TUTORIAL PHY 180 Prepared by

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"Universal Gravitation".

Serway 13-53

A ring on matter is a familiar structure in planetary and stelar astronomy. Consider a uniform ring of mass 2.36×10^{20} kg and radius 1.0×10^{8} m. An object of mass 1000 kg is placed at a point A on the axis of the ring, 2.0×10^{8} m from the center of the ring. When the object is released, the attraction of the ring makes the object move along the axis toward the center of the ring (point B).

a) Calculate the gravitational potential energy of the object-ring system when the object is at A.

b) Calculate the gravitational potential energy of the system when the object is at B.

c) Calculate the speed of the object as it passes through B.

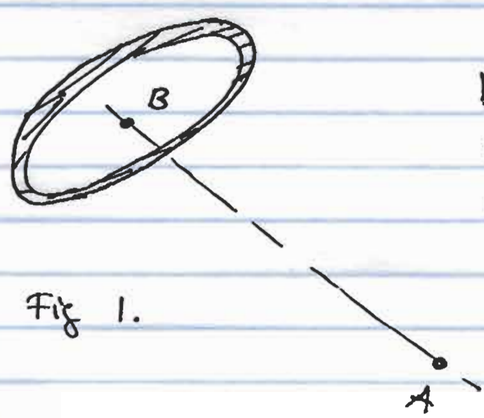


Fig 1.

Let $\overline{AB} = z$ the distance between points A ^ B, R the radius of the ring, M_R its mass and M_O the mass of the object.

The gravitational potential energy associated with two "particles" separated by a distance r is

$$U = -\frac{G m_1 m_2}{r}.$$

is the total mass of the ring. The final expression for U is

$$\boxed{U(z) = -\frac{G M_0 M_R}{\sqrt{R^2+z^2}}}.$$

a) When the object is at (A):

$z = \overline{AB} = 2.0 \times 10^8 m$, $R = 1.0 \times 10^8 m$, $M_0 = 10^3 kg$

$M_R = 2.36 \times 10^{20} kg$

$$U(2.0\times10^8) = -\frac{(6.67\times10^{-11} N\cdot m^2/kg^2)(10^3 kg)(2.36\times10^{20} kg)}{((1.0\times10^8 m)^2 + (2.0\times10^8 m)^2)^{\frac{1}{2}}}$$

$$U(2.0\times10^8 m) \equiv -7.03\times10^4 J$$

b) When the object is at B, $z=0$ and

$$\boxed{U(0) = -\frac{G M_0 M_R}{R}} = -15.74\times10^4 J$$

c) The speed of the object as it passes through B:

We use $\Delta k = -\Delta U$

$$\frac{1}{2} M_0 V_B^2 - \frac{1}{2} M_0 V_A^2 = -(U(B) - U(A)).$$

$V_A = 0$

$$V_B = \sqrt{\frac{2}{M_0}(U(A) - U(B))}$$

For a system with distributed mass, the expression above has to be modified:

$$U \equiv - \sum_{\substack{\{\text{ALL}\\ \text{elements}\\ \text{of mass}\}}} \frac{GM\Delta m}{r}$$

where r is the distance between the element of mass Δm and M. For the case of the ring we have (See Fig 2)

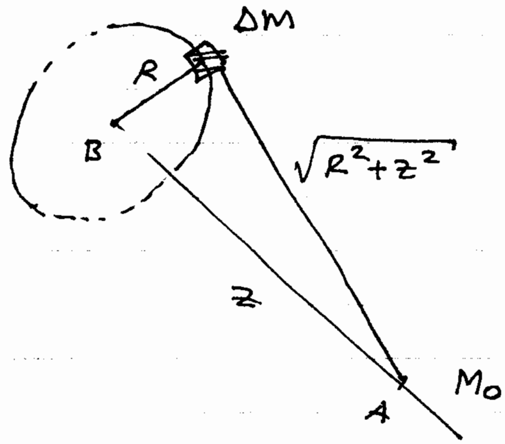


Fig 2.

The gravitational potential energy is therefore:

$$U = - \sum_{\{\text{ALL } \Delta m\}} \frac{GM_o\,\Delta m}{\sqrt{R^2+z^2}}$$

. Notice that all elements are located at the same distance $\sqrt{R^2+z^2}$, then this quantity can be considered as a "constant" in the integration process:

$$U \rightarrow U(z) = - \frac{GM_o}{\sqrt{R^2+z^2}} \sum_{\{\text{ALL } \Delta m\}} \Delta m \; .$$

$\sum_{\{\text{ALL } \Delta m\}} \Delta m \equiv M_{RING} \equiv M_R$: The sum of all elements of mass

Finally:

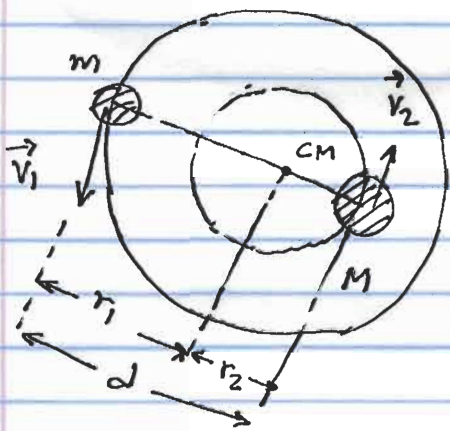
$$V_B = \sqrt{\frac{2 \times (-7.03\times10^4 - (-15.74\times10^4))}{10^3}}$$

$$\boxed{V_B = 13.2 \text{ m/s}}$$

Serway 13-69.

Two stars of masses M and m, separated by a distance d, revolve in circular orbits about their center of mass. (Fig. 3). Show that each start has a period given by

$$T^2 = \frac{4\pi^2 d^3}{G(M+m)}.$$



With respect to the center of mass:

i) For the star of mass m

$$\frac{GMm}{d^2} = m\omega^2 r_1.$$

ii) For the star of mass M

$$\frac{GMm}{d^2} = M\omega^2 r_2.$$

From the diagram $d = r_1 + r_2$. The "center of mass" condition leads to

$$Mr_2 = mr_1.$$

From (i) ^ (ii)

$$r_1 = \frac{GM}{d^2\omega^2} \quad ; \quad r_2 = \frac{Gm}{d^2\omega^2}$$

$$r_1 + r_2 = d = \frac{GM}{d^2\omega^2} + \frac{Gm}{d^2\omega^2} .$$

Obtaining ω^2 from the last expression:

$$\omega^2 = \frac{G(M+m)}{d^3} \equiv \left(\frac{2\pi}{T}\right)^2 . \quad \text{or}$$

$$\boxed{T^2 = \frac{4\pi^2 d^3}{G(M+m)}}$$

It's also possible to obtain the distances of m ^ M to the CM:

$$\boxed{r_1 = d\,\frac{M}{M+m}} \quad ; \quad \boxed{r_2 = d\,\frac{m}{M+m}}$$

Notice that $\frac{r_1}{r_2} = \frac{M}{m}$ which corresponds to the "center-of-mass" condition as an origin of coordinates.

Serway 13.52

(a) Show that the rate of change of the free-fall acceleration with distance above the Earth's surface is

$$\frac{dg}{dr} = -\frac{2GM_E}{R_E^3}.$$

This rate of change is called a "gradient". (b) If h is small in comparison to the radios of the Earth, show that the difference in free-fall acceleration between two points separated by vertical distance h is

$$|\Delta g| = \frac{2GM_Eh}{R_E^3}.$$

c) Evaluate this difference for $h = 6.0\,m$, a typical height for two-story building.

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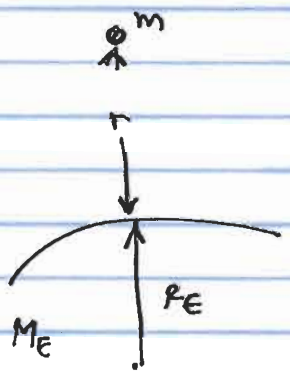

The gravitational field $g(r)$ is obtained from:

$$m\,g(r) \equiv \frac{GM_Em}{(R_E+r)^2}.$$

$$g(r) = \frac{GM_E}{(R_E+r)^2}.$$

This expression can be manipulated in the form:

$$g(r) = \frac{G M_E}{\left(R_E\left(1+\frac{r}{R_E}\right)\right)^2} \equiv \frac{G M_E}{R_E^2}\left(1+\frac{r}{R_E}\right)^{-2}$$

- For small values of $\frac{r}{R_E}$ (points located nearby of the Earth's surface:

$$g(r) \cong \frac{G M_E}{R_E^2}\left(1-\frac{2r}{R_E}\right) \qquad \left(r/R_E \ll 1\right)$$

$$g(0) = \frac{G M_E}{R_E^2}, \text{ and } \quad g(r) - g(0) = -\frac{2 G M_E}{R_E^3} r$$

In the limit where $r \to 0$ ∴

$$\lim_{r\to 0}\left(\frac{g(r)-g(0)}{r}\right) \simeq \boxed{\frac{dg(r)}{dr} = -\frac{2 G M_E}{R_E^3}}$$

b) For two points located at r and $r+h$ respectively, we have

$$g(r+h) - g(r) = \Delta g =$$

$$\left\{g(0) - \frac{2 G M_E}{R_E^3}(r+h)\right\} - \left\{g(0) - \frac{2 G M_E}{R_E^3} r\right\}$$

$$\boxed{\Delta g \equiv -\frac{2 G M_E h}{R_E^3}}$$

c) For $h = 6.0\,m$

$$\boxed{|\Delta g| = 1.85 \times 10^{-5}\ m/s^2}$$