Q1: A Simple Question on Potential Energy

The potential energy function for a system is given by:

$$U(x) = -x^3 + 2x^2 + 3x.$$

(a) Determine the force F_x as a function of x.

(b) For what values of x is the force equal to 0?

(c) Plot $U(x)$, and indicate points of stable and unstable equilibrium.

A:

(a) $F_x(x) = -\dfrac{dU}{dx} = 3x^2 - 4x - 3.$

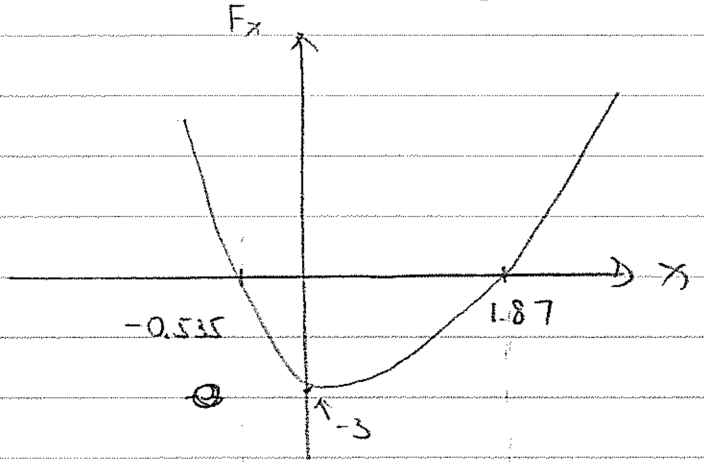
$\therefore \; F(x) = (3x^2 - 4x - 3)\,\hat{i}$

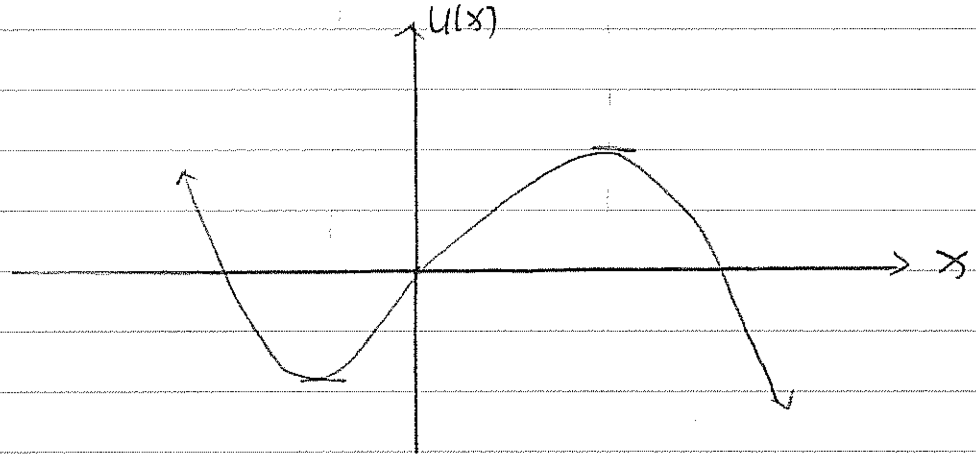
(b) $F_x = 0 \Rightarrow$

$$3x^2 - 4x - 3 = 0 \Rightarrow x = \frac{4 \pm \sqrt{52}}{6}$$

$$\Rightarrow x = 1.87 \text{ or } -0.535.$$

(c)



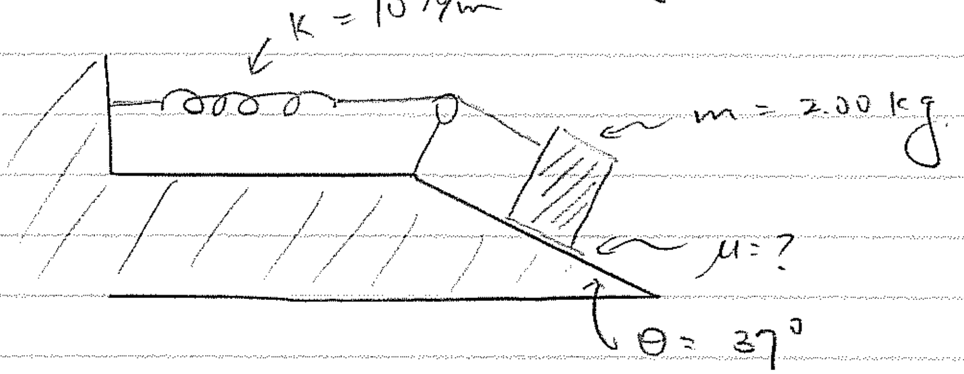


Q2: Mass on an incline with spring.

A 2.00 kg block is situated on a rough incline is connected to a spring of negligible mass having a spring constant $K = 100$ N/m. The pulley is frictionless. The block is released from rest when the spring is unstretched. The block moves 0.20 m down the incline before coming to rest. Find the coefficient of kinetic friction between block & incline.

③

A:



conservation of energy:

$$K_i + U_i - F_f \cdot d = K_f + U_f$$

where h_i, h_f = initial & final heights of the block.
d = amount by which the block moves downwards.
F_f = friction force.

$$0 + mgh_i - F_f \cdot d = mgh_f + \frac{1}{2}kd^2$$

Since $F_f = \mu N = \mu mg\cos\theta$
normal force

$$-\mu(mg\cos\theta)\,d = -mg(h_i - h_f) + \frac{1}{2}kd^2$$

$$\mu(mg\cos\theta)d = +mg(h_i - h_f) - \frac{1}{2}kd^2$$

$$\therefore \mu = \frac{mgd\sin\theta - \frac{1}{2}kd^2}{mg(\cos\theta)\cdot d}$$

after plugging in all values =

$m = 2.00$ kg
$k = 10$ N/m
$d = 0.2$ m

$\theta = 37^\circ$

$\mu = 0.114$

Q3: A Harder Question on Conservation of Energy.

A ball having mass m is connected by a strong string of length L to a pivot point. A wind exerting a constant force of magnitude F is blowing from left to right.

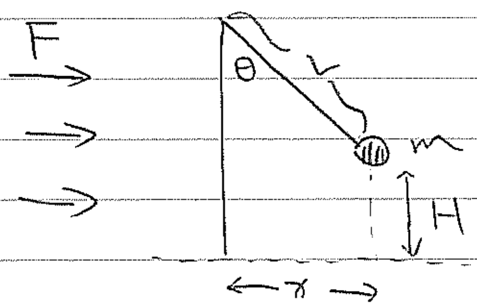
(a) Show that the max. height is

$$H = \frac{2L}{1 + (mg/F)^2}$$

if the ball is released from rest.

(b) Find the equilibrium height of the ball.

(c) Can the equilibrium point be greater than L?



(d) as a useful tool, we find x in terms of L & H:

$$(L-H)^2 + x^2 = L^2$$

$$\cancel{L^2} - 2LH + H^2 + x^2 = \cancel{L^2}$$

$$\therefore \quad x^2 = 2LH - H^2$$

now with conservation of energy:

$$K_i + U_i + Fx = K_f + U_f$$

$$0 + 0 + Fx = mgH$$

$$(Fx)^2 = (mgH)^2$$

$$F^2(2LH - H^2) = (mgH)^2$$

$$\therefore H\left[H(F^2 + (mg)^2) - F^2 \cdot 2L\right] = 0$$

$$\therefore \quad H = 0 \quad \text{or} \quad H = \frac{F^2 \cdot 2L}{F^2 + (mg)^2} = \underset{\substack{\uparrow \\ \text{max.}}}{\frac{2L}{1 + (mg/F)^2}}$$

For $H = 0$, we need $F \to 0^+$, i.e. no wind

$H = 2L$: : $F \to \infty^+$, i.e. a very strong wind.

(b) Force balance:

x - dir:

$$F - T\sin\theta = 0$$

↑ tension

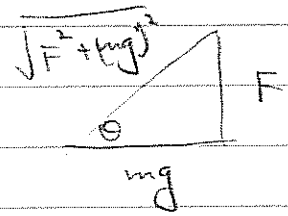
$$\therefore F = \frac{F}{\sin\theta}$$

y - dir:

$$T\cos\theta - mg = 0$$

$$\frac{F}{\sin\theta} \cdot \cos\theta - mg = 0$$

$$\therefore \tan\theta = \frac{F}{mg}$$



∴ From geometry:

equilibrium height

$$\cos\theta = \frac{L - H_{eq}}{L}$$

$$\therefore H_{eq} = L(1-\cos\theta) = L\left(1 - \frac{mg}{\sqrt{F^2+(mg)^2}}\right)$$

$$= L\left(1 - \frac{1}{\sqrt{1+(F/mg)^2}}\right)$$

(c) note since $F \in [0, \infty)$

$0 < \cos\theta \leq 1$

$\therefore 0 \leq H_{eq} < L$

$\therefore$ No, H_{eq} is never greater than L