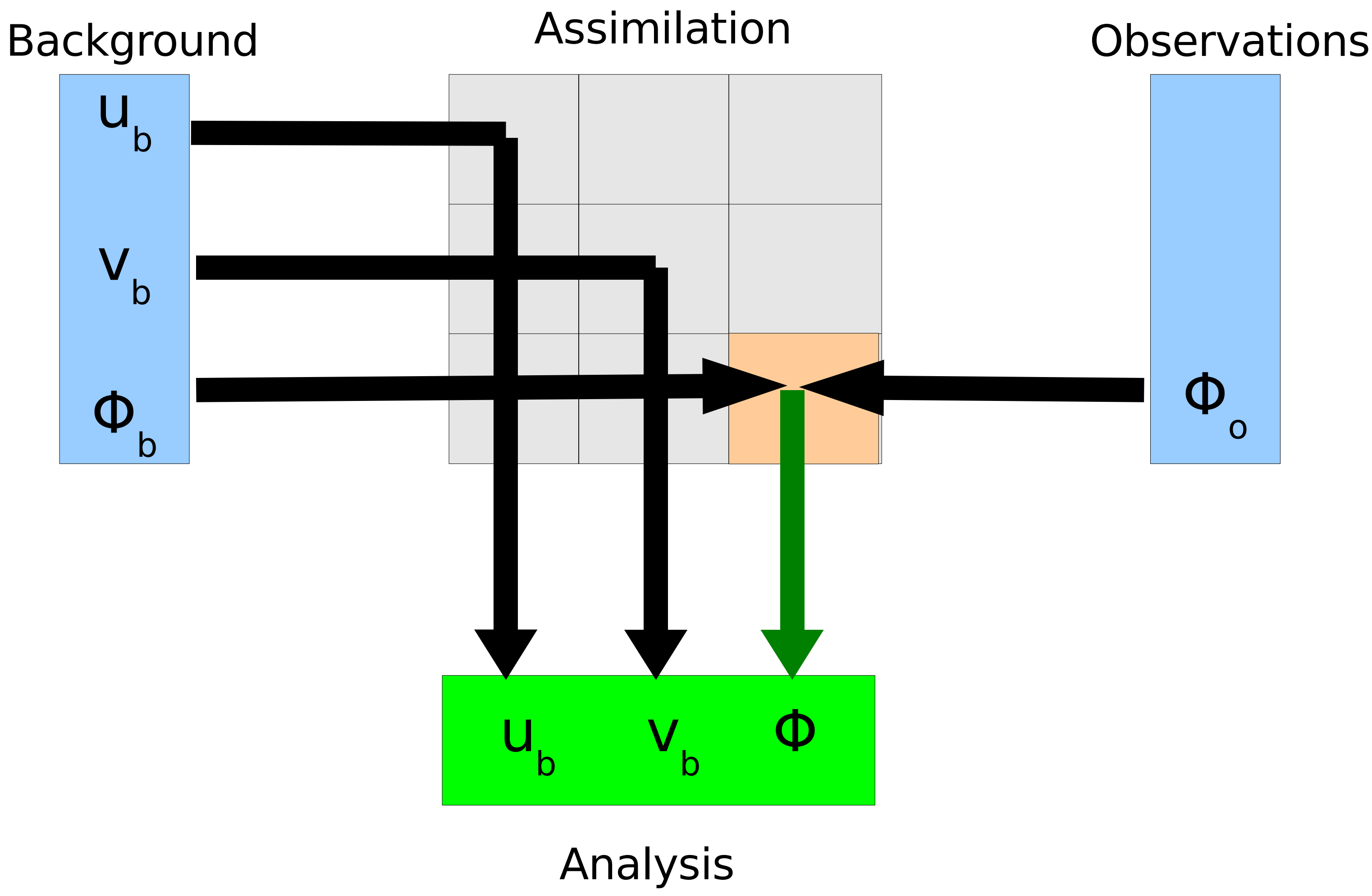
Multivariate analysis



Univariate analysis



Univariate analysis in the tropics



Mass/wind balance relationships in data assimilation

- Midlatitudes:
  - conventionally included in data assimiation
  - Geostrophy
  - Rossby waves

→

Hough modes with equivalent depth =10.000 m

- Equator:
  - Žagar et al. 2004 QJ RMS
  - Equatorial waves:
  - Kelvin, Rossby, Mixed Rossby-gravity, inertia-gravity waves

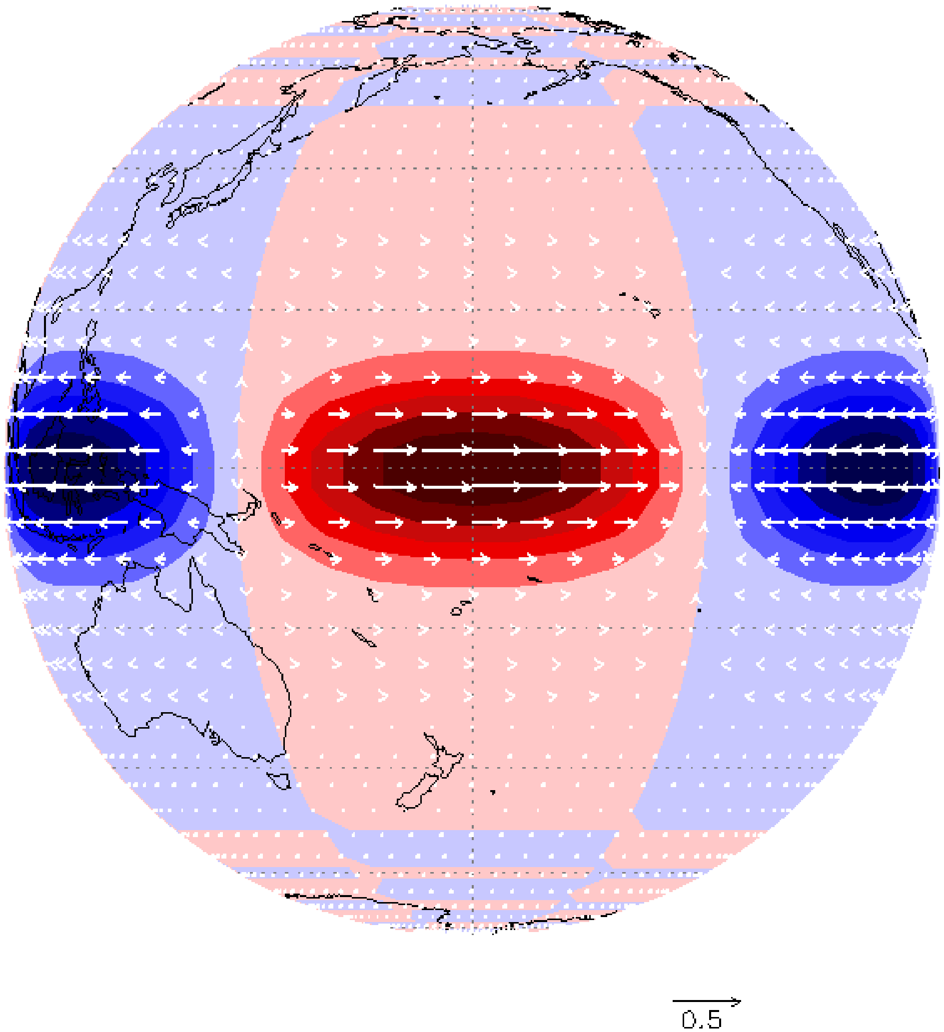
→

Hough modes with equivalent depth =25 m

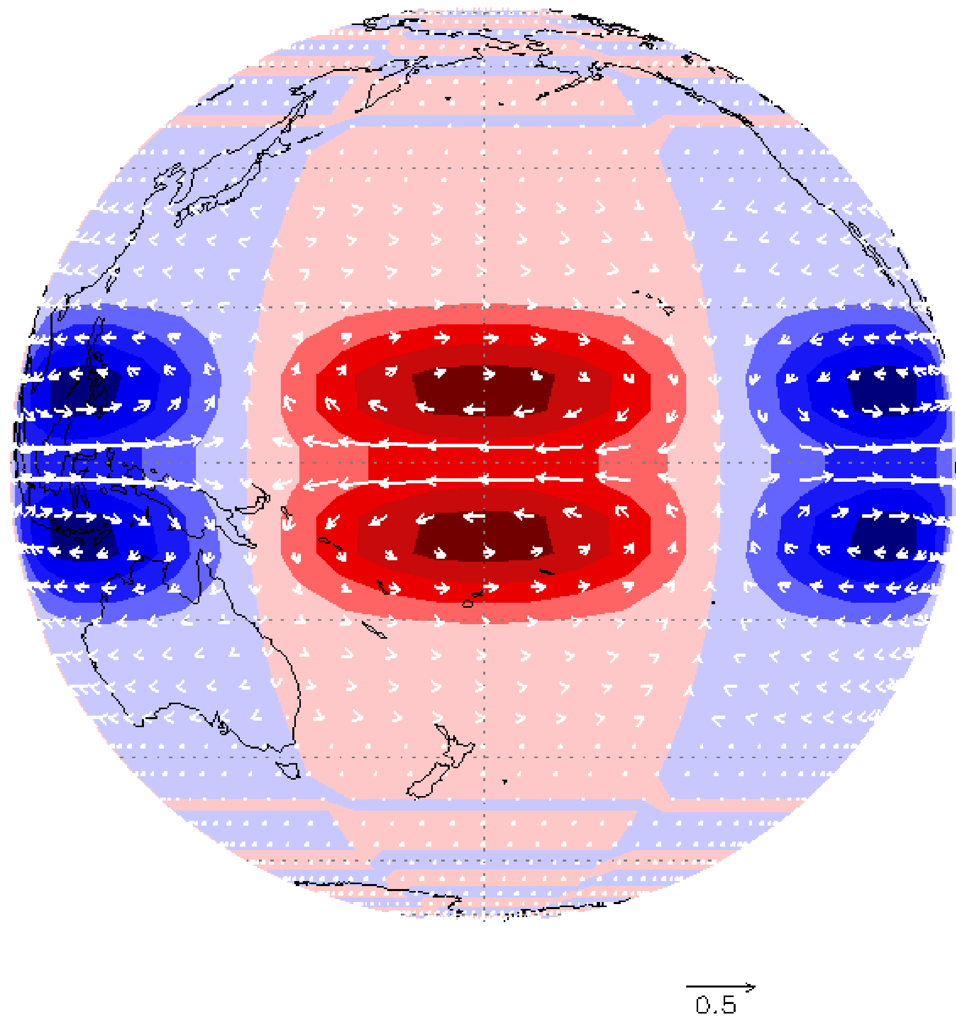
Combination of both: Körnich and Källén, Tellus 2007

Equatorially trapped wave modes

Kelvin mode



Rossby mode, n=1



Hough modes for equivalent depth H=25m and zonal wave number m=3.
Shadings: geopotential height (m), vectors: winds (m/s).

Characterization of equatorial waves

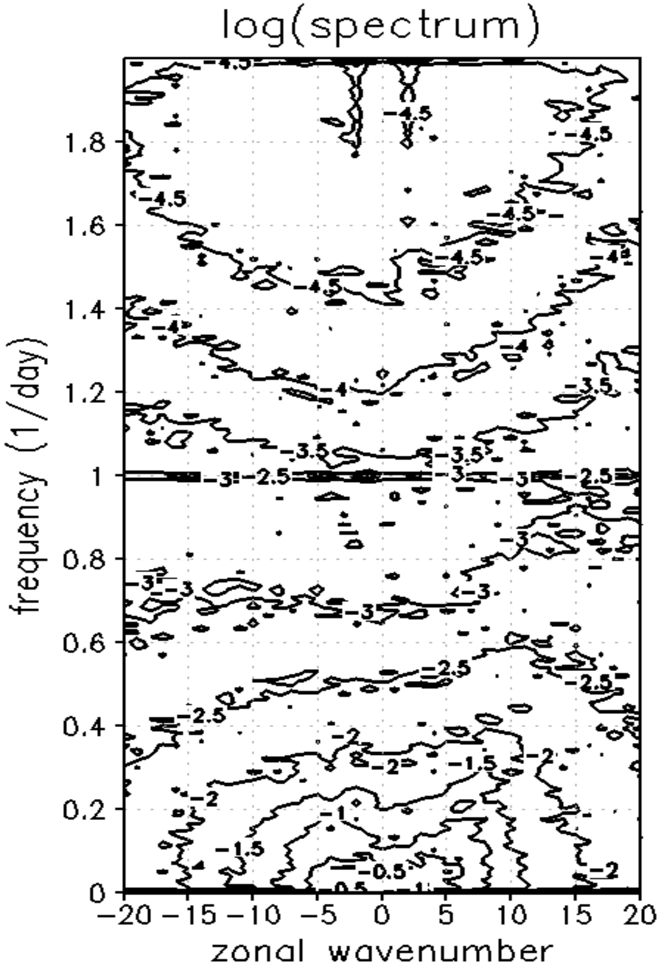
- Wavenumber-frequency spectra shows dispersion relation.
- Projection of data on equatorial waves yields the variance of each wave mode.

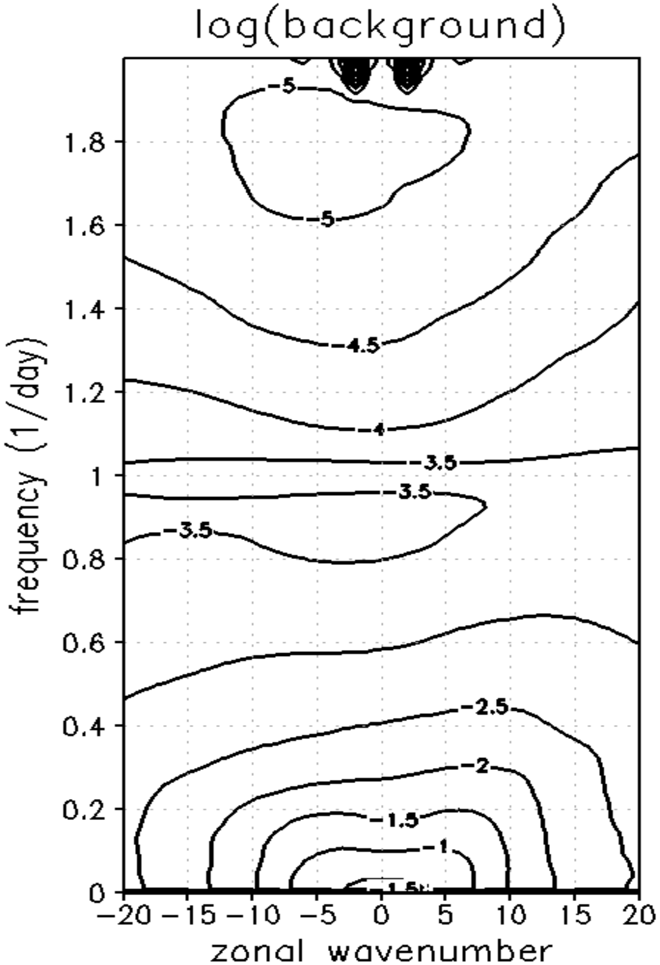
Data

- **CMAM**, T47 L72, from Thomas Birner. 3 years, continuous negative QBO-phase.
- **CMAM**, T47 L98, from James Anstey, 10 years with QBO.
- **GEM-BACH**, grid 240x120, 80 levels, 1 year, QBO- (aug-nov) QBO+ (jan-sept)
- **CMAM-DAS**, year 2002, T47 L72, from Shuzhan Ren, QBO+
- **ERA40 reanalysis**, 1991-2001 from ECMWF. 1996 QBO-, 1999 QBO+

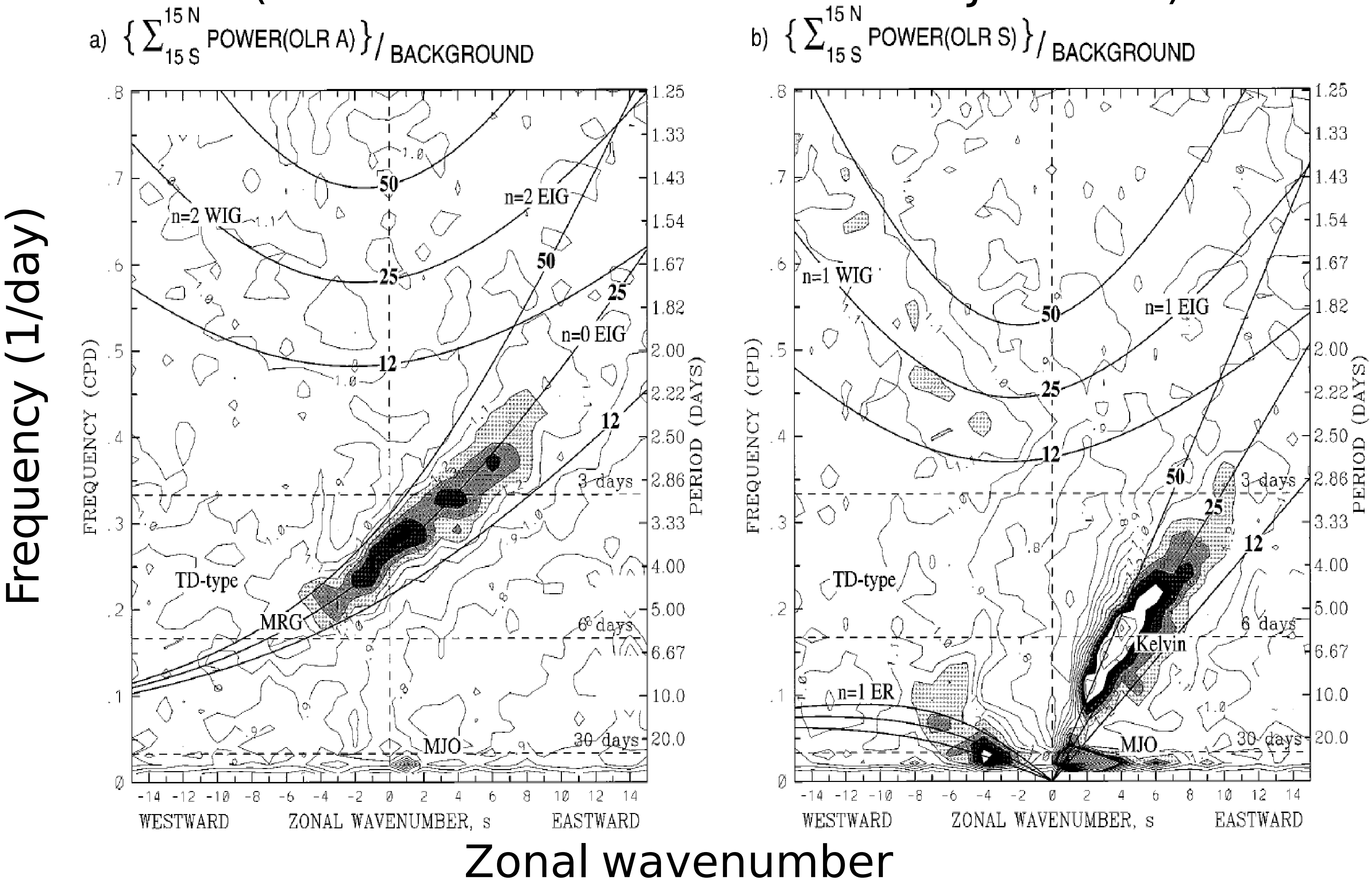
Wavenumber-frequency spectra

- Following Wheeler and Kiladis (JAS 1999)
- Using winds, and temperature from 15°S to 15°N
- Separate data in parts symmetric and antisymmetric to Equator
- 2D- Fourier transformation in longitude and time



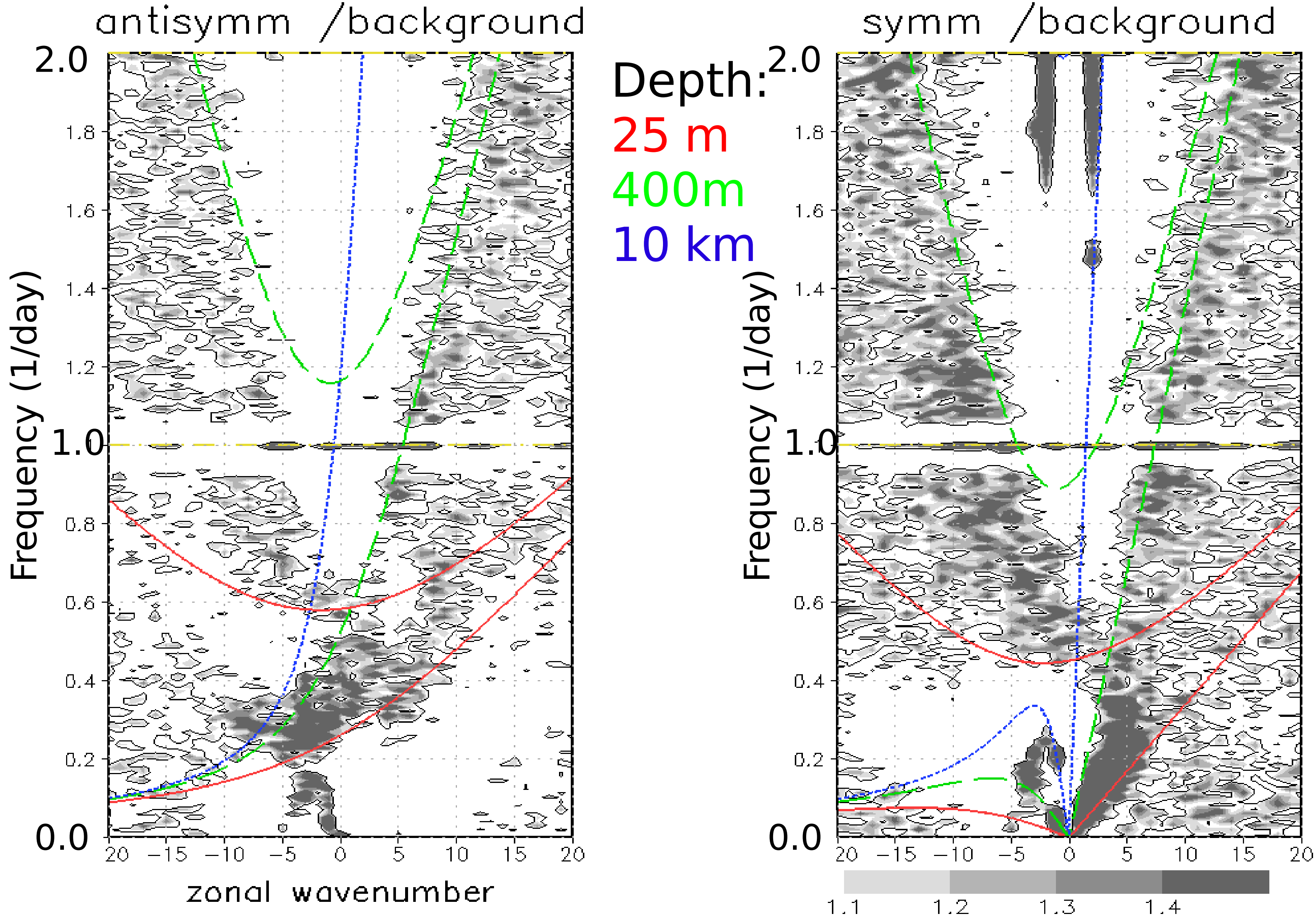


Spectra of the outgoing longwave radiation
(taken from Wheeler and Kiladis JAS 1999)



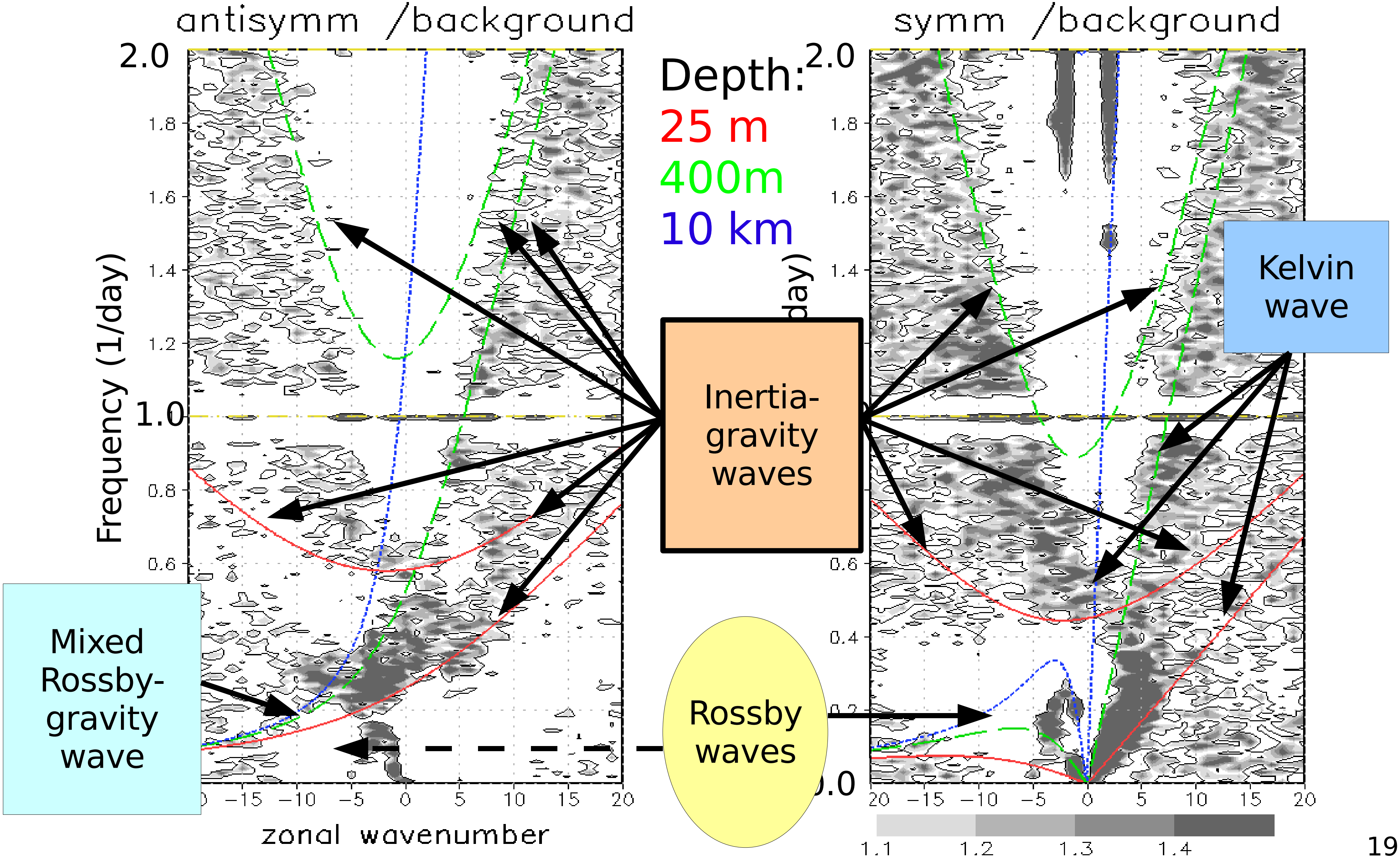
Wavenumber-frequency spectrum, CMAM zonal wind u, eta=0.052 (~50hPa)

CMAM T47L72



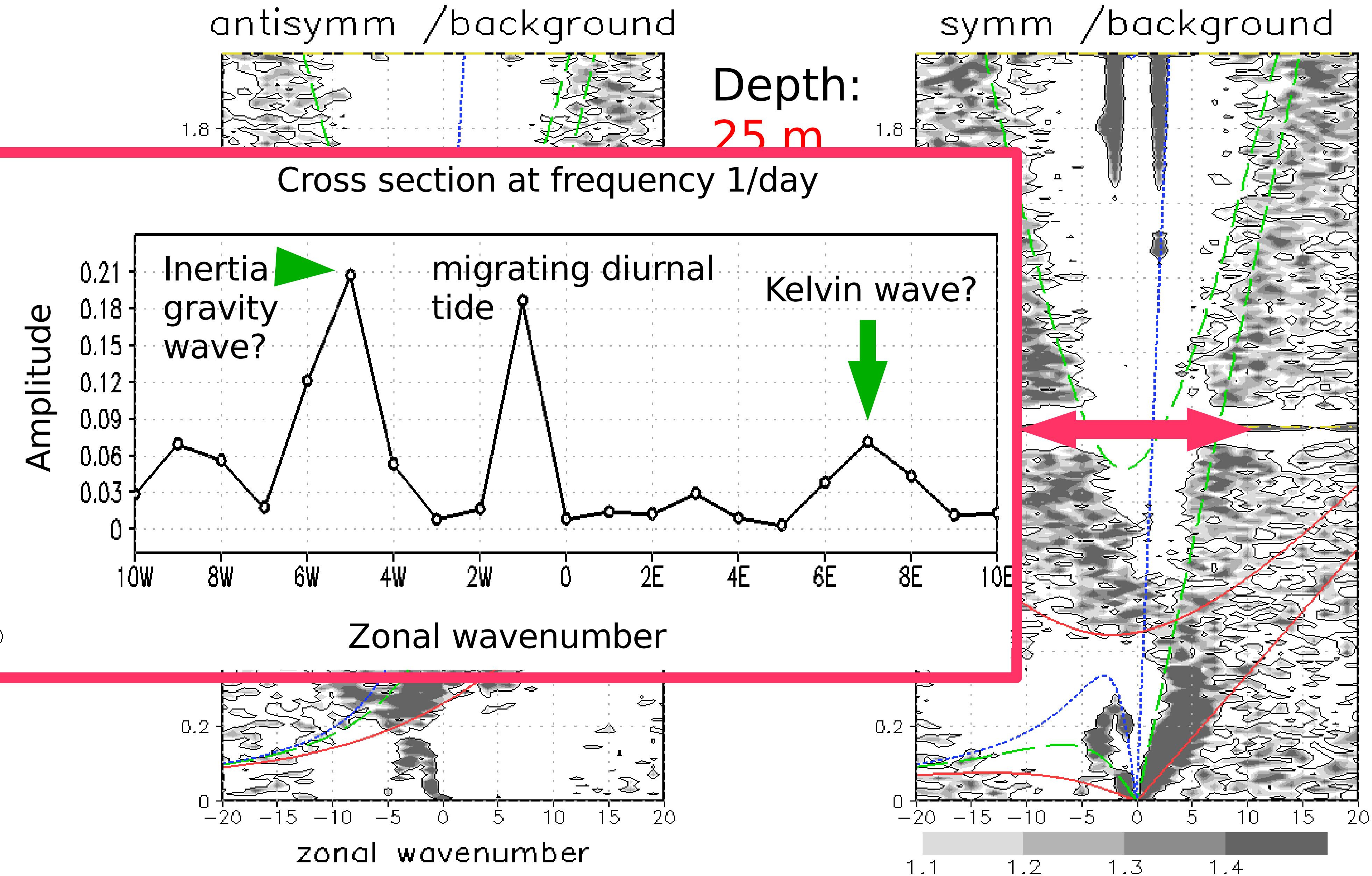
CMAM T47L72

Wavenumber-frequency spectrum, CMAM zonal wind u, eta=0.052 (~50hPa)



Wavenumber-frequency spectrum, CMAM zonal wind u, eta=0.052 (~50hPa)

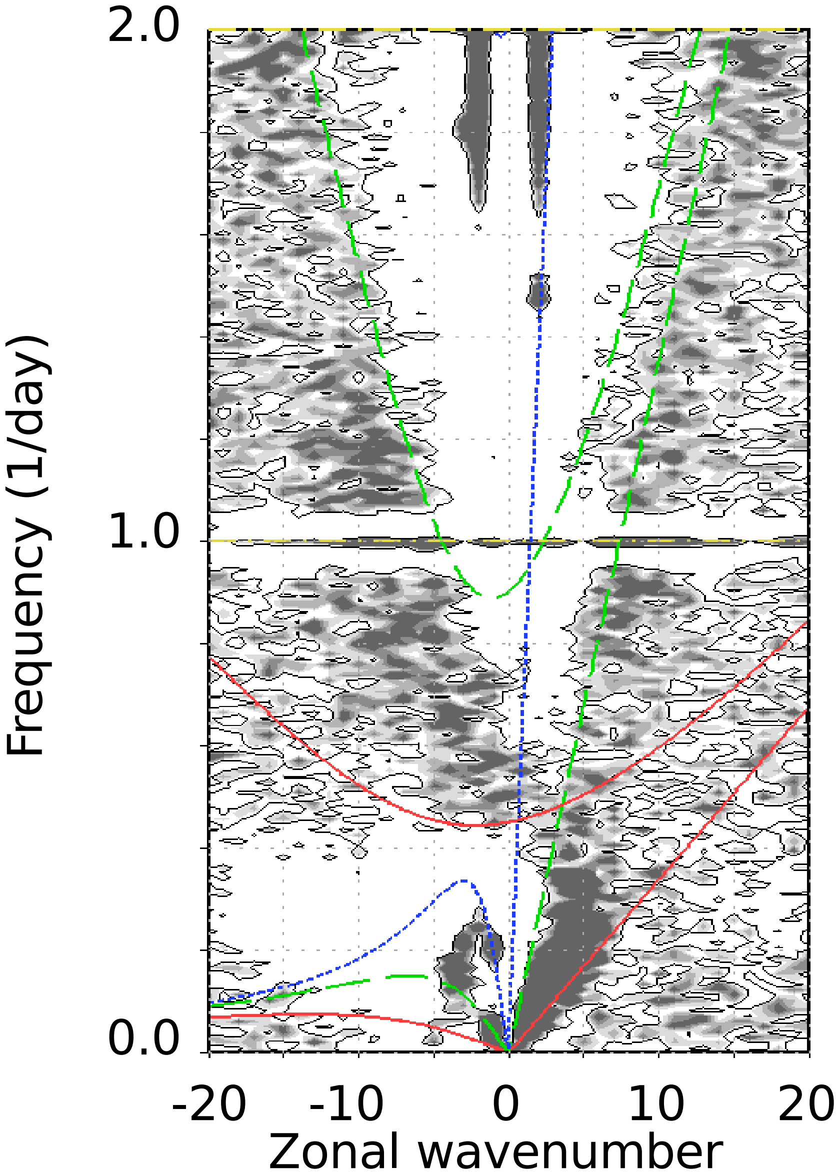
CMAM T47L72



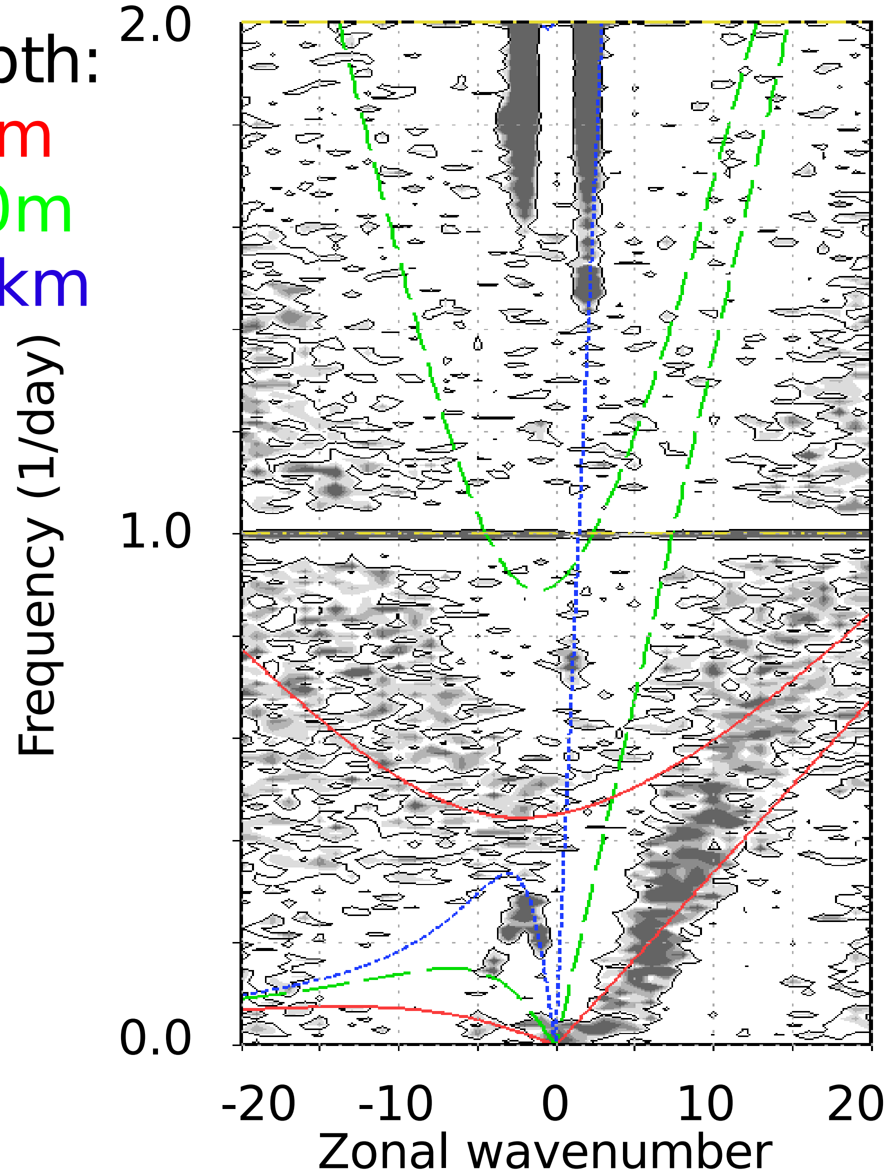
CMAM
T47L72

Symmetrical zonal-wind spectra, CMAM,

at eta=0.052 (~50hPa) and eta=0.474 (500hPa)



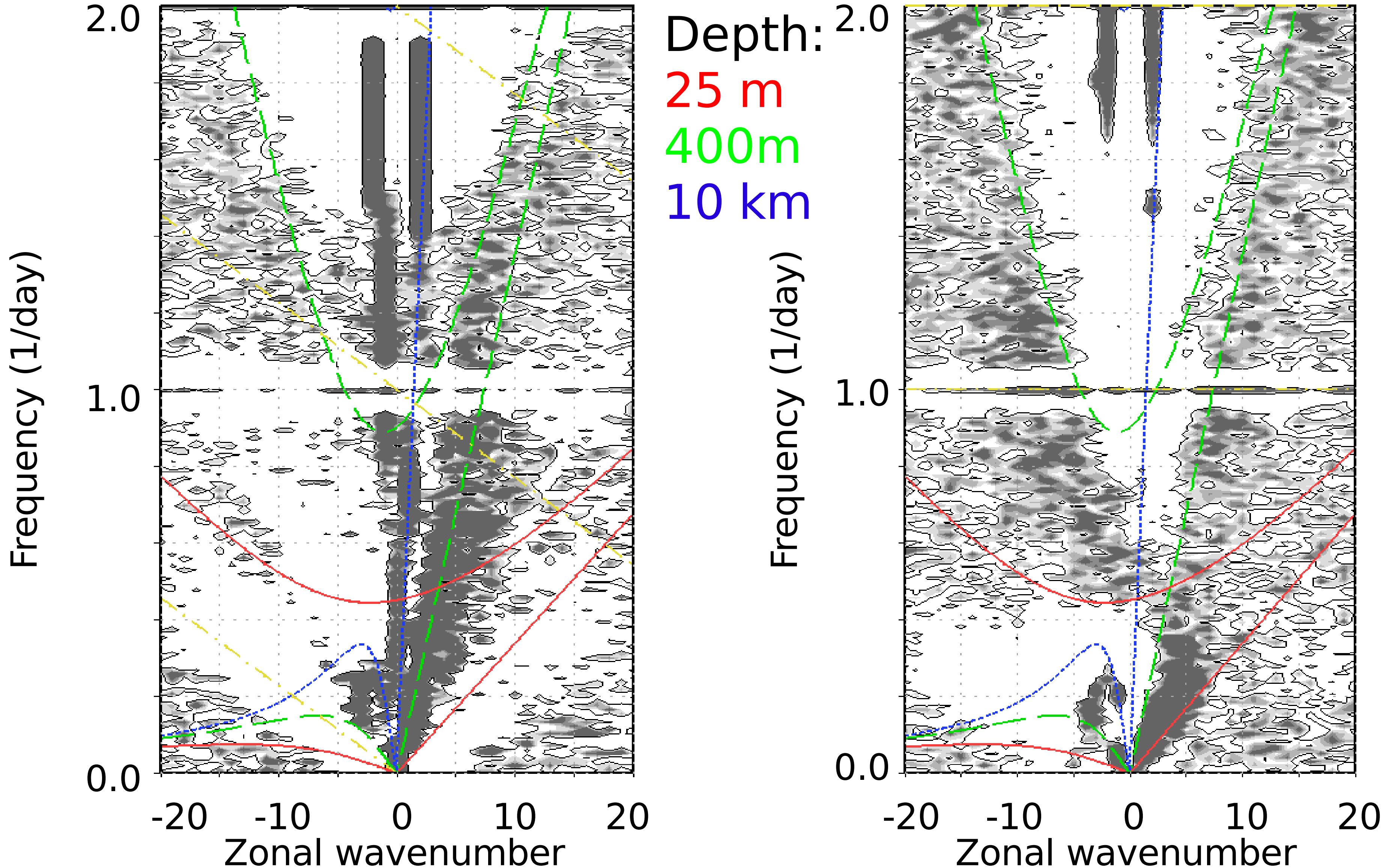
Depth:
25 m
400m
10 km



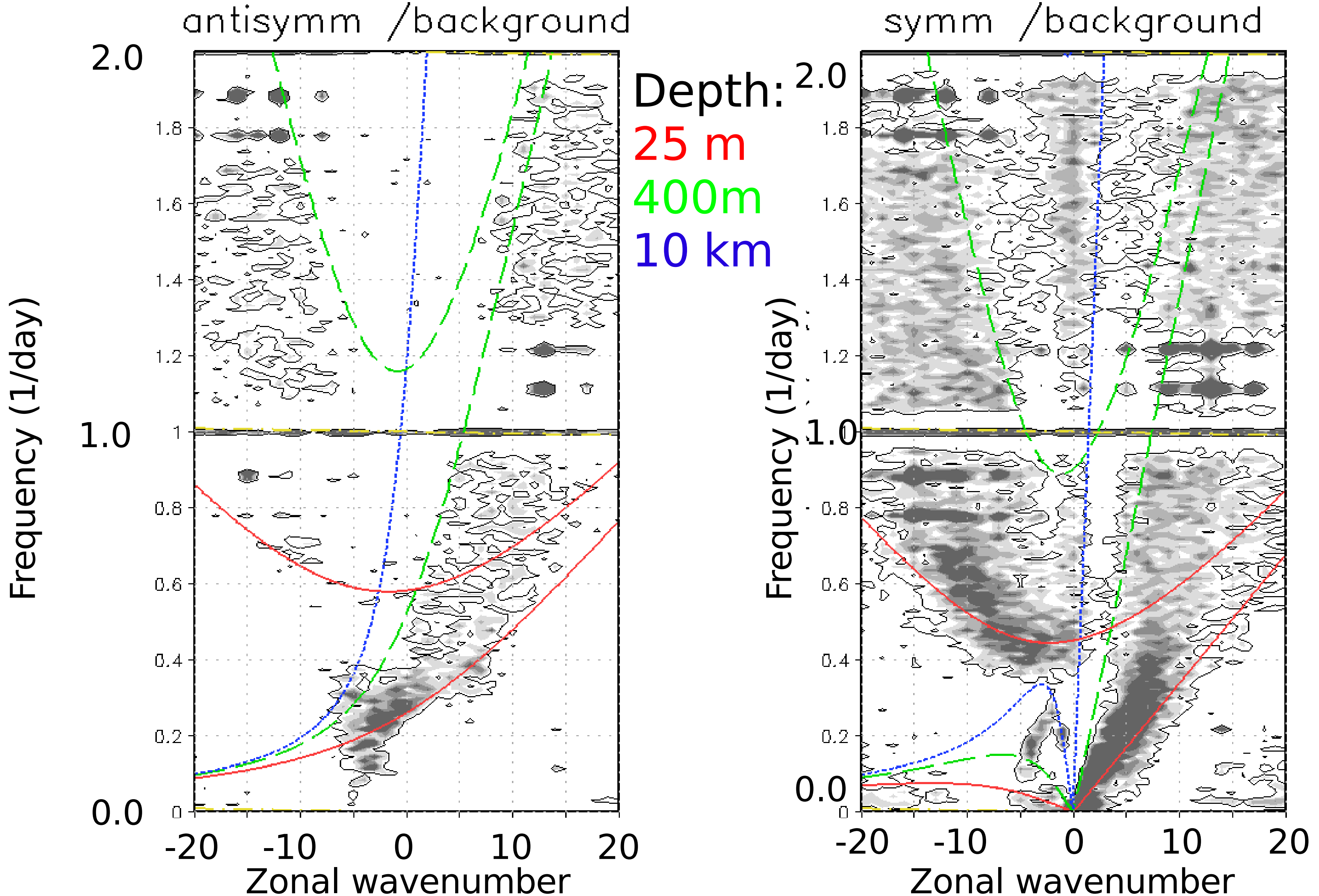
Model comparison:

Symmetrical u-Spectra in free model runs, eta≈0.05 (50hPa)

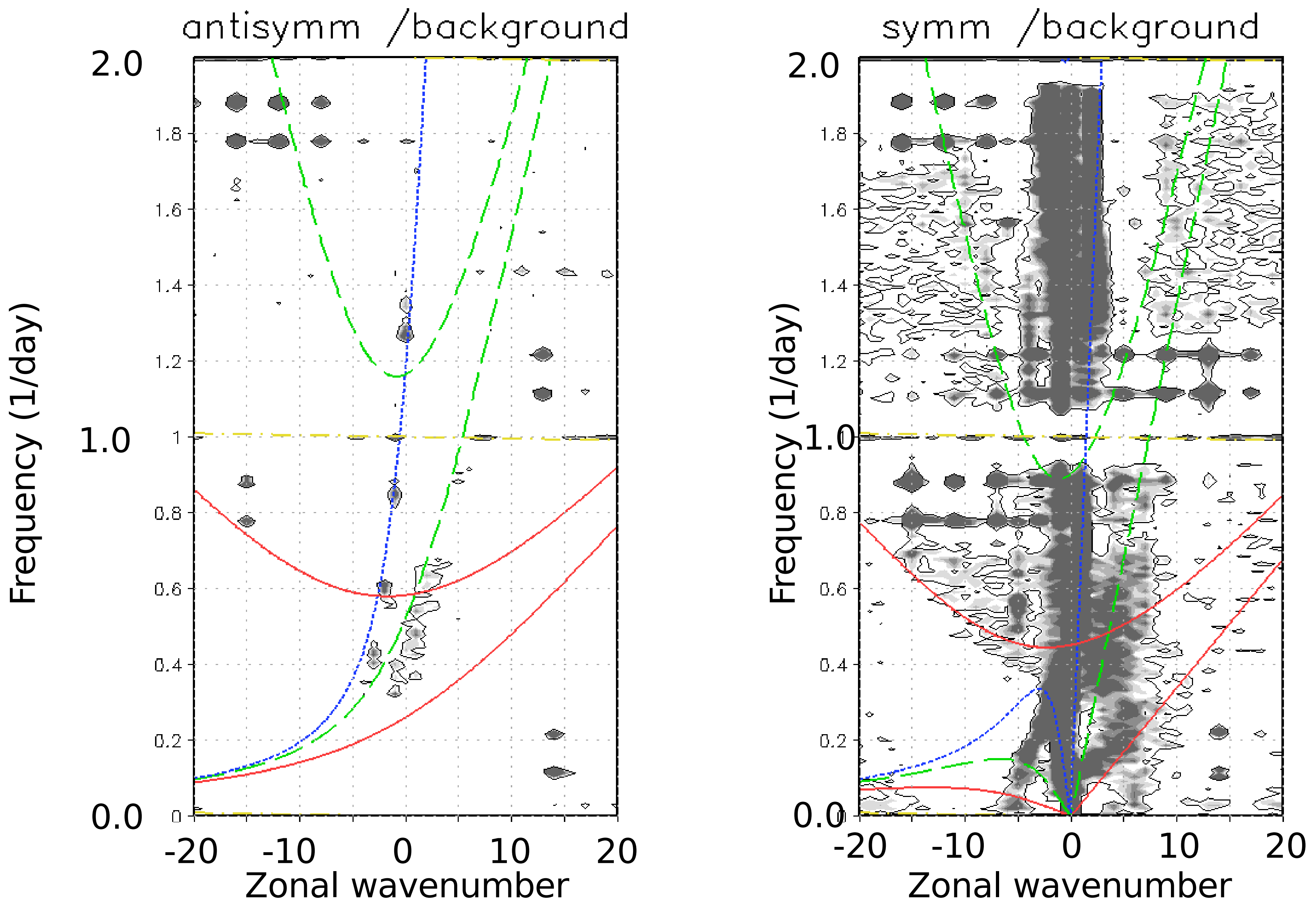
GEM-BACH and CMAM



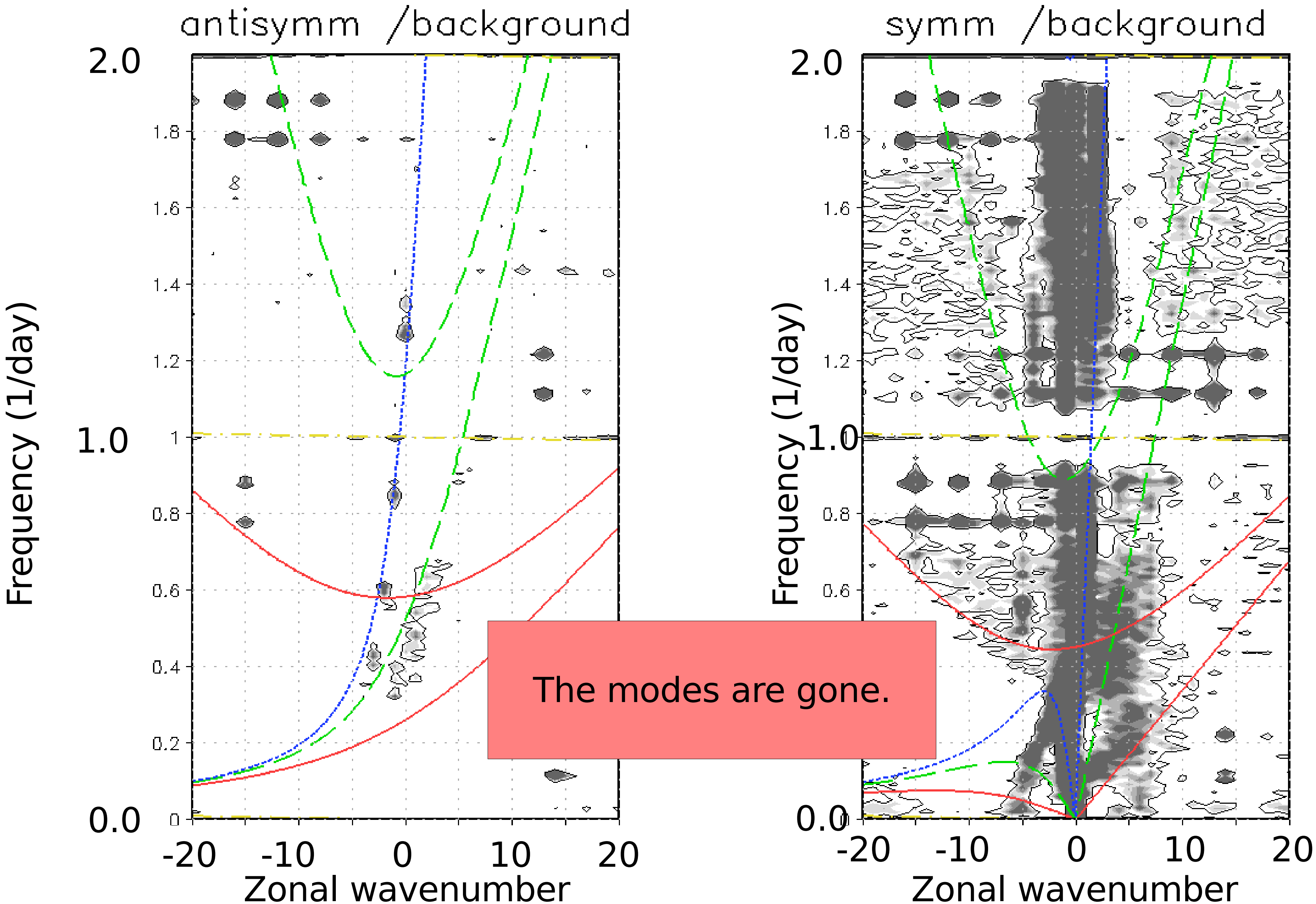
u-Spectra, 100hPa, ERA40-reanalysis



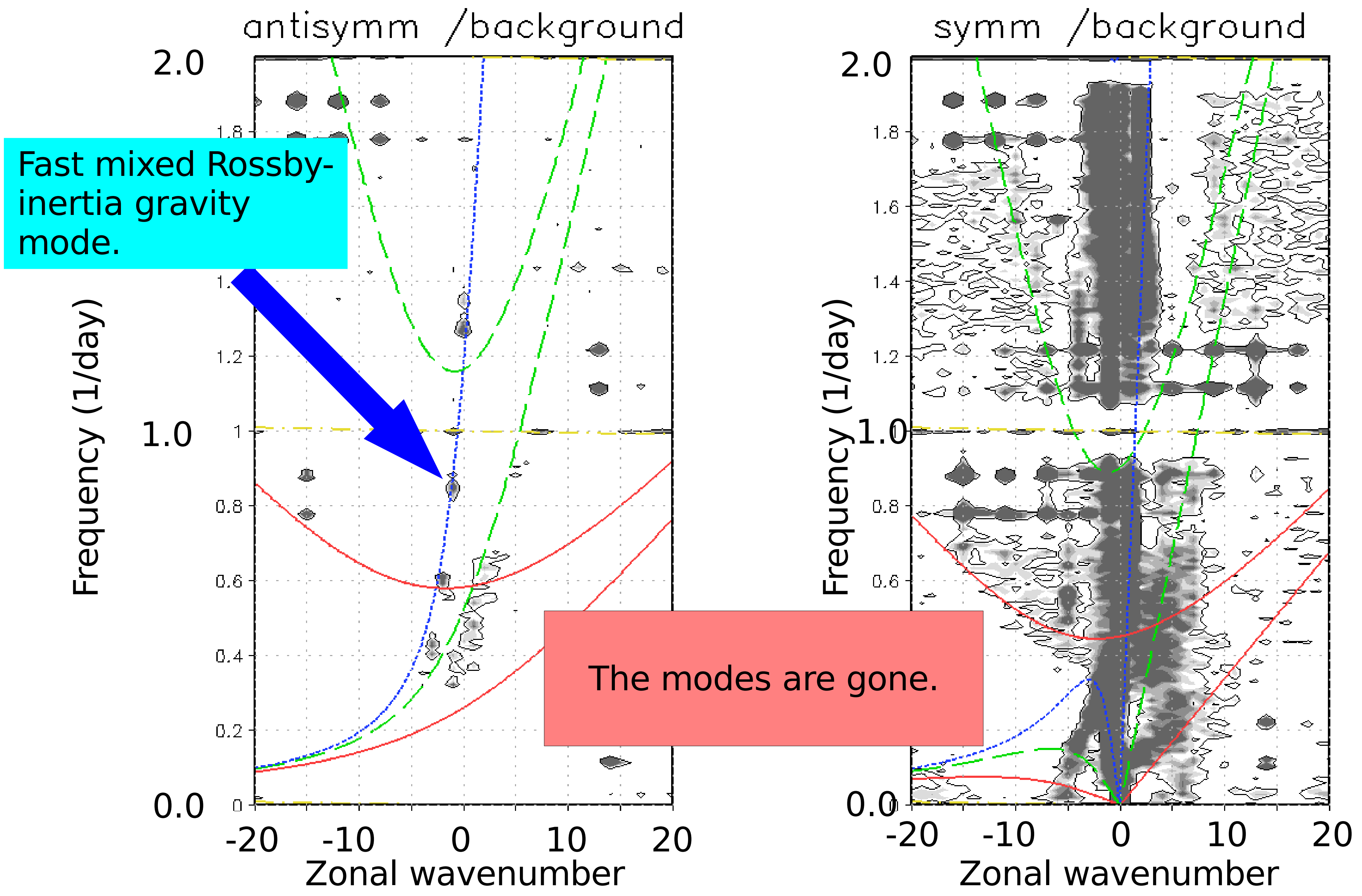
Z-Spectra, 100hPa, ERA40-reanalysis



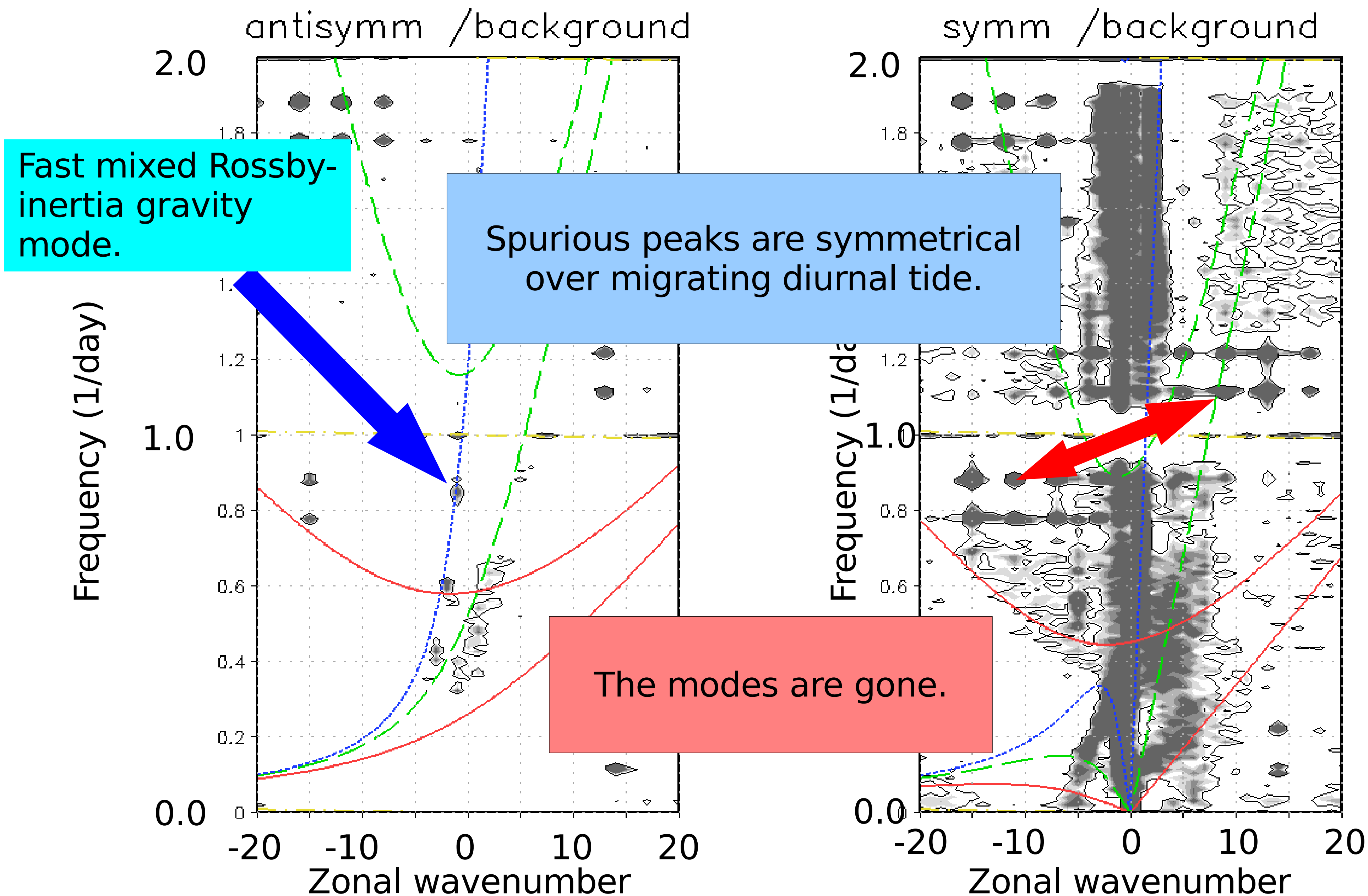
Z-Spectra, 100hPa, ERA40-reanalysis



Z-Spectra, 100hPa, ERA40-reanalysis



Z-Spectra, 100hPa, ERA40-reanalysis



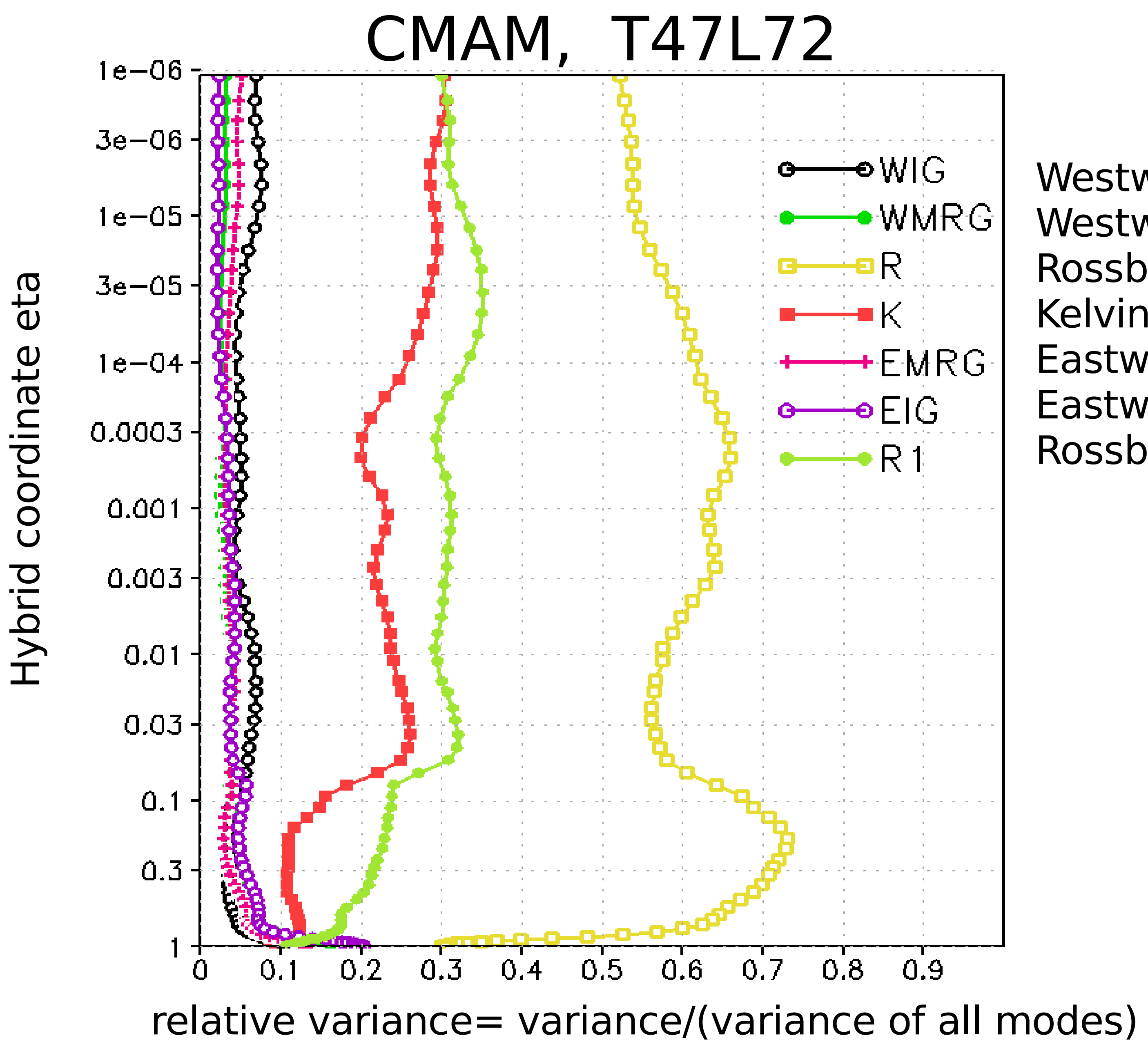
Data shows equatorial waves.

Quantification of variance for equatorial modes

➔ needed for background covariance matrix

- **Selection rule**: only equatorially trapped modes with periods longer than 6 hours
- Data D(x,y,t), Hough modes M(x,y), amplitude of modes A(t)
- Amplitudes $A = D^T M$

Relative variance of equatorial modes

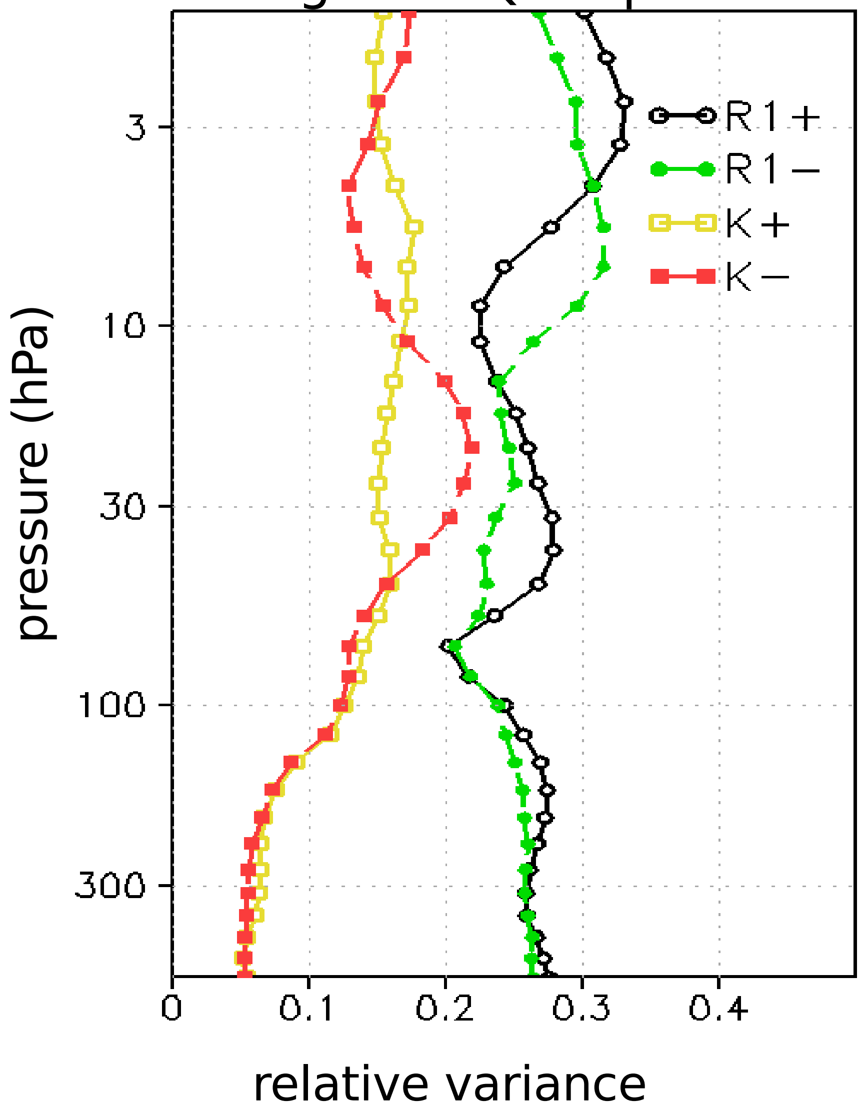


Westw. Inertia-gravity mode
Westw. mixed Rossby-gravity
Rossby wave
Kelvin wave
Eastw. inertia-gravity (n=0)
Eastw. inertia-gravity m.
Rossby wave (n=1)

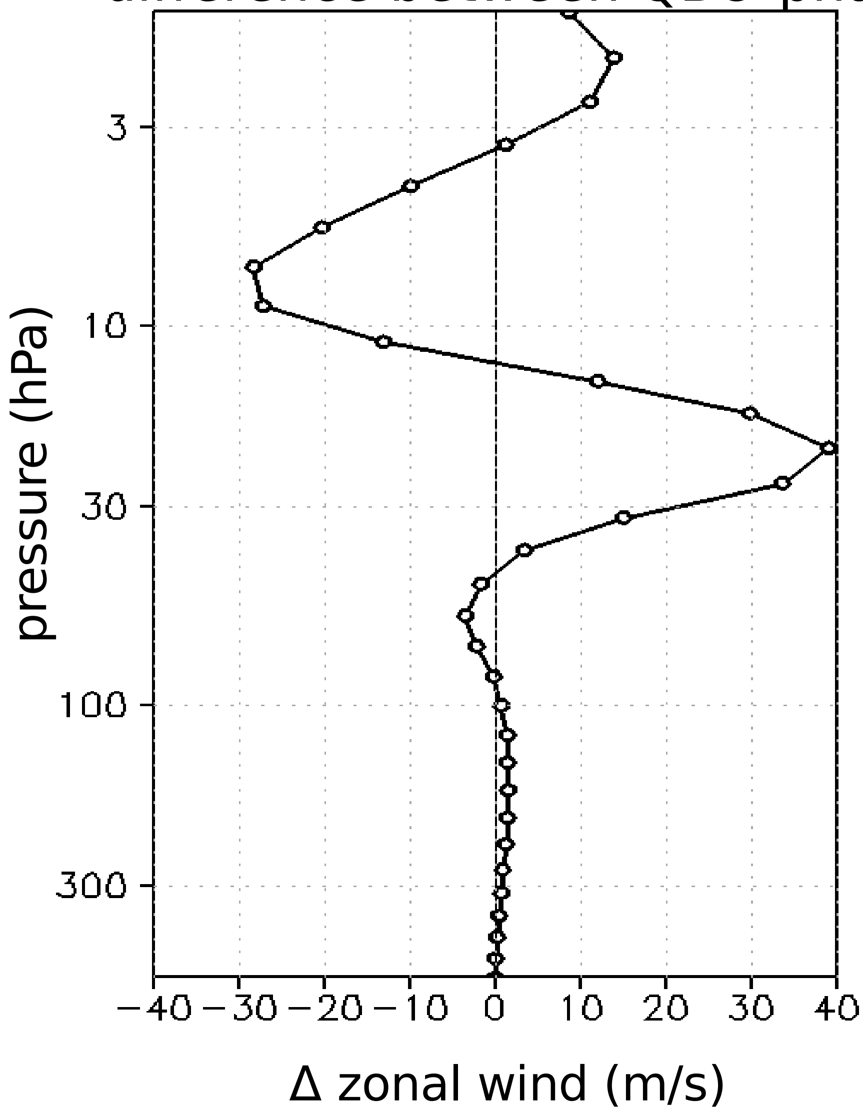
Effect of QBO on relative variances for Rossby (n=1) and Kelvin waves

Data:
CMAM T47L90
free model run

Relative variances for positive and negative QBO-phase



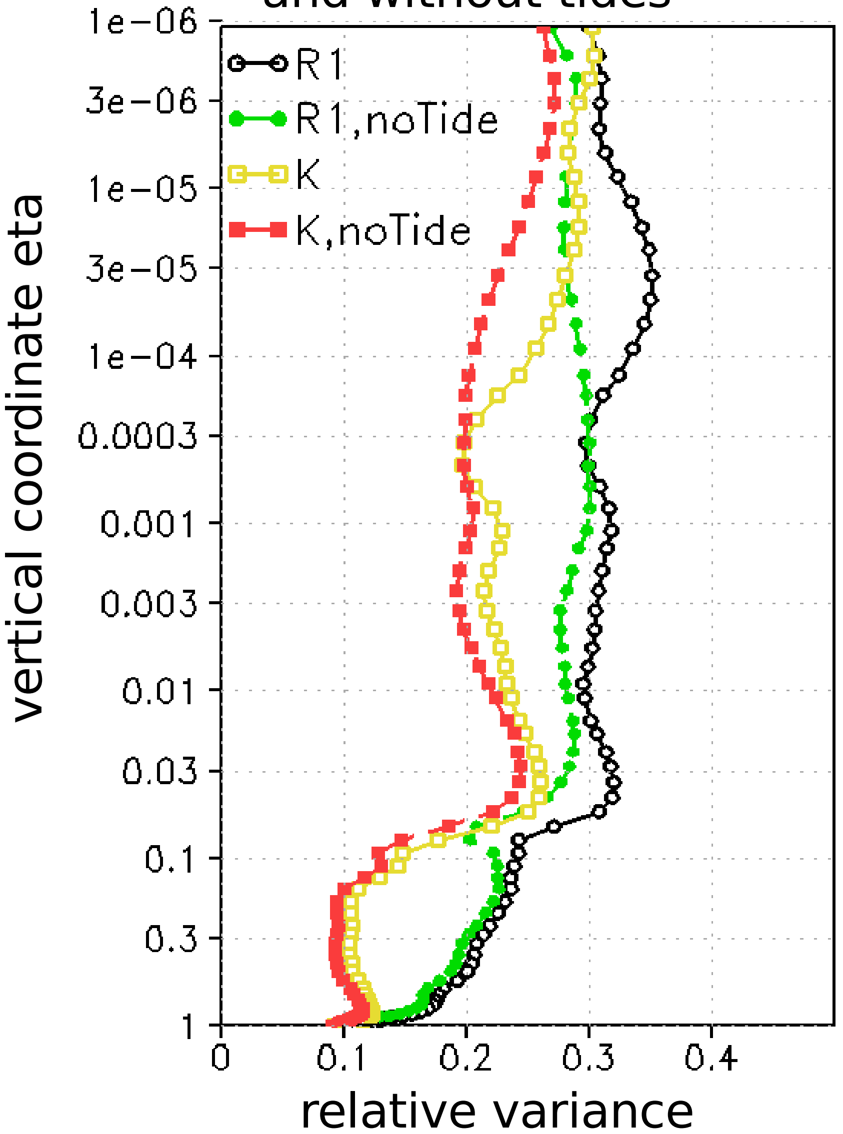
Zonal mean zonal wind difference between QBO-phases



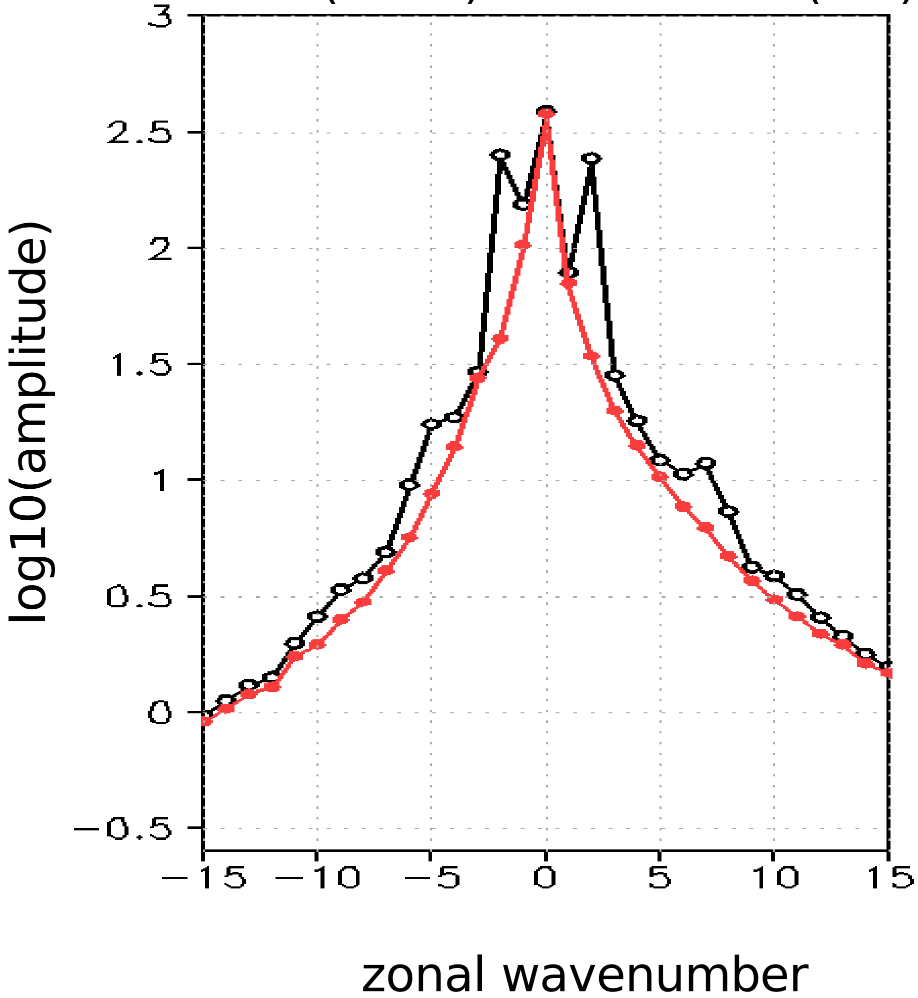
Contribution of tides to variances of Rossby (n=1) and Kelvin waves

Data:
CMAM T47L72
free model run

Relative variance with and without tides



Geopotential spectrum at eta=0.028 (30hPa). With tides (black)without tides (red).

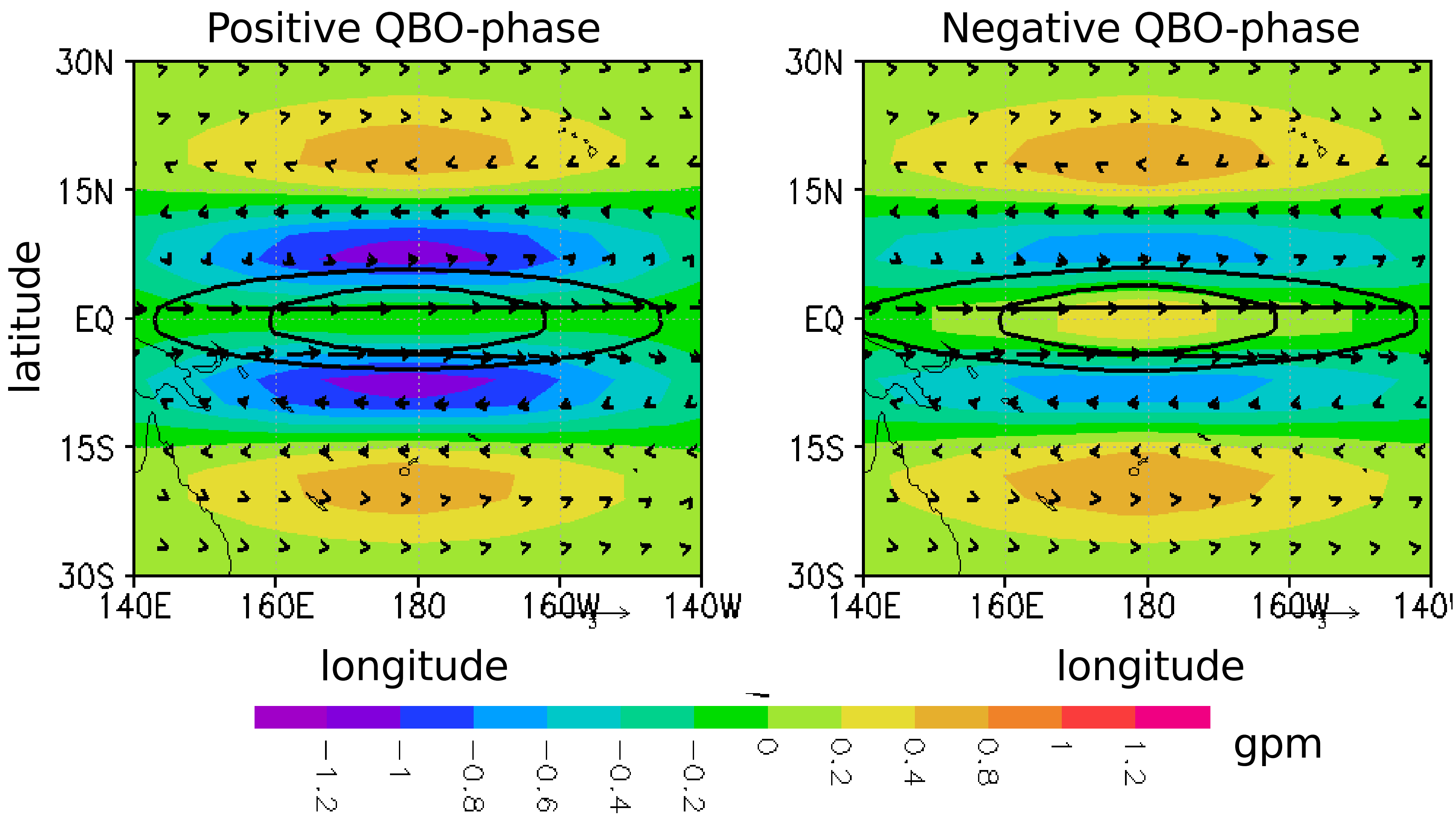


Data assimilation

- 3D-var data assimilaton with a global shallow water model
- single observation experiments with derived variances of modes
- Comparison of explained variance with nonlinear balance

Impact of the QBO on single zonal wind observation at the Equator and 26 hPa

Using equatorial mode variances from CMAM T47L98



winds: 3 m/s windspeed: contour interval 1 m/s

Equatorial waves vs. Charney balance

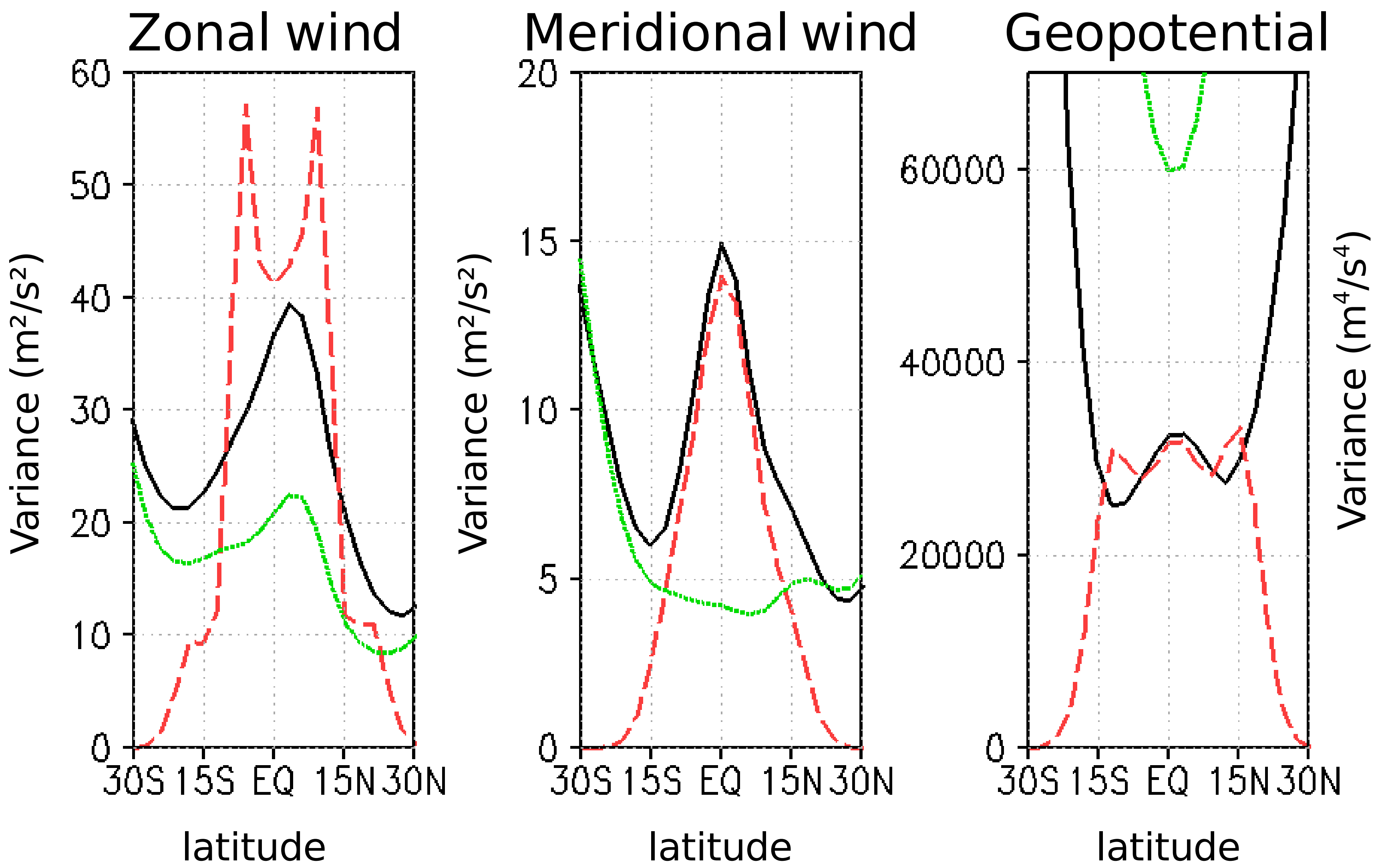
- Variance of the part explained by equatorial waves: $E = M A^T$
  - where M and A contain the modes and their amplitudes.
- Charney nonlinear balance :

$$\nabla^2 \Phi_B = \nabla \cdot (f \nabla \Psi) + \frac{2}{a^2} J\left(\frac{1}{\cos(\phi)} \frac{\partial \Psi}{\partial \lambda}, \frac{\partial \Psi}{\partial \phi}\right)$$

  - assume: given streamfunction Ψ

Total variance (black solid), explained variance by equatorial waves (red dashed) and by Charney's balance (green dotted) at eta=0.07 (70 hPa)

GEM-BACH



Conclusions

- Equatorial wave theory yields a mass/wind balance relationship for the tropical atmosphere.
- Variances of wave modes depend on height, model, quasi-biennial oscillation, and tides.
- Useful application is only given with tropical wind observations.
- Equatorial waves explain more tropical variance than the Charney nonlinear balance.

Acknowledgment

- Ted Shepherd and Saroja Polavarapu
- Thomas Birner, James Anstey, and Shuzhan Ren
- Matt Reszka

Thank you!