Inertial Instability in the Equatorial Middle Atmosphere

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OUTLINE

- Inertial instability
- Axisymmetric inertial stability in hydrostatic β-plane system
- Observational and model evidence
- Application of Hamiltonian methods

Inertial instability

- Steadily rotating flows have balance between centrifugal and pressure gradient forces

- Flow is *inertially unstable* if radial perturbations amplified by imbalance of forces

- Rayleigh criterion for stability:

absolute angular momentum
everywhere increases with distance
from axis of rotation

Equatorial β-plane

- Model equatorial dynamics with β-plane instead of spherical shell

- Simplifies math., but retains latitude dependence of absolute angular momentum

- Zonal flows in rotating, spherical shell conserve

$$m_0 = ur\cos\phi + \Omega r^2 \cos^2\phi$$

- Construct symmetric β-plane equations to conserve largest terms in m_0:

$$m = u - \tfrac{1}{2}\beta y^2$$

- Rayleigh criterion for inertial stability in symmetric β-plane system is

$$\beta y(\beta y - U_y) > 0$$

- Can solve linearized system for simple cases such as linear meridional shear in zonal velocity:

- Basic state is unstable in $0 < y < \frac{\lambda}{\beta}$

Features of solution

- Unstable solutions exist for all nonzero λ for large enough vertical wavenumber $\mu \geq \frac{4N\beta}{\lambda^2}$

- Growth rate faster for higher meridional shear and smaller vertical scale

- Overturning *Taylor vortex* cells in vertical-meridional plane, centred on $y = \frac{\lambda}{2\beta}$

- Oppositely signed zonal jets stacked vertically on equator side of unstable region

- *Pancake structures* in temperature perturbation field on the sides of unstable region

Observational evidence

- Signatures of inertial instability have been observed in equatorial ocean and middle atmosphere

- *Hua et al (1995)*: stacked zonal jets in equatorial ocean

- *Hitchman et al (1987)*,
  Hayashi et al (1998):

  pancake structures in satellite observations of temperature in middle atmosphere

- and in numerical simulations and GCM's: *Hunt (1981)*, *Semeniuk and Shepherd (2000)*, etc.

Hamiltonian analysis

- By casting equations in Hamiltonian form, can analyze more general basic states for stability criteria

- Can estimate *saturation bounds* on instability to quantify total effect of (nonlinear) adjustment to stable state

  (Energy and momentum conservation restrict the growth - energy release - of instabilities)

- Potentially develop parameterization of inertial instability for numerical models that cannot resolve small enough vertical scales

Summary

- Inertial instability must happen in middle atmosphere but is difficult to observe or simulate realistically

- Linear analysis gives intuitive picture of response of dynamical fields to unstable basic state, and compares well with observation

- Numerical models cannot resolve small enough scales, so large inertial cells appear at grid-scale and (for example) overestimate vertical motion

- Hamiltonian methods may lead to parameterization scheme