Zonal jet formation and Equatorial Super-rotation in the Destabilization of Short Mixed Rossby-Gravity Waves

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OUTLINE

1. Equatorial zonal jets

2. Destabilization of short westward MRG wave

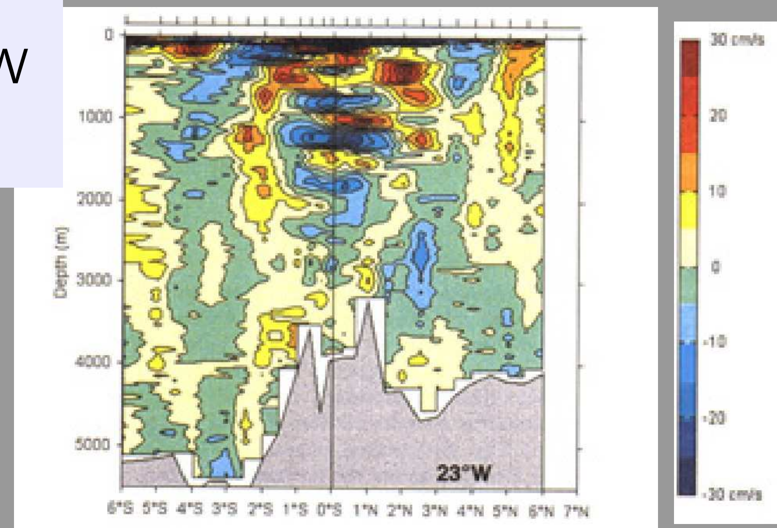
3. Inertial Instability and PV Homogenization

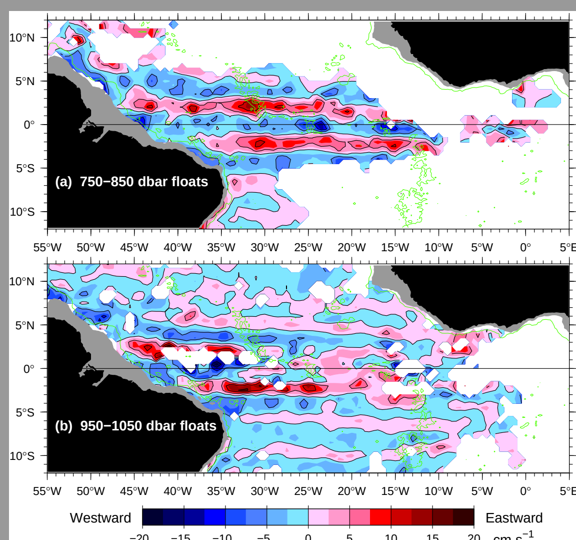
4. Super-rotation and the non-traditional Coriolis force

Gouriou et al. 2001 ⟹

– instantaneous merid. sect. 23° W

– strongly barotropic jets





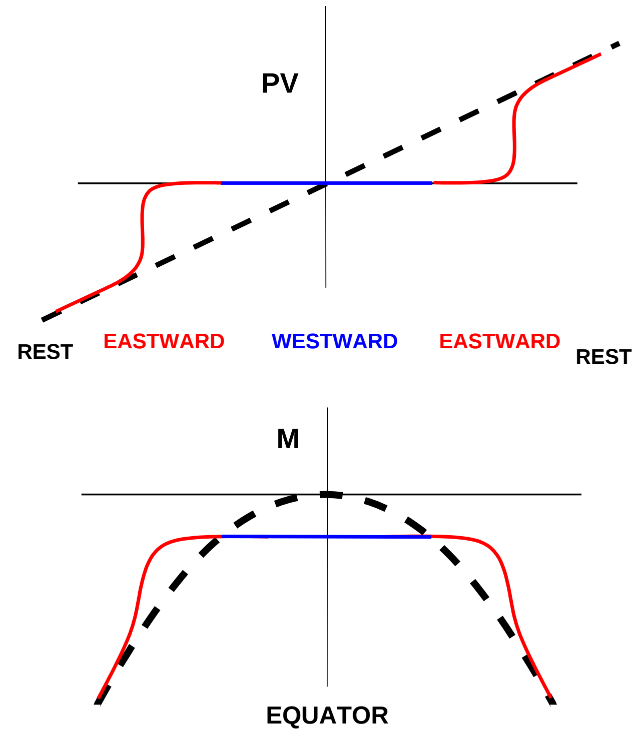
⟸ Ollitraut et al. 2006

– zonal currents near 1000 m depth

– multiple, equally spaced jets

- Realistic reproduction of zonal jets possible in basin simulations (d’Orgeville et al. 2006, Ménesguen et al. 2008)

- Westward flow at equator and eastward jets at $\pm 2^\circ$ latitude (Gouriou et al. 2001)
- Often corresponds to angular momentum $M \equiv U - \frac{1}{2}\beta y^2$ (and hence $PV \propto -M_y$) homogenized about the equator surrounded by PV barriers.



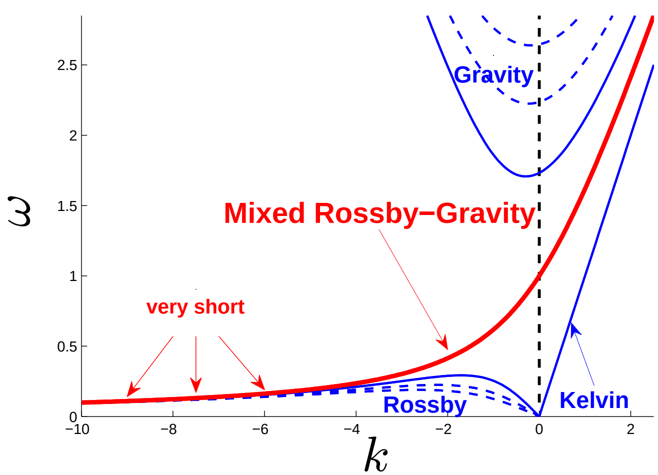
But what causes the homogenization of PV in the first place?

- Propose mechanism for generating jets through barotropic instability of short equatorial waves and subsequent adjustment due to inertial instability.
- Also explains "equatorial deep jets" (Hua et al. 2008)

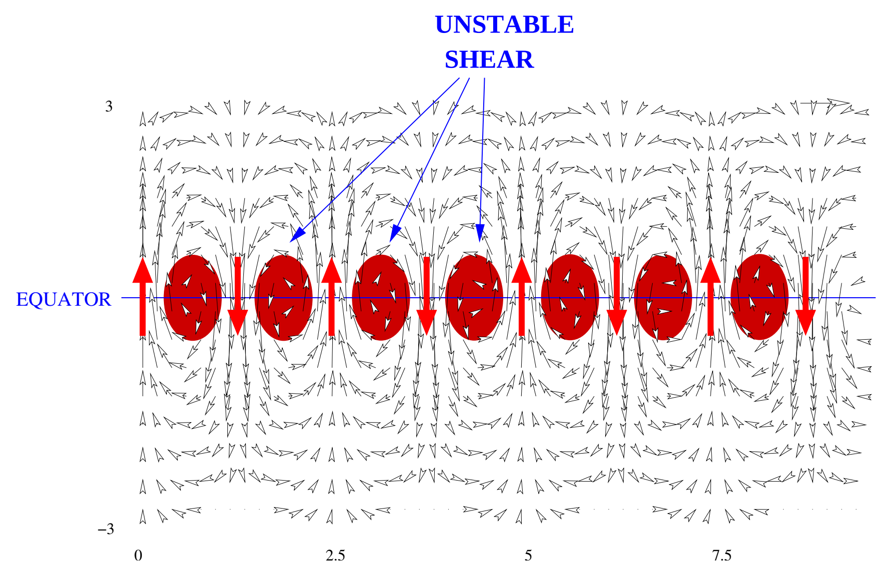
2. Destabilization of short westward MRG wave

MRG wave in large negative k limit:

DISPERSION DIAGRAM FOR EQUATORIALLY TRAPPED WAVES



MRG WAVE VELOCITY FIELD



- Cases considered: $-6 < k < -16$
- Oceanic vertical mode 1 $\Rightarrow$ 50 - 200 day period, 1.2 $^\circ$ - 3° wavelength.
- i.e. very low frequency: $\omega \ll 1$, very short wavelength: $|k| \gg 1$

$\Rightarrow$ β (rotation) and ω (wave propagation) unimportant.

MERIDIONAL SECTION MOVIE Simulations at $0.1^\circ \times 0.1^\circ$, 100-200 vertical levels using ROMS model:

Amplitude 0.36 cm, $k = -6.3$ (3.3° wavelength)

MERIDIONAL SECTION MOVIE

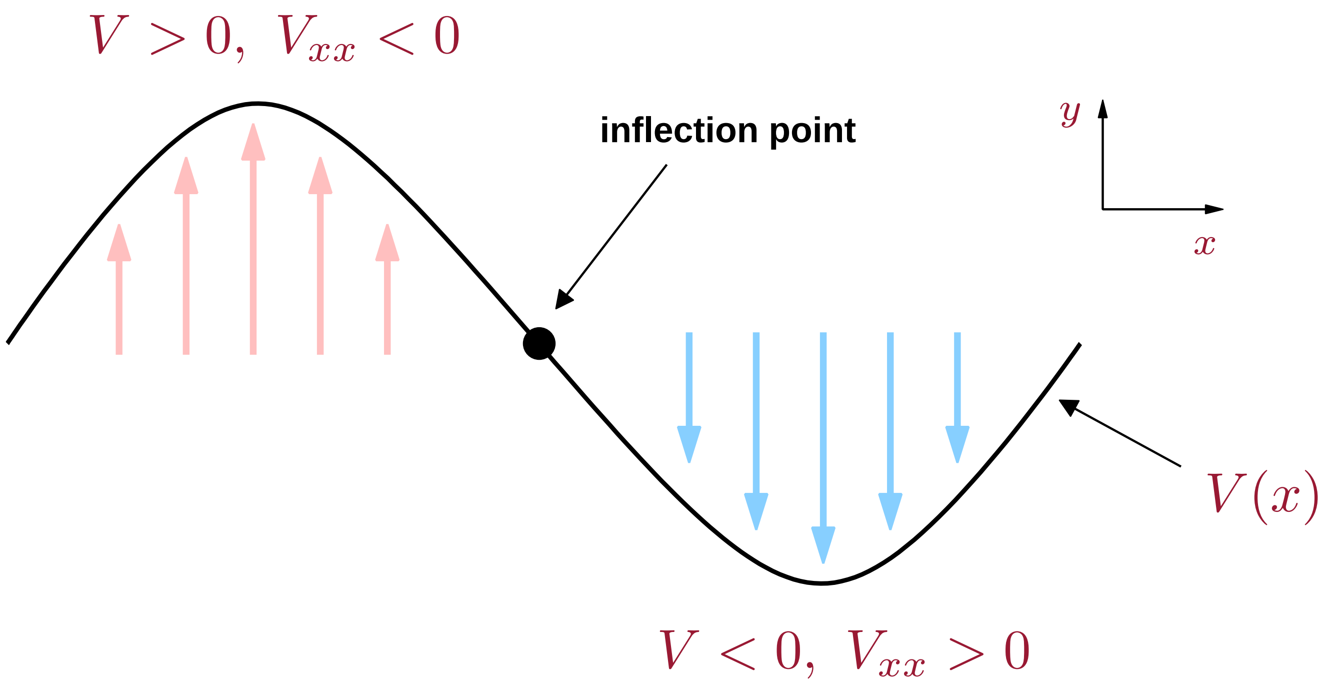
still of last frame

EQUATORIAL SECTION MOVIE

EQUATORIAL SECTION MOVIE

still of last frame

Barotropic Instability Mechanism:

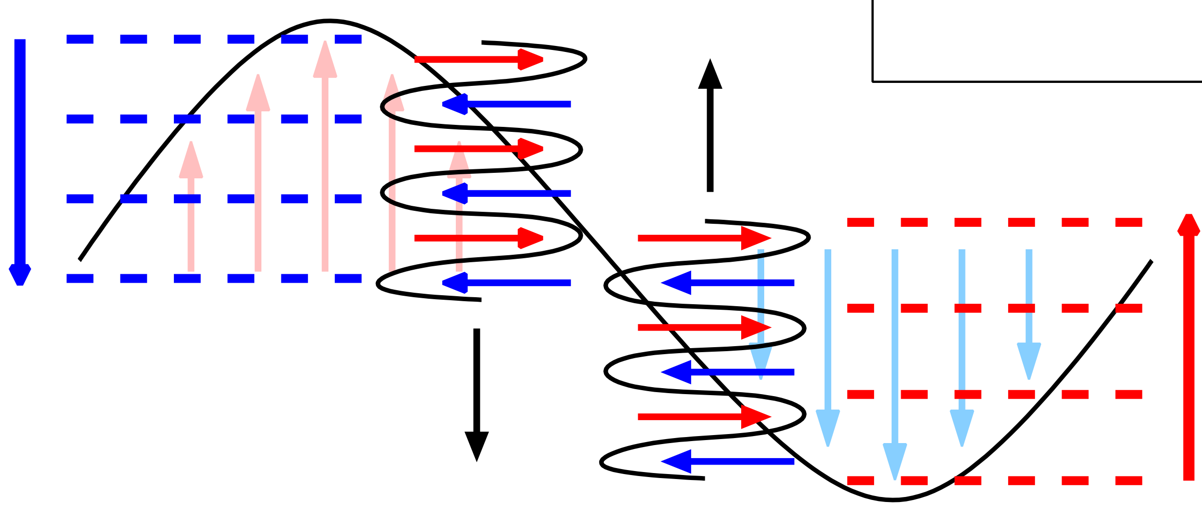


- Linearized barotropic vorticity equation: $\zeta'_t + V\zeta'_y + \underbrace{V_{xx}u'}_{\text{“}\beta\text{” term}} = 0$

Barotropic Instability Mechanism

$$\zeta'_t + V\zeta'_y + V_{xx}u' = 0$$

$V > 0,\ V_{xx} < 0$



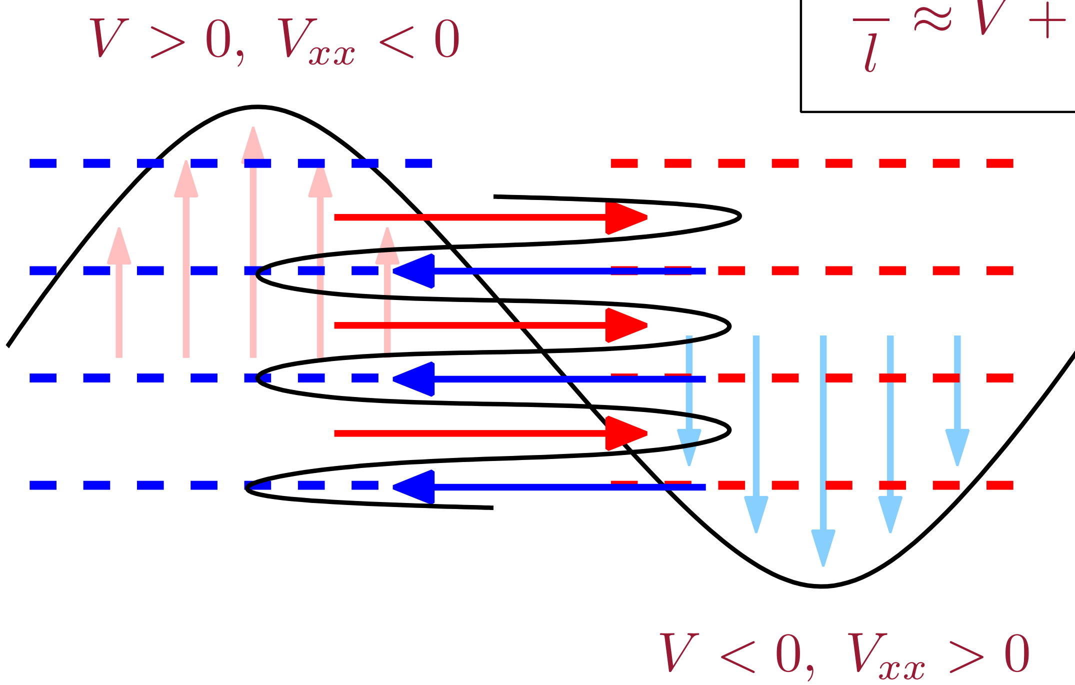
$V < 0,\ V_{xx} > 0$

- Supports "Rossby" waves: $$\frac{\omega}{l} \approx V + \frac{V_{xx}}{k^2 + l^2}$$

  with intrinsic meridional phase speed of same sign as V_{xx}.

Barotropic Instability Mechanism

$$\frac{\omega}{l} \approx V + \frac{V_{xx}}{k^2 + l^2}$$



- Because of opposite sign of V in two regions, for small enough l (i.e. long enough scale in y), waves can phase-lock and amplify one another.

Linear Theory in $k \ll -1$ limit:

Scale length and time like $(x, y) = k^{-1}(\xi, \eta)$ and $t = (kV_0)^{-1}\tau$.

Velocity components of basic state wave:

$$\begin{aligned} V &\sim V_0 \exp\left(\frac{-\eta^2}{2k^2}\right)\cos(z)\cos(\xi), \\ U &\sim k^{-1}V_0\frac{\eta}{k^2}\exp\left(\frac{-\eta^2}{2k^2}\right)\cos(z)\sin(\xi), \\ W &\sim k^{-2}V_0\left(\frac{-\eta^2}{k^2}\right)\exp\left(\frac{-\eta^2}{2k^2}\right)\sin(z)\cos(\xi), \\ Z &\sim -kV_0\exp\left(\frac{-\eta^2}{2k^2}\right)\cos(z)\cos(\xi), \end{aligned}$$

Look for horizontally non-divergent perturbations $u' = -\psi'_\eta$, $v' = \psi'_\xi$.

Perturbation vorticity equation is approximately

$$\boxed{\tilde{\zeta}'_\tau + V\tilde{\zeta}'_\eta + k^{-1}Z_\xi u' = \nabla^2\psi'_\tau + \cos(z)\exp\left(\tfrac{-\eta^2}{2k^2}\right)\cos(\xi)\left(\nabla^2\psi'_\eta + \psi'_\eta\right) = 0}$$

Next order in k^{-1}: time dep. of MRG wave, advection by U, and β effect.

Case 1: η small, $\exp\left(\frac{-\eta^2}{2k^2}\right) \approx 1$

Special case of problem of Gill (1974) for barotropic Rossby wave.

$$\boxed{\nabla^2\psi'_\tau + \cos(z)\exp\left(\frac{-\eta^2}{2k^2}\right)\cos(\xi)\left(\nabla^2\psi'_\eta + \psi'_\eta\right) = 0}$$

- Linear partial differential equation with coefficients periodic in ξ.
- Seek separable solution with same periodicity as coefficients (Floquet theorem with exponent $n_0 \equiv 0$).

$$\psi'(\xi,\eta,z,t) = \Re\Big\{\underbrace{e^{\mu\cos z\,\tau}e^{il\eta}}_{(\eta,z,\tau)\text{ dependence}} \times \underbrace{e^{in_0\xi}\sum_{n=-\infty}^{\infty}\psi'_n e^{in\xi}}_{\xi\text{ dependence}}\Big\}$$

- *l.h.s.* has terms like $\cos(\xi)\psi'_n e^{i(n_0+n)\xi} = \frac{1}{2}\psi'_n\left(e^{i(n_0+2n)\xi} + e^{i(n_0)}\right)$

$\Rightarrow$ set $f_n\left\{e^{i(n_0+n)\xi}\right\}$ closed under *l.h.s.* operator.

$\Rightarrow$ infinite eigenvalue problem for ψ', μ.

- Approximate solution by truncated series:

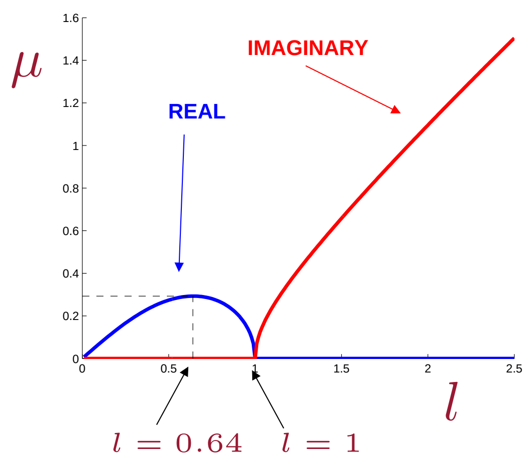
$$\psi' = \Re\left\{e^{\mu\cos(z)\tau}e^{il\eta}\left(\psi'_{-1}e^{-i\xi}+\psi'_0+\psi'_1e^{i\xi}\right)\right\}$$

ψ'_0 is zonal jet, $\psi'_{\pm 1}$ zonally short wave. Leads to:

$$\mu\begin{pmatrix}\psi'_{-1}\\ \psi'_0\\ \psi'_1\end{pmatrix} = \begin{pmatrix}0 & \frac{il}{2}\frac{l^2-1}{l^2+1} & 0\\ -\frac{il}{2} & 0 & -\frac{il}{2}\\ 0 & \frac{il}{2}\frac{l^2-1}{l^2+1} & 0\end{pmatrix}\begin{pmatrix}\psi'_{-1}\\ \psi'_0\\ \psi'_1\end{pmatrix}$$

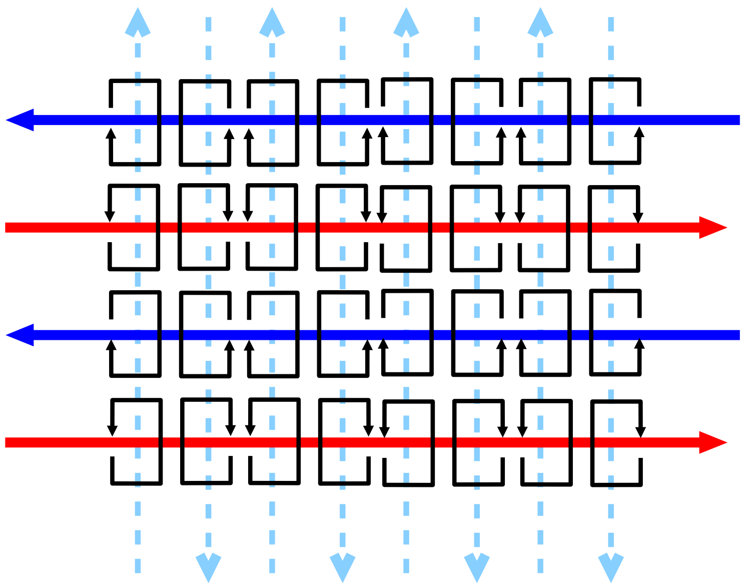
Instability if

$$\Re\{\mu\} = \Re\left\{\frac{l}{\sqrt{2}}\sqrt{\frac{1-l^2}{l^2+1}}\right\} > 0$$

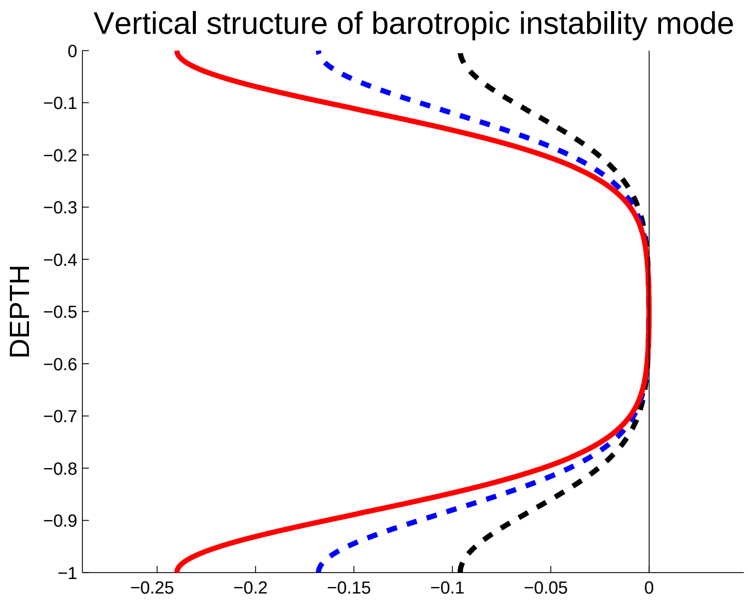


$\Rightarrow$ Instability if $|l| < 1 \Rightarrow$ perturbation of larger scale than original wave (expected on grounds of simultaneous conservation of energy and enstrophy). Fastest growth rate for $\boxed{l = \sqrt{\sqrt{2}-1} \approx 0.64}$.

Horizontal structure



Vertical structure



- Zonal jets alternating in direction in latitude, and small scale cells stirring within jet centres.
- No preferred latitude for jet alignment.

Case 2: Solution valid on $-\infty < \eta < \infty$

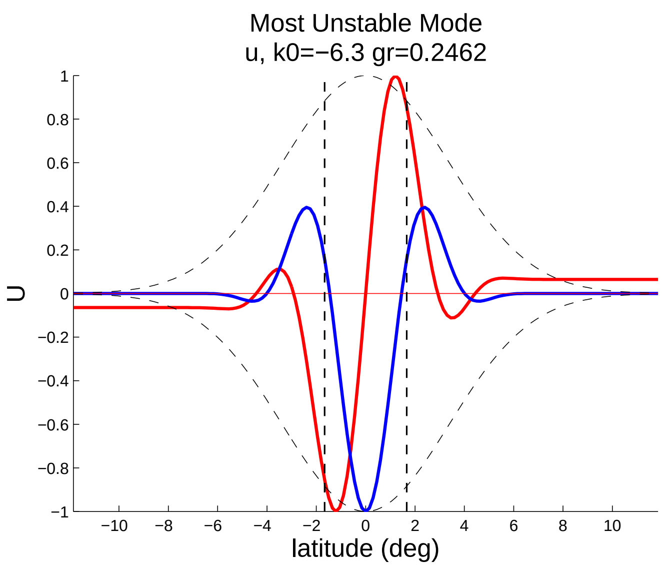
- Again look for truncated series solution:

$$\psi' = \Re\left\{e^{\mu\cos(z)\tau}\left(\psi'_{-1}(\eta)e^{-i\xi} + \psi'_0(\eta) + \psi'_1(\eta)e^{i\xi}\right)\right\}$$

- Solve numerically. Most unstable mode:

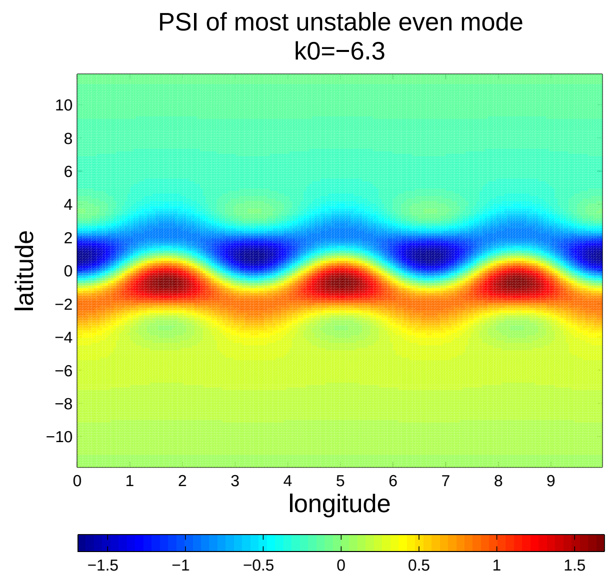
Zonal mean U

(even and odd modes)

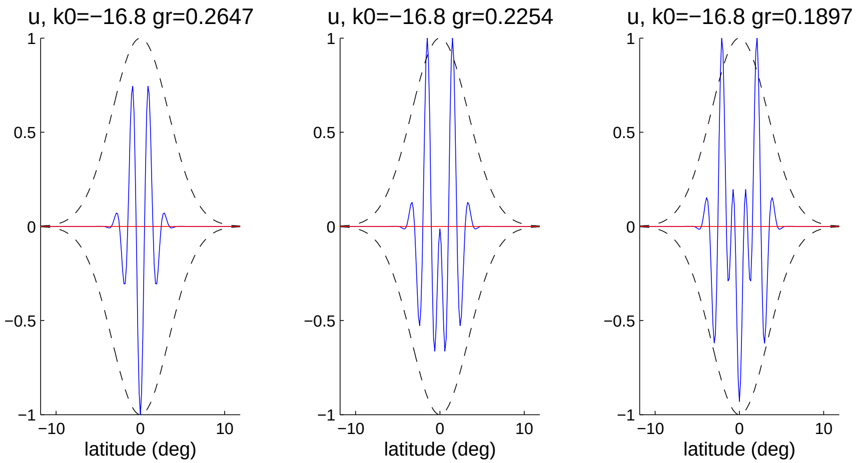


Streamfunction

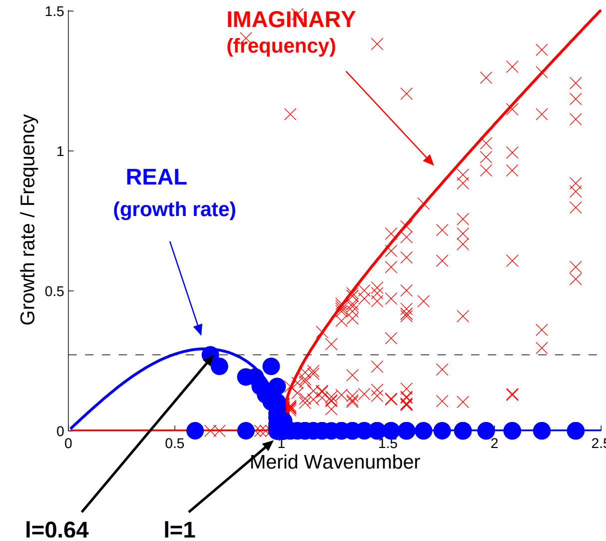
(of even mode)



$u_0(y)$ for fastest growing even eigenmodes

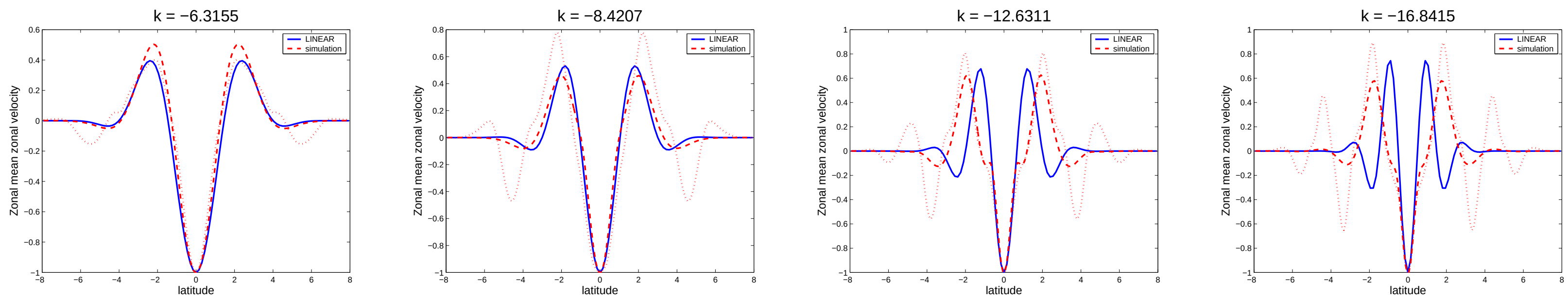


Growth rate vs. meridional wavenumber

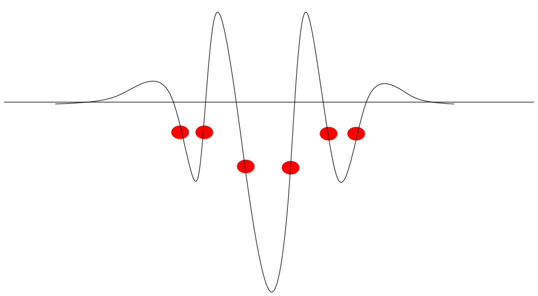


- Jet spacing near equator well predicted by Gill barotropic plane wave case.
- Jet spacing wider than wavelength of MRG wave $l < 1$.

Comparison with simulations

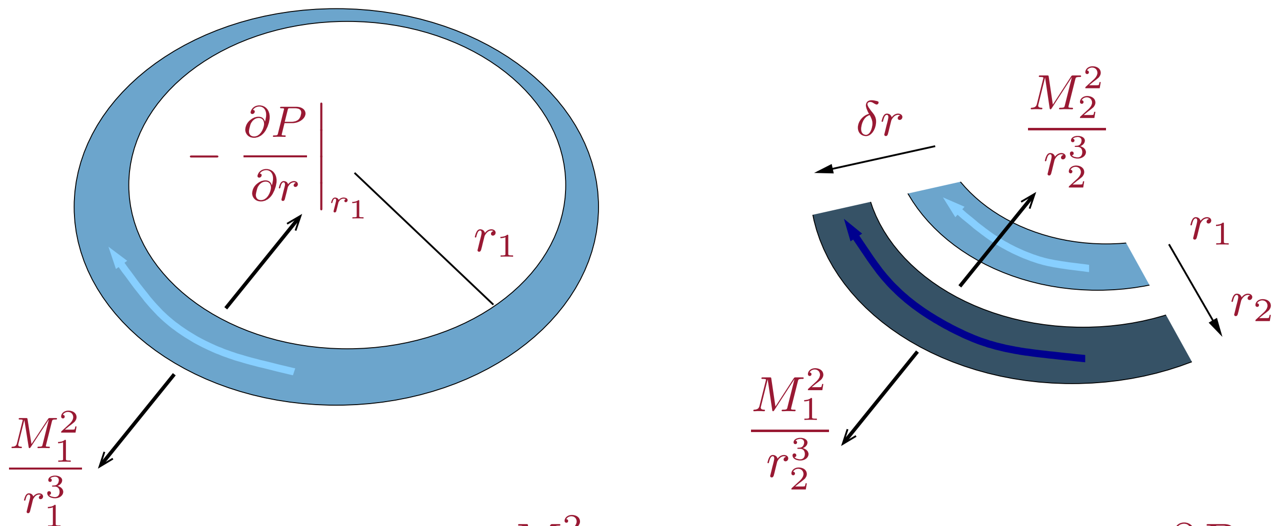


- U even about equator since odd modes inertially unstable
- Equatorial jet always westward. Probably due to mixing of planetary angular momentum by zonally short modes (strongest at equator) biasing simulations towards westward flow at equator.
- Extra-equatorial jet positioning poleward of that in most unstable linear mode: barotropic instability?



barotropic instability possible when $U_{yy} = \beta$ (does not lead to meridional jets due to Coriolis effect).

3. Inertial Instability and PV Homogenization

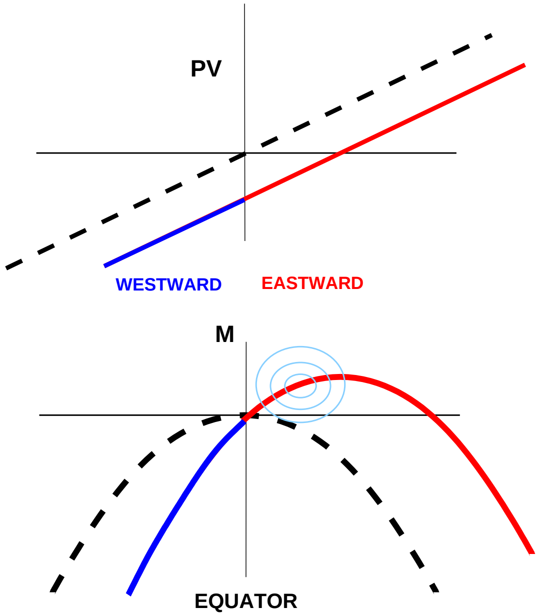


- Balanced centrifugal $\frac{M^2}{r^3}$ and pressure gradient $-\frac{\partial P}{\partial r}$ forces.
- Ring of fluid displaced from r_1 to r_2 conserving its angular momentum without disturbing the pressure field feels radial force

$$\frac{M_1^2 - M_2^2}{r_2^3} \approx -\frac{1}{r_2^3} \left.\frac{\partial\left(M^2\right)}{\partial r}\right|_{r_2} \delta r$$

$\Rightarrow$ An axially symmetric flow is inertially stable if $\frac{\partial\left(M^2\right)}{\partial r} > 0$

Equatorial shear inertial instability

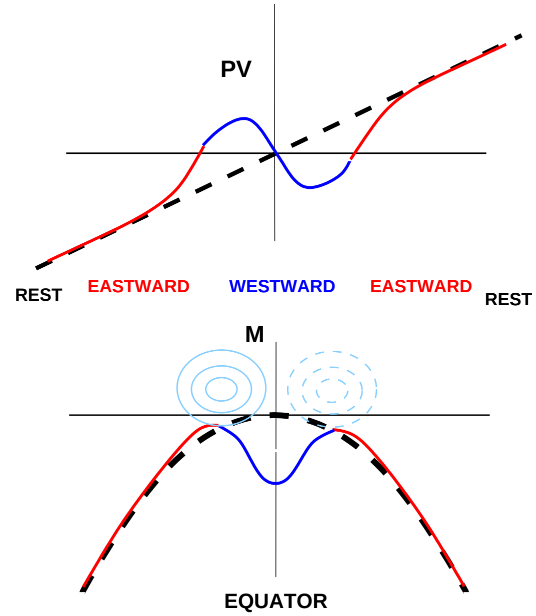


(Dunkerton 1981)

$U = \lambda y$, unstable for all $\lambda \neq 0$

Adjustment mixes in latitude and depth.

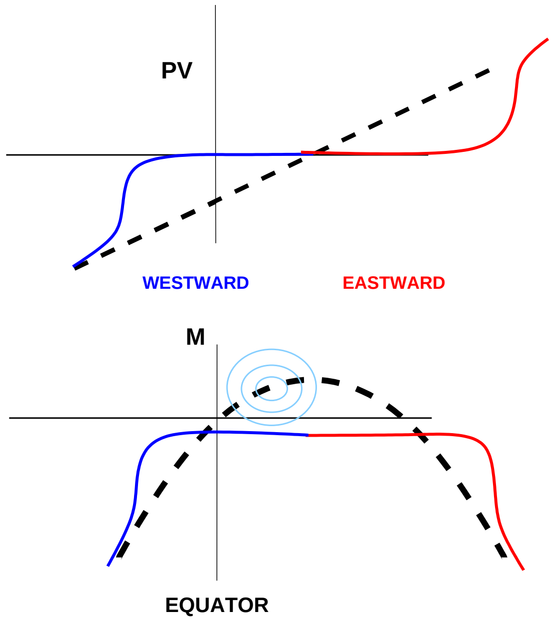
“Curvature” inertial instability



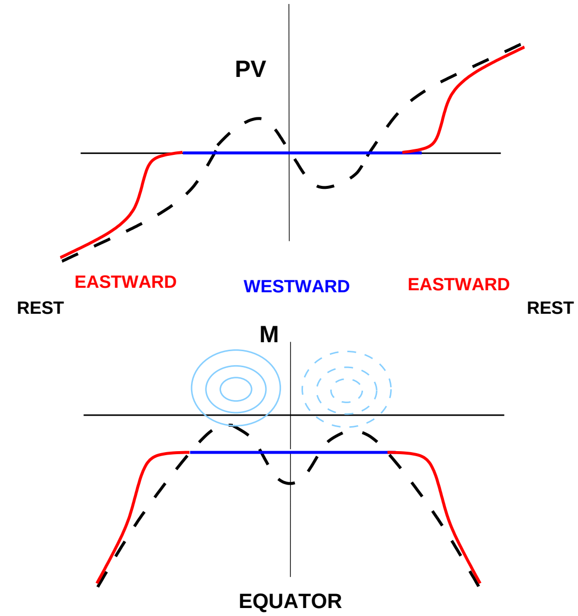
$U = (-U_0 0 + \frac{1}{2} b y^2) e^{-\frac{1}{2} y^2}$,

unstable for $b > \beta$

Equatorial shear inertial instability

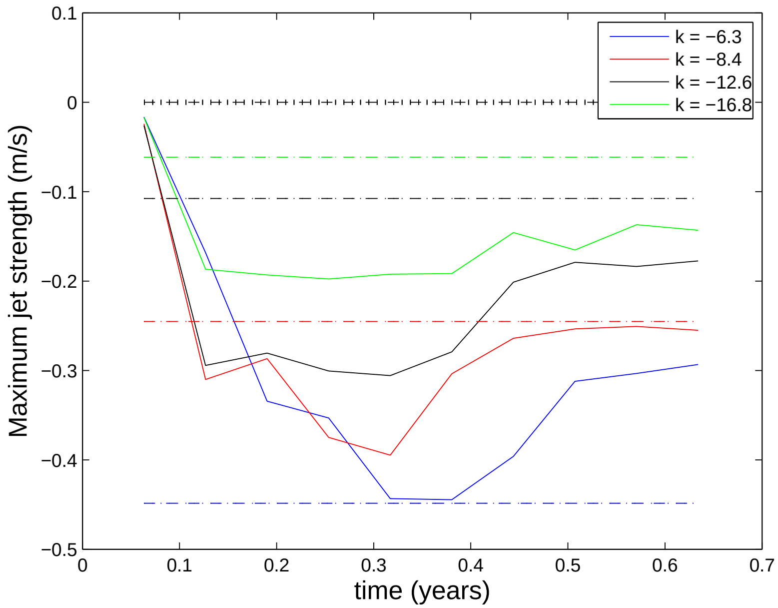


"Curvature" inertial instability



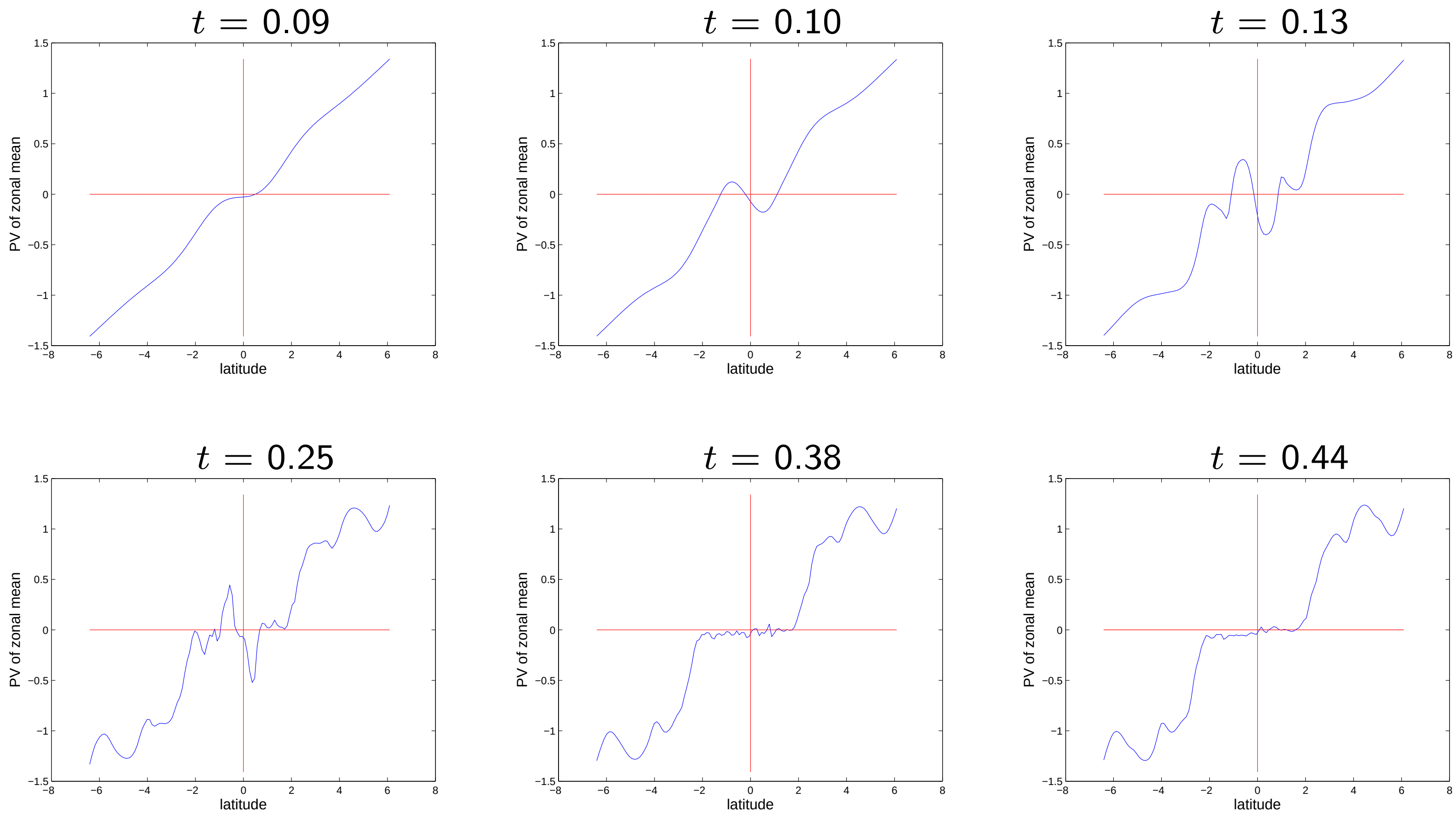
- inertial adjustment takes place over wider latitude interval than covered by initial instability (Hua et al. 1997, Griffiths 2003).
- Results in wide area of zero PV.

Equatorial jet amplitudes from simulations and inertial instability thresholds of linear modes:



- Curvature inertial instability threshold lower for shorter wavelength (and hence more unstable) initial waves.
- Peak and equilibrated jet amplitudes weaker for shorter wavelength (more unstable) initial waves..

Time evolution of zonal mean PV:



Note equatorially symmetric inertial instability ($t > 0.10$) and subsequent adjustment to uniform zero PV over interval wider than initially unstable interval (implies a widening and/or weakening of westward jet).

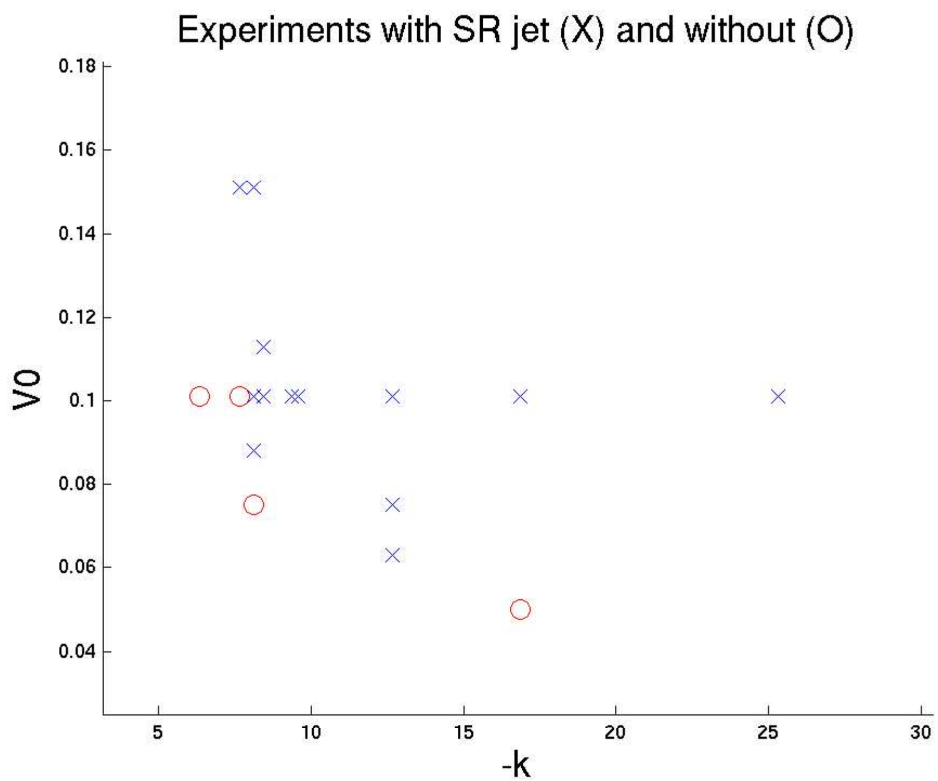
4. Super-rotation and the non-traditional Coriolis force

Super-rotation movie

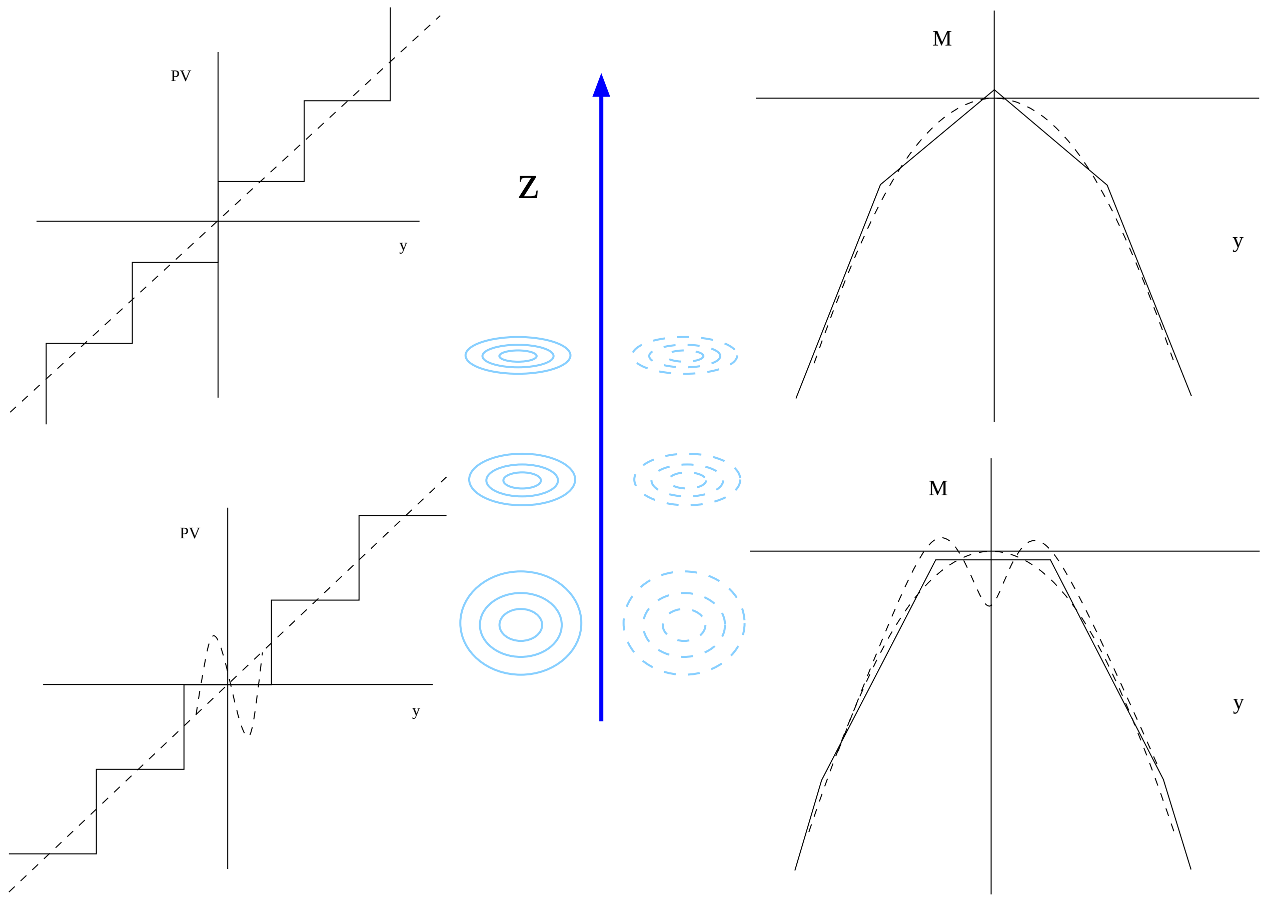
Amplitude 0.36 cm, $k = -8.4$ (2.5° wavelength)

- Super-rotation implies nonlocal and nondiffusive transport of zonal mean angular momentum. How to explain and predict it?
- Occurs for larger $|k|$ (shorter initial waves) and larger initial amplitudes

:



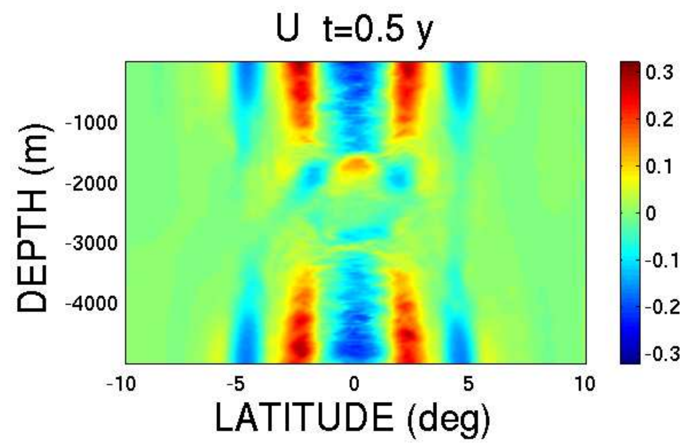
- Explanation in terms of lateral mixing of PV extending beyond unstable depth range in both depth and latitude?

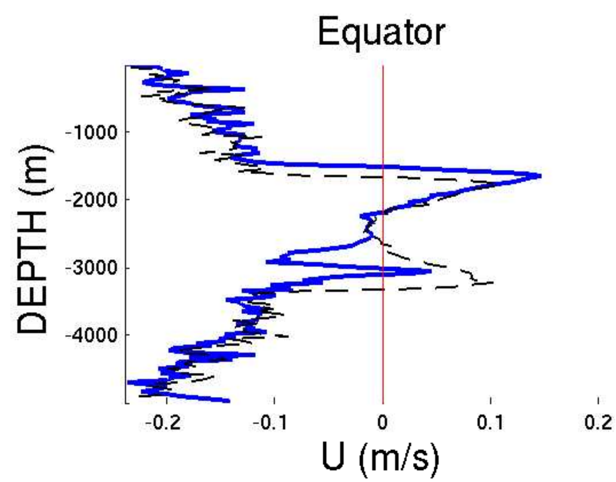


- Clue may be effect on super-rotating jet of non-traditional Coriolis force terms:

$$
\begin{aligned}
u_t &= \beta y v - \gamma w - p_x, \\
v_t &= -\beta y u - p_y, \\
p_z &= -(g/\rho_m)\rho' + \gamma u, \\
\rho'_t &= -(rho_m/g)N^2 w, \\
0 &= u_x + v_y + w_z
\end{aligned}
$$

- Zonally symmetric version conserves $M_\gamma \equiv U - \frac{1}{2}\beta y^2 + \gamma z$.
- Linear waves modified by $\mathcal{O}((\Omega/N)^2)$.
- Effect much greater (here $\Omega/N \approx 0.1$) (note vertical symmetry breaking).





Summary

- Zonal jets with realistic spatial scales can be generated by the barotropic destabilization of mixed Rossby-gravity waves of frequencies consistent with forcing on equatorial ocean.
- Considerations of inertial stability and planetary potential vorticity mixing explain sign and position of equatorial jet.
- Shape of equatorial westward jet well described by truncated Floquet solution to linearized barotropic vorticity equation.
- Equatorial jet peaks at lower amplitude for more unstable initial waves – consistent with transition due to inertial instability.
- Super-rotating equatorial jets form for very short waves, phenomenon particularly sensitive to traditional Coriolis force approximation.