

# Barotropic and Super-Rotating Jet Formation in the Evolution of Very Short Mixed Rossby-Gravity Waves

Mark Fruman

Lien Hua, Richard Schopp

LPO, Ifremer

Brest, France

IUGG XXIV GENERAL ASSEMBLY

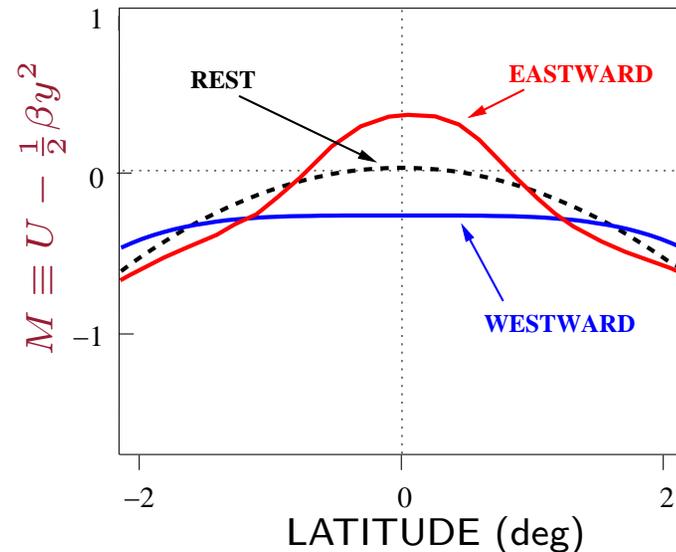
Perugia, Italy, July 2<sup>nd</sup> - 13<sup>th</sup>, 2007

## OUTLINE

1. Equatorial super-rotation
2. Super-rotation in the destabilization of a short westward MRG wave
3. Effect of non-traditional Coriolis force

# 1. Equatorial super-rotation

- **Super-rotation** refers to steady zonally symmetric **eastward flow** at the equator.



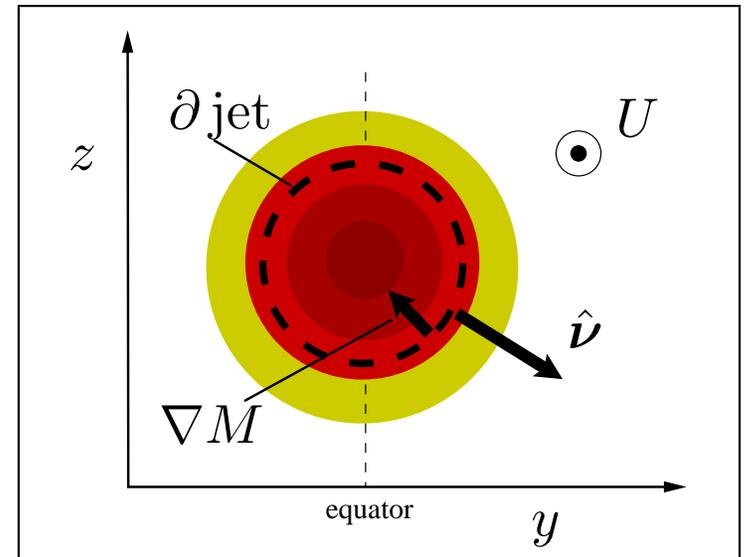
- Eastward flow at the equator implies absolute angular momentum greater than anywhere in the **rest state**.  
 $\Rightarrow$  cannot come about through simple 2-D (zonally symmetric) rearrangement of zonal-mean angular momentum.
- Observed in stratosphere (**QBO**), other planets (global super-rotation on Venus: **Gierasch, 1975**), troposphere models (**Suarez & Duffy, 1992**; **Saravanan, 1993**; **Williams, 2003**; etc.)

- **Hide's Theorem:** If departures from zonal symmetry (“eddies”) can be assumed to dissipate zonal-mean angular momentum **down-gradient**, then an isolated jet at the equator cannot exist in the steady state.

$$\frac{\partial}{\partial t} \iint_{\text{jet}} M \, dy \, dz = \mu \int_{\partial \text{jet}} \nabla M \cdot \hat{\nu} \, dl = 0$$

$$M \equiv U - \frac{1}{2}\beta y^2, \quad U = \text{zonal velocity}$$

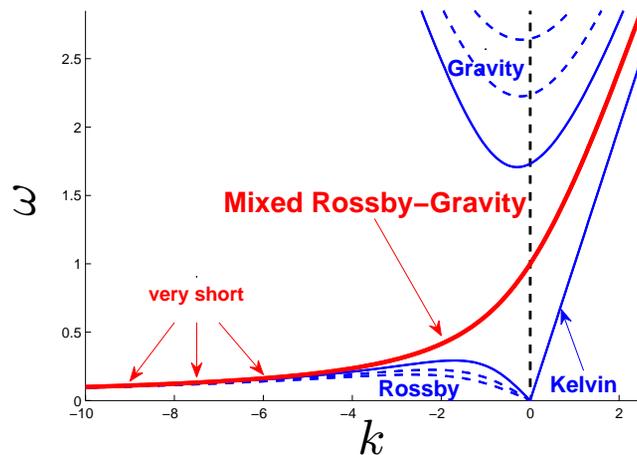
$$\Rightarrow \text{steady state} \Leftrightarrow \nabla M = 0, \text{ i.e. no jet}$$



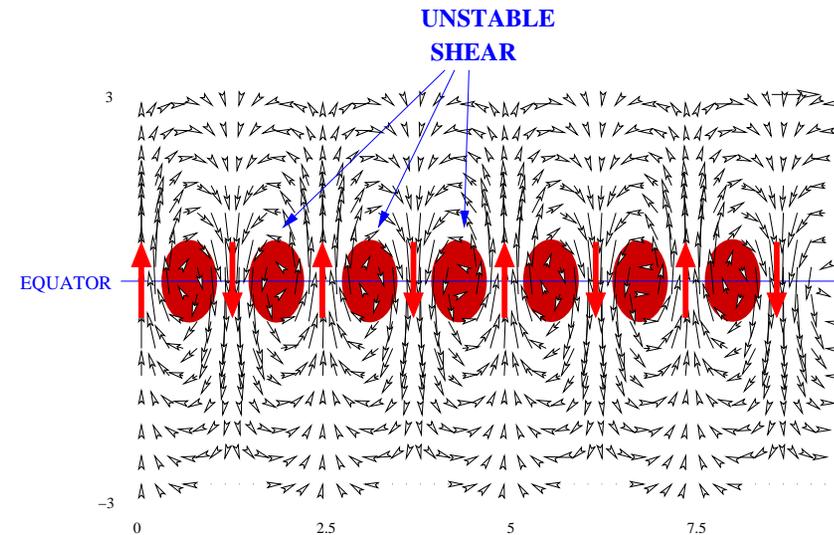
- Eddies must be transferring angular momentum **up-gradient**.
- Key to maintaining super-rotation is **non-local** angular momentum transfer by waves (cf. QBO)
  - $\Rightarrow$  Eastward jets might be sources of **Rossby waves** or sinks of **Kelvin waves**.

## 2. Super-rotation in the destabilization of a short westward MRG wave

### DISPERSION DIAGRAM FOR EQUATORIALLY TRAPPED WAVES



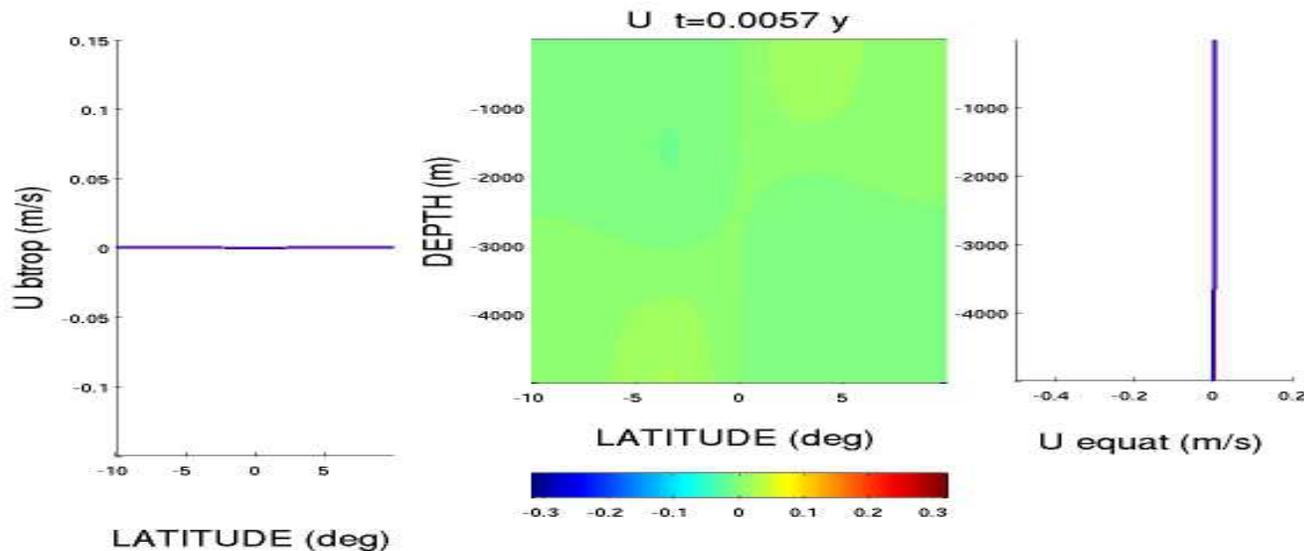
### MRG WAVE VELOCITY FIELD



- Gill (1974) – short wavelength limit barotropic Rossby waves  $\rightarrow$  barotropic instability due to  $\partial V/\partial x \rightarrow$  zonal jet formation.
- Hua et al. (2007) – MRG wave destabilization leads to extra-equatorial jets spaced by  $\mathcal{O}(k^{-1})$  (explained by linear theory for non-divergent perturbations).

- MRG wave has same mean zonal angular momentum as **rest state**.  
⇒ Any eastward jet at equator in long time steady state implies up-gradient transfer of mean angular momentum transfer.
- We performed high-resolution ( $0.1^\circ \times 0.1^\circ \times 200$  levels) simulations using **ROMS** for  $k \lesssim -6k_R$ , where  $k_R^{-1}$  is deformation radius.
- Two configurations of simulations:
  - \*  $10^\circ$  long,  $30^\circ$  wide equatorial  $\beta$ -plane channel with **periodic boundary conditions** in  $x$  and lowest vertical mode MRG waves initialized over **entire length**.
  - \*  $150^\circ$  long channel with MRG wave initialized over central  $60^\circ$ .

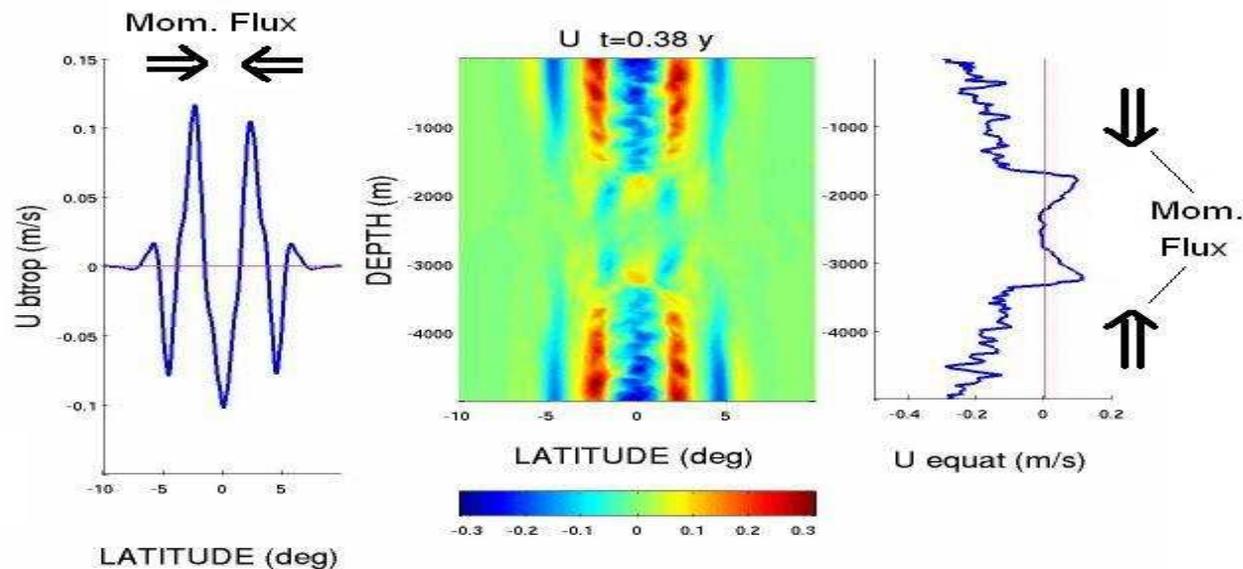
## 10° channel simulation



- Barotropic signal (left):
  - Westward flow at equator (Reason for bias – Hide’s Theorem?)
  - Westward momentum flux from equator to extra-equatorial jets
- Equatorial profile (right):
  - Westward flow at depths of maximum initial wave amplitude
  - Twin eastward jets straddling depths of initial wave nodes.

10° channel simulation

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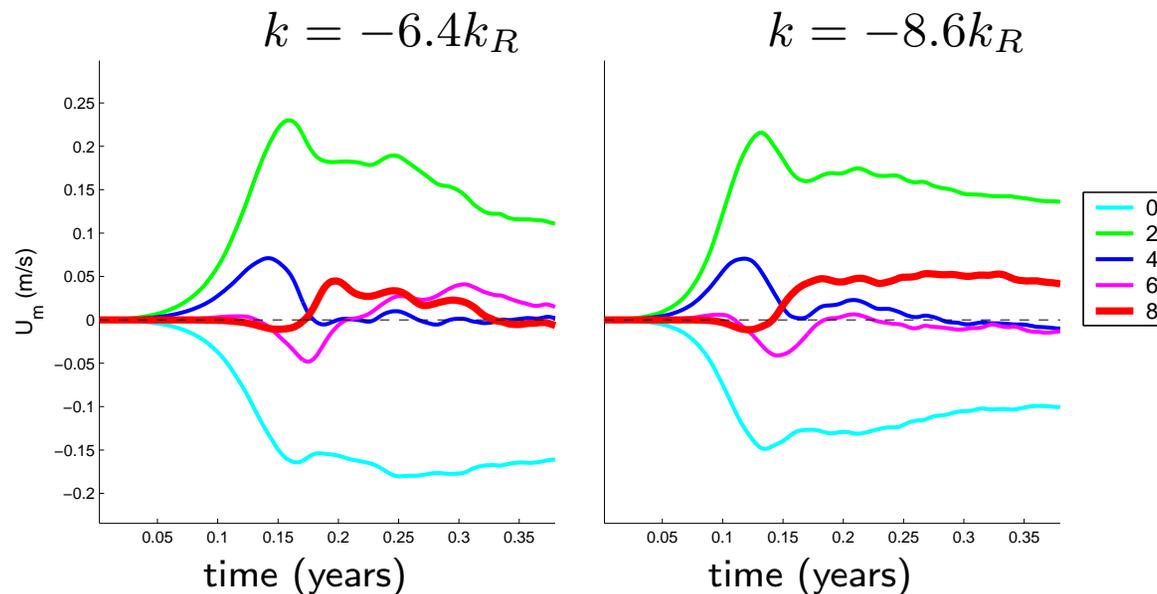


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## 10° channel simulations

- Super-rotating jets appear for large enough  $-k$  and high enough amplitude of the initial wave.

### VERTICAL MODE AMPLITUDES

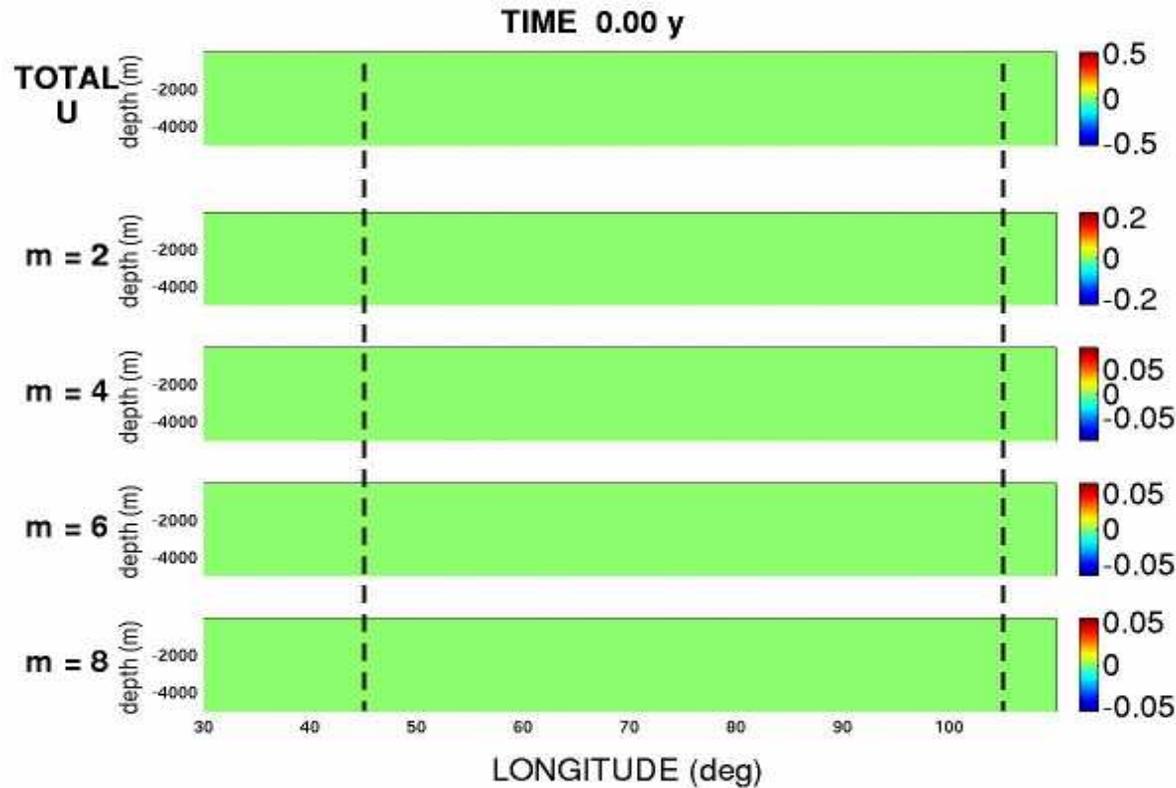


LEFT: No super-rotation

RIGHT: Super-rotation

- Super-rotating state distinguished by strong vertical mode 8
- **Modes 8 and 6** amplify at expense of **modes 2 and 4**.

## 150° channel simulation

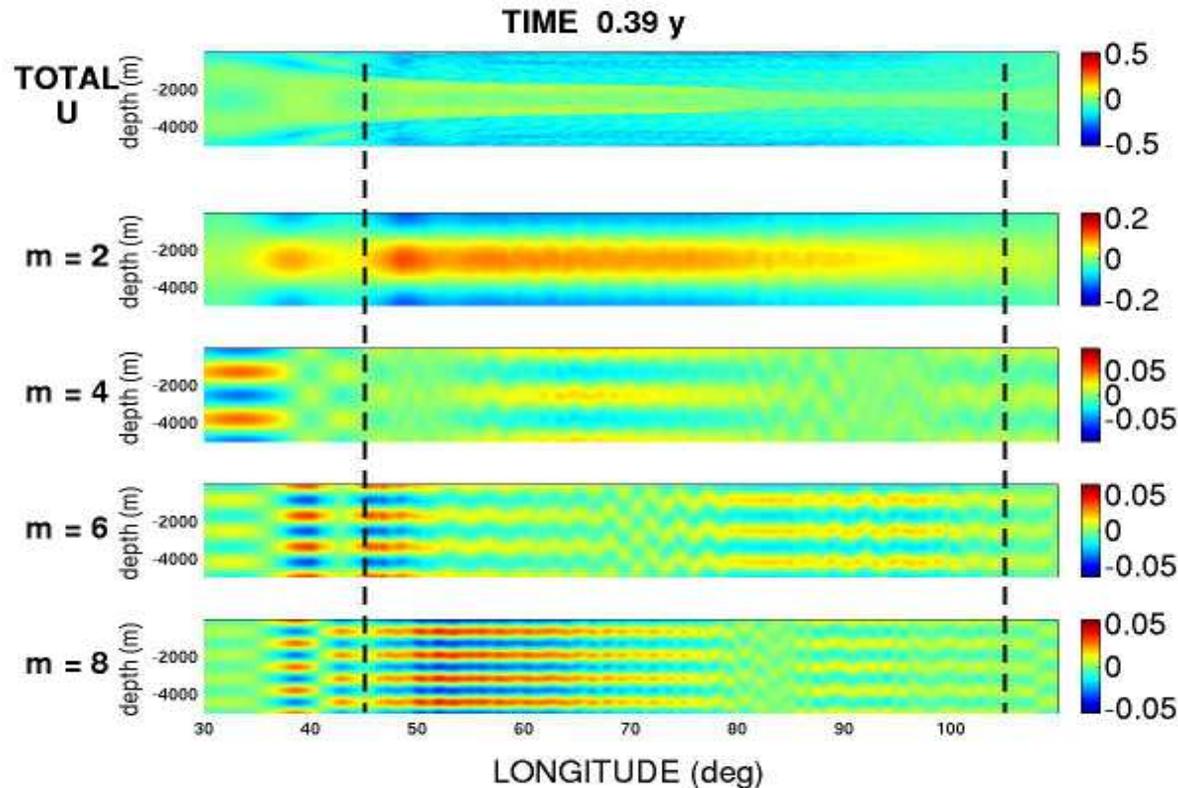


- Mostly westward (group) propagation of low vertical modes (Rossby waves?)
- Spatial correspondence of weakened modes 2 and 4 and intensified modes 6 and 8, with mode 8 predominating.

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### 3. Effect of non-traditional Coriolis force

- Coriolis force terms associated with  $\gamma \equiv 2\Omega \cos \theta$  neglected in “traditional approximation” (Eckhart, 1960):

$$Du/Dt = \beta yv - \gamma w - p_x$$

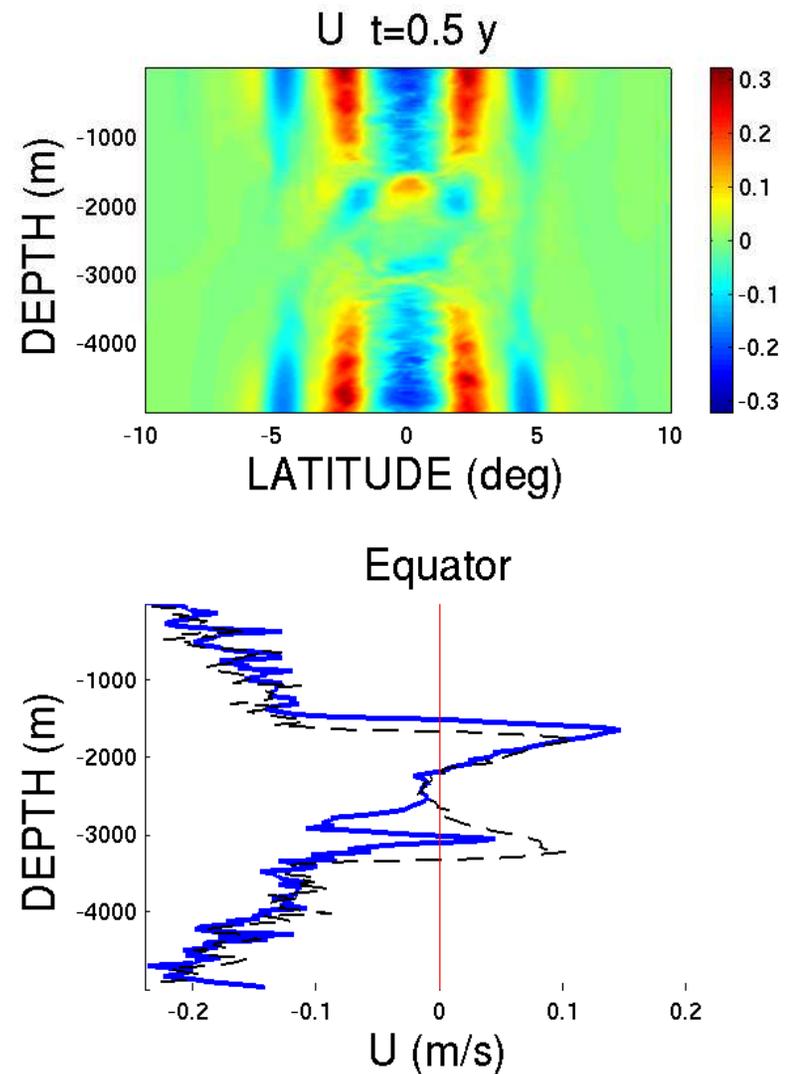
$$(Dw/Dt) = \gamma u - g\rho' - p_z$$

- Non-traditional terms couple zonal and vertical velocities and complicate interactions with bottom boundary (where  $w = 0$ ).
- Significant for small aspect ratio (deep atmosphere - e.g. Jupiter), and near equator (White & Bromley, 1995; de Verdère & Schopp, 1994)
- Effect on equatorial wave solutions is  $\mathcal{O}(2\Omega/N)$  (which is 10% in our simulations) in magnitude of velocity components.

- Effect on super-rotating jet much greater than 10%.
- Symmetry breaking in vertical: stronger upper jet, weaker lower
- What is reason for asymmetric vertical redistribution of angular momentum

$$M_\gamma \equiv U - \frac{1}{2}\beta y^2 + \gamma z ?$$

- For higher vertical mode initial states, symmetry is broken for each pair of eastward jets.



## Summary

- Short westward propagating **MRG wave destabilization** can lead to sustained low vertical mode super-rotating jets, implying **up-gradient** angular momentum transfer.
- Eastward “**super-rotating**” jets appear to be associated with westward propagating modes of the instability (long Rossby waves?)
- **Non-traditional Coriolis force** acts to break vertical symmetry through coupling of  $u$  and  $w$ , strengthening upper branches of super-rotating jets.

## Continuing Work

- Analysis of vertical and meridional momentum fluxes, diagnosis of **momentum source for eastward jets**.
- Quantification of effect of **non-traditional Coriolis force** terms and assessment of relevance to equatorial ocean dynamics.