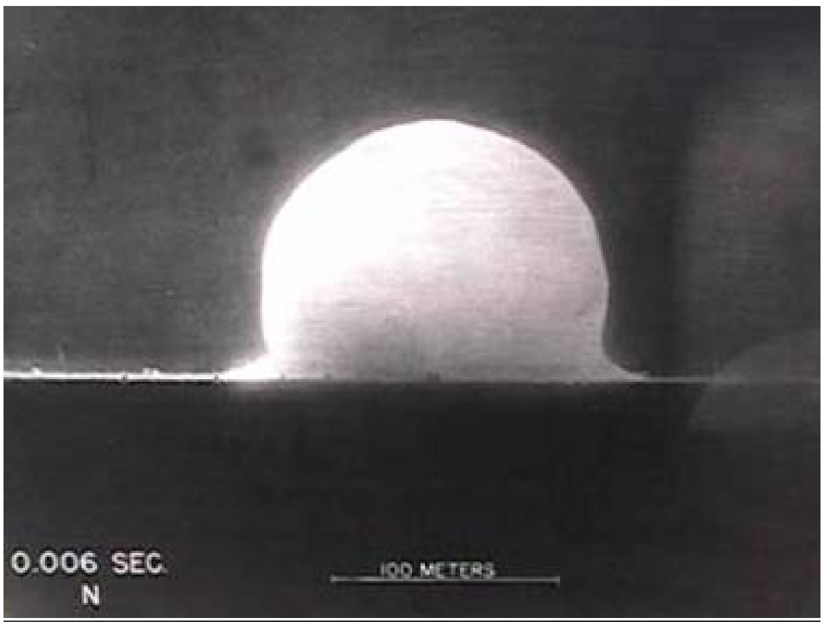
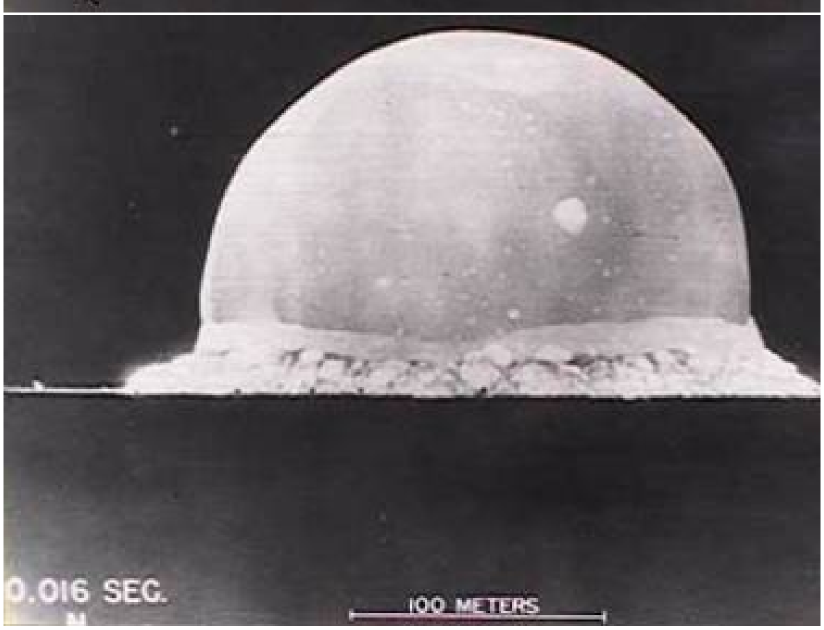
Estimate of the energy released in the first Atomic Bomb explosion.

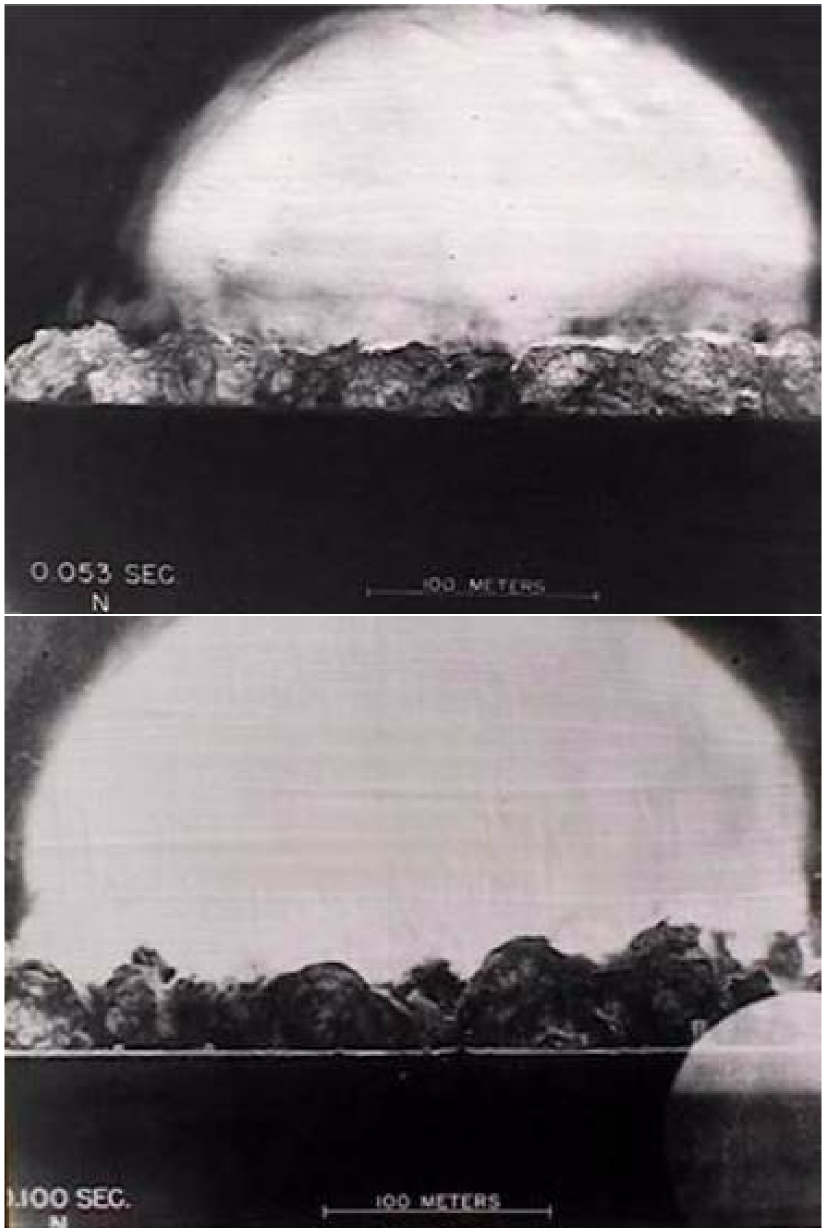
This document is adapted after the URL: `http://www.pa.uky.edu/~sps/Month1.htm` (snapshot as of Sept. 10, 2004).
My only contribution was typesetting the above mentioned web document in LaTeX.

The first explosion of an atomic bomb was the Trinity test in New Mexico in 1945. Several years later a series of pictures of the explosion, along with a size scale, and time stamps were released and published in a popular magazine. Based on these photographs a British physicist named G. I. Taylor was able to estimate the *power released by the explosion* (which was still a secret at that time).

How can the following pictures be used to make this estimate?







First two assumptions need to be made:

1. The energy (E) was released in a small space.
2. The shock wave was spherical.

We have the size of the fire ball (R as a function of t) at several different times. How does the radius (R) depend on:

- energy (E)
- time (t)
- density of the surrounding medium (ρ – initial density of air)

Let's perform a dimensional analysis of the problem:

- $[R] = L$:radius is determined by a distance
- $[E] = ML^2/T^2$:energy is determined by a mass times a distance squared divided by time squared.
- $[t] = T$:Time is determined by the time.
- $[\rho] = M/L^3$:density is determined by a mass divided by a distance cubed.

We can say

$$[R] = L = [E]^x[\rho]^y[t]^z$$

Substituting the units for energy, time and density that we listed above we have:

$$[R] = L = M^{(x+y)}L^{(2x-3y)}T^{(-2x+z)}$$

M is to the $x + y$ power because energy and density are both dependent on M.

L is to the $2x - 3y$ power because energy is dependent on the square of distance and density is dependent on one over the cube of distance.

T is to the $-2x + z$ power because energy is dependent on one over the square of time and time is dependent on time. This provides three simultaneous equations:

$$\begin{aligned} x + y &= 0, \\ 2x - 3y &= 1, \\ -2x + z &= 0, \end{aligned}$$

yielding the results:

$$x = 1/5, \quad y = -1/5, \quad z = 2/5.$$

The radius of the shock wave is therefore:

$$R = E^{1/5}\rho^{-1/5}t^{2/5} * constant$$

Let's assume the constant is approximately 1.

Solving the equation for E we get:

$$E = (R^5\rho)/t^2.$$

At $t = .006$ seconds the radius of the shock wave was approximately 80 meters. The density of air is $\rho = 1.2\,kg/m^3$. Plugging these values into the energy equation gives:

$$\begin{aligned} E &= (80^5) \times 1.2/(.0062)\,kg * m^2/s^2 \\ &= 1 \times 10^{14}\,kg * m^2/s^2 \\ &= 1 \times 10^{21}\,ergs \end{aligned}$$

Now, 1 g of TNT $= 4 \times 10^{10} erg$, and hence

$$E = 25\,\text{kilo} - \text{tons of TNT}$$

We would like to thank Prof. Wolfgang Korsch for presenting this problem and its solution.