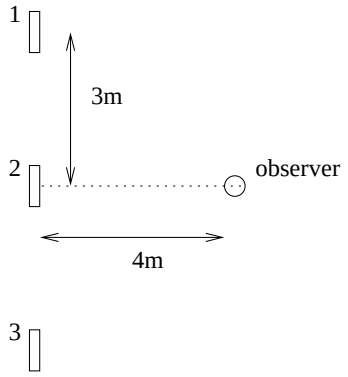
3-source interference

This is the solution to problem # 73 from Chapter 21 of the Knight textbook. The setup is depicted in the figure.



The wave produced by the top [1] and bottom [3] speakers at the location of the observer is

$$A_1 = A_3 = a\sin(5\pi - \omega t), \tag{1}$$

while speaker 2 produces a wave given by

$$A_2 = a\sin(4\pi - \omega t). \tag{2}$$

In this problem it is easier to just substitute the numerical values for distances and k (– the wave number), rather than carry on the whole algebraic expression ($\ldots\sin(kx - \omega t)$, etc.). Namely,

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{2\pi 170}{340} = \pi\ m^{-1}.$$

In (1) and (2) the coefficients 4 and 5 are just the distances from the speakers to the observation point ($5 = \sqrt{3^2 + 4^2}$). The resultant amplitude due to the superposition of the waves is

$$A = A_1 + A_2 + A_3 = 2a\sin(5\pi - \omega t) + a\sin(4\pi - \omega t) = 2a\sin(\omega t) - a\sin(\omega t) = a\sin(\omega t). \tag{3}$$

The answer looks like an oscillation of amplitude a – this is the answer for part a).

For part b) let us notice that the wavelength of the waves is $\lambda = v/f = 2m$. Waves from speakers 1 and 3 arrive at the observation point with a 5π phase shift (relative to the phase at the source) – this is due to the distance source-observer of 2.5 wavelengths, while the wave from speaker 2 arrives with a phase of 4π (2.0 wavelengths). Hence, it is obvious we need to shift speaker 2 to the left by $1.0m$ (half of λ), so that the phase at arrival is also 5π, and therefore constructive interference at the observation point. Note that we could have shifted speaker 2 to the right by $1.0m$ with the same result (but they specifically asked us to move the speaker to the left).

For part c) the answer is straightforward – we use equation (3) but with $a\sin(4\pi - \omega t)$ replaced by $a\sin(5\pi - \omega t)$, and obtain

$$A^* = 3a\sin(5\pi - \omega t).$$

The intensity of the sound is proportional to the square of the amplitude of oscillation, so the answer for this part is that the sound intensity is **9** times greater under the constructive interference from the 3 speakers scenario, compared to the sound intensity from a single speaker.

As a final observation, please note that we assumed the amplitude of the wave from the 2-nd speaker to be still a at the observation point, even after we shifted the speaker to the left! Well, is this reasonable ... ? Perhaps not, but otherwise we don't get the answer from the textbook :-)

Last revised:November 27, 2006; solution by Sorin Codoban.