Secondary instabilities in the breaking of an inertia-gravity wave

Mark D. Fruman[1] and Ulrich Achatz – Institut für Atmosphäre und Umwelt, Goethe-Universität, Frankfurt, Germany

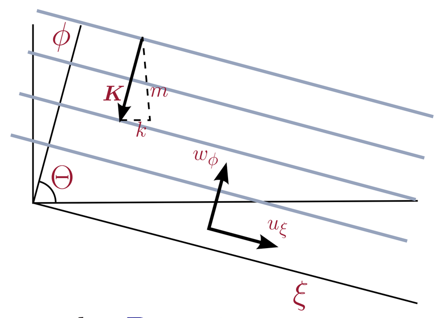
GOETHE UNIVERSITÄT FRANKFURT AM MAIN

[1]fruman@iau.uni-frankfurt.de

Introduction

- Gravity wave breaking is an important process in driving atmospheric circulations but its detailed dynamics are still not fully understood.
- Parameterizations are typically based on a Richardson number criterion, and do not consider horizontal gradients.
- But even statically stable gravity waves can break and degenerate into turbulence if perturbed by finite amplitude disturbances.
- Identification of the fastest growing disturbance (normal mode) and of the disturbance whose energy grows most in a given time (singular vector) of a monochromatic plane gravity wave in the Boussinesq equations are time-independent, two-dimensional, linear problems.
- Three-dimensionalization of the turbulent development can be studied systematically using singular vector analysis of the equations linearized about the evolving two-dimensional state of the wave and a leading primary perturbation (cf. [3, 5]).

Inertia-Gravity Waves



- We investigate the breakdown of a plane inertia-gravity wave (IGW) propagating in the xz-plane at an angle Θ to the horizontal, with upward vertical group velocity.
- The IGW is a time-independent solution to the Boussinesq equations in a reference frame propagating with the phase velocity of the wave:

$$\frac{\partial u_\xi}{\partial \xi} + \frac{\partial v}{\partial y} + K\frac{\partial w_\phi}{\partial \phi} = 0$$

$$\frac{Du_\xi}{Dt} - f\sin\Theta\ v + \frac{\partial p}{\partial \xi} + \cos\Theta\ b = \nu\nabla^2 u_\xi$$

$$\frac{Dv}{Dt} + f(\sin\Theta\ u_\xi + \cos\Theta\ w_\phi) + \frac{\partial p}{\partial y} = \nu\nabla^2 v$$

$$\frac{Dw_\phi}{Dt} - f\cos\Theta\ v + K\frac{\partial p}{\partial \phi} - \sin\Theta\ b = \nu\nabla^2 w_\phi$$

$$\frac{Db}{Dt} + N^2(-\cos\Theta\ u_\xi + \sin\Theta\ w_\phi) = \mu\nabla^2 b,$$

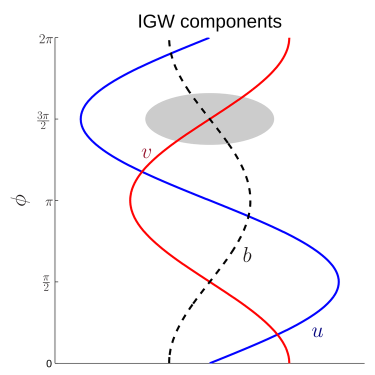
where b is buoyancy, f is the Coriolis parameter and N is the constant Brunt-Väisälä frequency

- The IGW solution is the real part of

$$[u_\xi, v, w_\phi, b] = a\left[\frac{iK\omega}{km}, \frac{f}{k}, 0, -\frac{N^2}{m}\right]e^{i\phi},$$

where

$$\omega = -\sqrt{f^2\sin^2\Theta + N^2\cos^2\Theta}$$



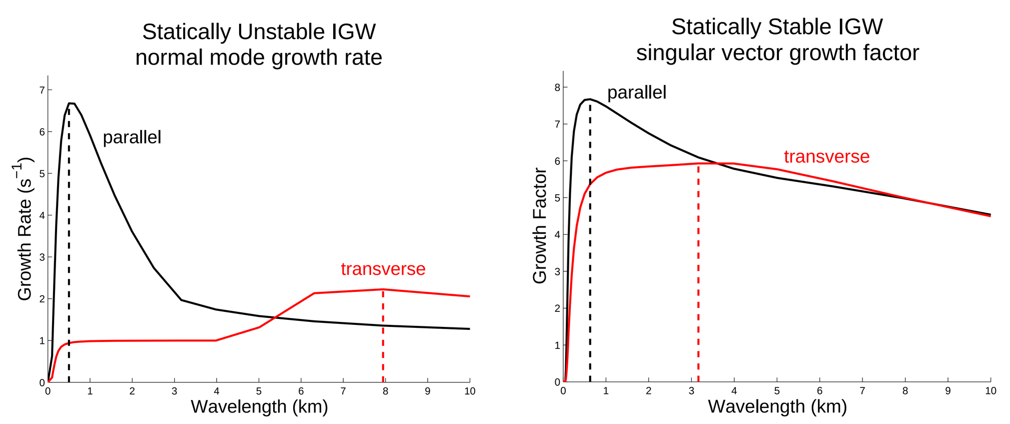
- The IGW is normalized so that the point of minimum static stability (grey oval) of a wave of amplitude $a = 1$ is neutrally statically stable.
- As $\Theta \to 90°$, the horizontal velocity becomes circularly polarized and the wave frequency approaches f (but slowly, since $N \gg f$).
- We consider an IGW with $\Theta = 89.5$ at latitude $70N$, implying a wave period of $2\pi/\omega = 8$ hours, and with wavelength 6 km. We consider both a "statically stable" ($a = 0.87$) wave and a "statically unstable" ($a = 1.2$) wave.

Normal Modes and Singular Vectors

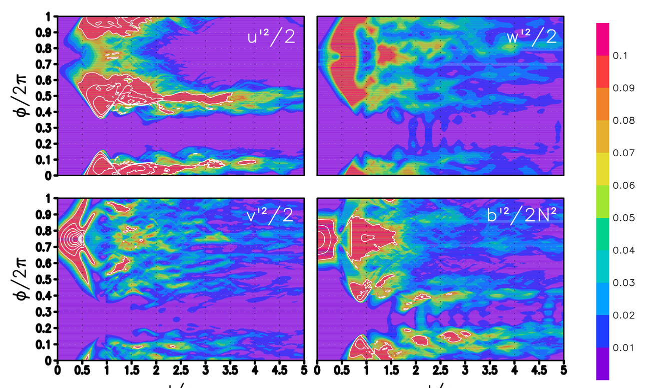
- Normal Modes (NM) are eigenvectors of the governing equations linearized about a time-independent basic state (such as the IGW in the co-moving reference frame)
  - NM have fixed complex spatial structure and oscillate and/or grow or decay exponentially with time.
  - For long times, the normal mode with eigenvalue having the largest real part (the growth rate) dominates the linear solution.
  - NM are not well defined when the basic state is time dependent.
- Singular vectors (SV) are the initial perturbations to the equations linearized about a (possibly time-dependent) solution whose energy increases by the largest growth factor in a given optimization time τ.
  - Their spatial structure changes with time and depends on τ
  - SV with large growth factors can exist even in the absence of exponentially growing normal modes
  - Their calculation requires the adjoint of the tangent linear model used – here developed using the TAMC [4].

Primary Instabilities

- The calculation of the leading perturbation to the IGW is one-dimensional in Fourier space, with the IGW phase ϕ as spatial coordinate and the horizontal wavevector of the perturbation as a parameter (see [1]).
- The limiting cases are parallel and transverse with respect to the horizontal wavevector of the IGW.
- For statically unstable IGW, we calculate leading normal modes and for statically stable IGW the leading singular vectors with $\tau = 5$ min.:



- The leading perturbations are initially concentrated near the point of minimum static stability, maximum shear in v, and minimum u_ξ.
- Shown is a nonlinear (2-D) integration initialized with a statically stable IGW and the leading transverse singular vector with amplitude such that the maximum SV energy density equals the IGW energy density (Fig. from [2]).



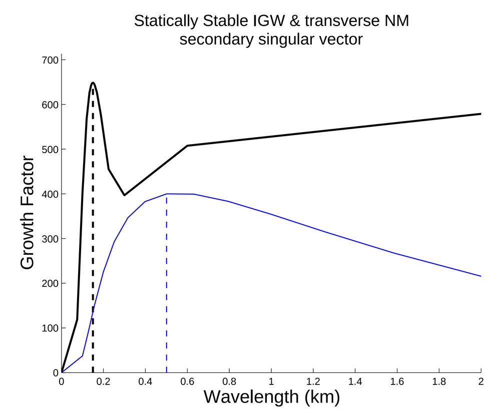
- The initial growth is via a statically triggered Orr mechanism due primarily to vertical shear in v.
- Later growth is due to a roll mechanism associated with vertical shear in u above and below the point of initially weakest stability.

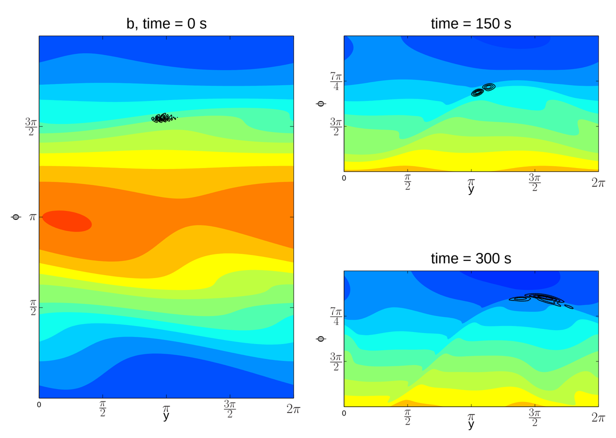
Secondary Instabilities

- Secondary instabilities are investigated using the equations linearized about a nonlinear integration of the full equations initialized with the IGW and a leading primary perturbation.
- The resulting tangent-linear model is two-dimensional with time-dependent coefficients and admits singular-vector analysis with the wavelength λ in the direction perpendicular to both the IGW and primary perturbation as a parameter.
- We calculate the leading SV with optimization time $\tau = 5$ minutes for both the unstable and stable perturbed IGW.

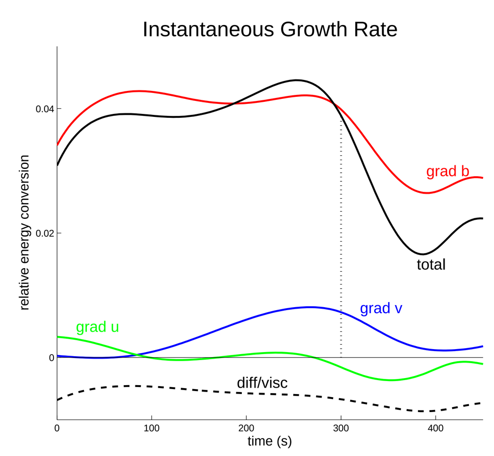
Statically unstable IGW

- The growth factor as a function of λ for perturbations to the unstable IGW plus leading transverse NM is shown to the right. The initial perturbation with maximum growth factor has $\lambda = 150$ m.
- By comparison, the leading "parallel" primary normal mode has wavelength 500 m.



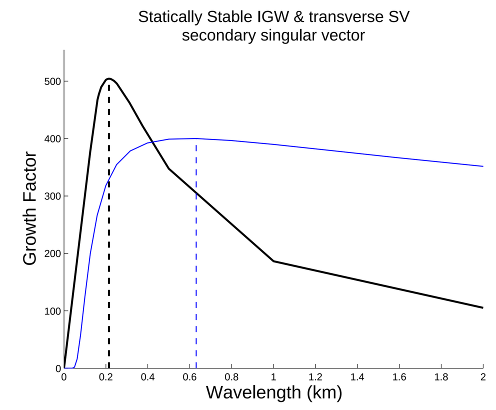


- Contours of perturbation vertical velocity w'_ϕ for the leading secondary SV are shown over the basic state buoyancy field b at three times.
- Initially concentrated near the maximum negative buoyancy gradient, the perturbation is advected by (v, w_ϕ) of the basic state velocity field as it grows.
- The relative energy conversion terms (instantaneous growth rate) decomposed into contributions from gradients in basic state show the perturbation grows mainly due to ∇b_0.
- Shear in v_0 becomes important later in the development.

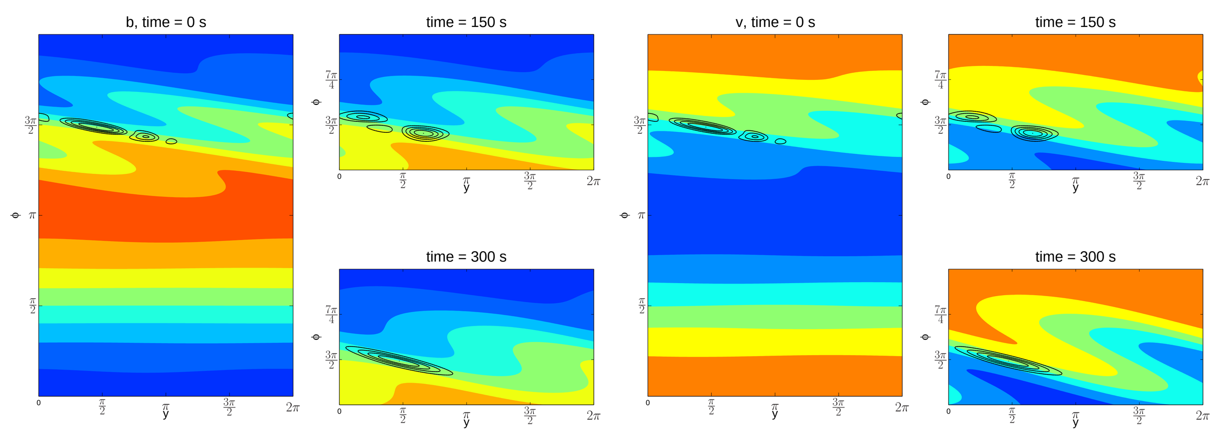


Statically stable IGW

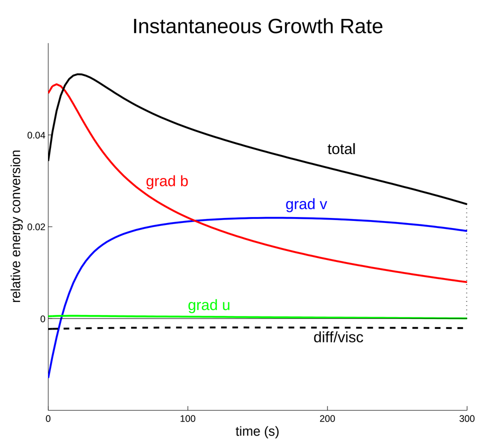
- The optimal perturbation to the stable IGW plus leading transverse SV with normalized amplitude 0.1 has wavelength $\lambda = 215$ m compared to the leading parallel primary singular vector, which has wavelength 630 m.



- Again, the leading secondary SV is concentrated near the point of maximum buoyancy gradient, but here it is located near a node in the (v, w_ϕ) field and is therefore not advected.
- Furthermore, the perturbation is located at a point of strong vertical shear in v_0 and aligns itself perpendicular to ∇v_0.



- Initially, the perturbation grows due to the buoyancy production terms, but as it develops, it draws energy mostly from the shear in v_0.
- Since the perturbation varies in the ξ direction but grows due to shear in the perpendicular velocity component v_0, this is known as a (statically triggered) roll mechanism.



- The secondary SV in the this case resembles the parallel primary singular vector of the IGW, but modified by the spatial structure of the transverse primary SV.
- Perturbation energy growth is approximately exponential with time and the growth factor increases exponentially with optimization time between one and five minutes.

Outlook

- Further explore parameter space of primary perturbation amplitude and secondary perturbation optimization time
- Three-dimensional nonlinear simulations initialized with IGW, primary SV/NM, and secondary SV – using a triply periodic domain with sides equal to respective wavelengths
- Investigate secondary instabilities of a high-frequency gravity wave (HGW), which has shallower propagation angle so that simple static and dynamic stability criteria are less applicable due to horizontal gradients in the wave.

References

[1] U. A. Achatz. The primary nonlinear dynamics of modal and nonmodal perturbations of monochromatic inertia-gravity waves. *J. Atmos. Sci.*, 64:74–95, 2007.

[2] U. A. Achatz. Gravity-wave breaking: Linear and primary nonlinear dynamics. *Adv. Space Res.*, 40:719–733, 2007.

[3] D. C. Fritts, J. R. Isler, and Ø Andreassen. Gravity wave instability dynamics at high Reynolds numbers. Part I: Wave field evolution at large amplitudees and high frequencies. *J. Atmos. Sci.*, 66:1126–1148, 2009.

[4] R. Giering and T. Kaminski. Recipes for adjoint code construction. *ACM Trans. Math. Software*, 24:437–474, 1998.

[5] G. P. Klaassen and W. R. Peltier. The onset of turbulence in finite-amplitude Kelvin-Helmholtz billows. *J. Fluid Mech.*, 155:1–35, 1985.