Free falling objects

These notes present the solution of the problem #**52** from Ch. 2 of Serway&Jewett.

"A rock is dropped from rest into a well. (a) If the sound of the splash is heard $2.40s$ later, how far below the top of the well is the surface of the water? The speed of sound in air is $336m/s$. (b) If the travel time for the sound is ignored, what percentage error is introduced when the depth of the well is calculated?"

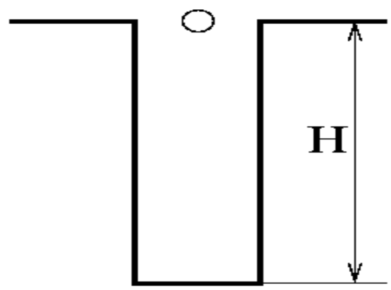

Figure 1: The setup.

Let us denote the total time by τ and the depth of the well by H and take $g = 9.80m/s^2$.

It is clear that the total time is the sum of the time of free fall τ_f plus the time τ_c it takes the sound to propagate "back". For the free fall we have

$$H = y = \frac{g}{2}\tau_f^2 \Rightarrow \tau_f = \sqrt{\frac{2H}{g}}, \tag{1}$$

while for the sound travel we get $\tau_c = \dfrac{H}{c}$. Hence, the total time is given by

$$\tau = \sqrt{\frac{2H}{g}} + \frac{H}{c}. \tag{2}$$

We need to solve for H. From (2) one obtains

$$0 < \sqrt{\frac{2H}{c}} = \left(\tau - \frac{H}{c}\right). \tag{3}$$

We outlined the positiveness of both sides for reasons to be explained in a moment. From (3) we derive a quadratic equation in H with the solutions

$$H_{1,2} = c\left\{\tau + \frac{c}{g} \pm \frac{c}{g}\sqrt{1 + \frac{2g\tau}{c}}\right\}. \tag{4}$$

Interesting enough, both solutions in (4) are positive! Which solution should we choose?

The relief is brought by the equation (3) which implies $H < c\tau$. This condition rules out the "+" solution (please check for yourself!). We are left with

$$H = c\left\{\tau + \frac{c}{g} - \frac{c}{g}\sqrt{1 + \frac{2g\tau}{c}}\right\}. \tag{5}$$

Numerically, $H = 26.4m$.

(b) If the travel time for sound is neglected the depth $\tilde{H}$ we estimate is

$$\tilde{H} = \frac{g}{2}\tau^2, \tag{6}$$

which numerically gives us $\tilde{H} = 28.2m$. Therefore, the percentage error is

$$\varepsilon = \frac{\tilde{H} - H}{H} = 0.068 = 6.8\% \tag{7}$$

One may ask how does this percentage error changes with the depth, or for that matter, with the time τ that we measure. In Fig. 2 the error ε was plotted against the time τ. We see that for $\tau = 30s$ we get $\varepsilon = 0.75$. Observe that I plotted only $\tau < 33s$. If there is no friction then $33s$ is about the time when the speed of free fall equals the speed of sound; arguably, at this speed the formulae for subsonic fall cease to be valid :-)

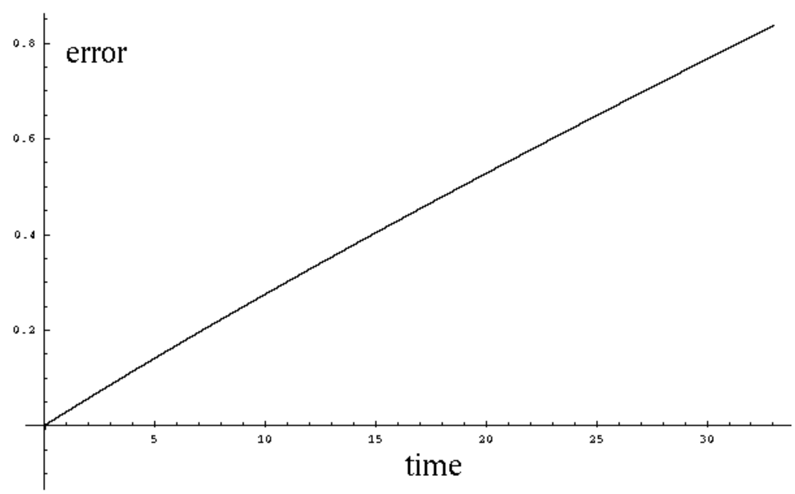

Figure 2: The error ε plotted vs. measured time τ (see text for details).

Last revised: September 25, 2003