TUTORIAL PHY 180F

Week Nov 06 - 10 /06

Ch9 - Ch10.

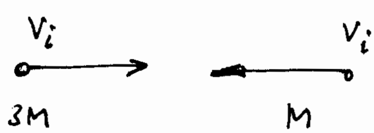
- Two particles with masses M and $3M$ are moving toward each other along the x axis with the same initial speeds v_i. Particle M is traveling to the left, while particle $3M$ is traveling to the right. They undergo an elastic glancing collision such that particle M is moving downward after the collision at right angles from its initial direction.

(a) Find the final speeds of the two particles.

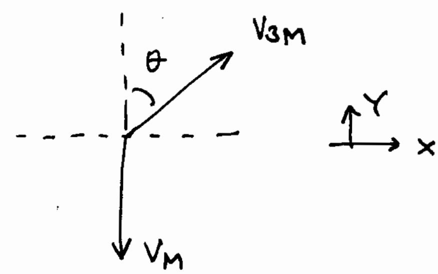
(b) What is the angle θ at which the particle $3M$ is scattered?

Sol.

(a)



before



after

Conservation of linear momentum: (x axis)

$$3M V_i \,(\hat{\imath}) + M V_i \,(-\hat{\imath}) = 3M V_{3M} \sin\theta \,(\hat{\imath}) \qquad (1)$$

(y axis)

$$0 = 3M V_{3M} \cos\theta \,(\hat{\jmath}) + M V_M \,(-\hat{\jmath}). \qquad (2)$$

Conservation of kinetic energy:

$$\frac{1}{2} M V_i^2 + \frac{1}{2}(3M) V_i^2 = \frac{1}{2} M V_M^2 + \frac{1}{2}(3M) V_{3M}^2 \qquad (3)$$

From Eqs (1) , (2) :

$$\cos\theta = \frac{V_M}{3V_{3M}} \quad ; \quad \sin\theta = \frac{2V_i}{3V_{3M}}$$

or , from $\cos^2\theta + \sin^2\theta = 1$;

$$\boxed{9V_{3M}^2 = V_M^2 + 4V_i^2} \quad (4)$$

From Eq. (3) :

$$\boxed{4V_i^2 = V_M^2 + 3V_{3M}^2} \quad (5)$$

By combining (4) and (5), we get :

(a) $$\boxed{V_{3M} = \sqrt{\frac{2}{3}}\, V_i} \qquad \boxed{V_M = \sqrt{2}\, V_i}$$

(b) Also, from Eqs (1) , (2), we can get the scattered angle:

$$\frac{\sin\theta}{\cos\theta} \equiv \tan\theta = \frac{\frac{2V_i}{3V_{3M}}}{\frac{V_M}{3V_{3M}}} = 2\frac{V_i}{V_M} = \sqrt{2}$$

$\tan\theta = \sqrt{2}$, $\boxed{\theta = 54.7^0}$

2. A rocket for use in deep space is to be capable of boosting a total load (payload plus rocket frame and engine) of of 3.00 metric tons to a speed of 10,000 m/s

(a) It has an engine and fuel designed to produce an exhaust speed of 2,000 m/s. How much fuel plus oxidizer is requiered?

(b) If a different fuel and engine design could give an exhaust speed of 5,000 m/s, what amount of fuel and oxidizer would be requiered for the same task?

Sol)

(a). The speed of the rocket in terms of its "instantaneous" mass M is given by the expression:

$$V(M) = V_0 + V_e \ln\left(\frac{M_0}{M}\right), \qquad (1)$$

where V_0 is a "reference" speed, and V_e is the exhaust speed. M_0 is the mass of the system when the velocity is V_0. Therefore, the ~~total~~ mass M as a function of the speed is:

$$M(V) = M_0 \, e^{-(V-V_0)/V_e} \qquad (2).$$

From Eq (2), we notice that the total mass (M_{total}) can be obtained when $V=0$; i.e. no fuel has been burnt and the system is about to be launched.

mathematically:

$$\boxed{M_{total} = M_0\, e^{\frac{V_0}{V_e}}} \qquad (3)$$

For the specific case : $M_0 =$ 3 metric tons, $V_0 = 10^5$ m/s
and (a) $V_e = 2\times10^3$ m/s.

$$M_{total} = 445.2 \text{ metric tons}.$$

Now, $M_{total} = M_0 + M_{fuel}$ (M_0 : $M_{payload} + M_{frame} + M_{engine}$)

then $M_{fuel} = M_{total} - M_0$

$$M_{fuel} = M_0\left(-1 + e^{V_0/V_e}\right) = (445.2 - 3) \text{ metric tons}$$

$$\boxed{M_{fuel} = 442.2 \text{ metric tons}} \qquad (4)$$

(b) In similar way, $V_e = 5$ m/s $(\times10^3)$ and

$$M_{fuel} = M_0\left(e^{V_0/V_e} - 1\right) = 3\times\left(e^2 - 1\right) = 19.2 \text{ metric tons}$$

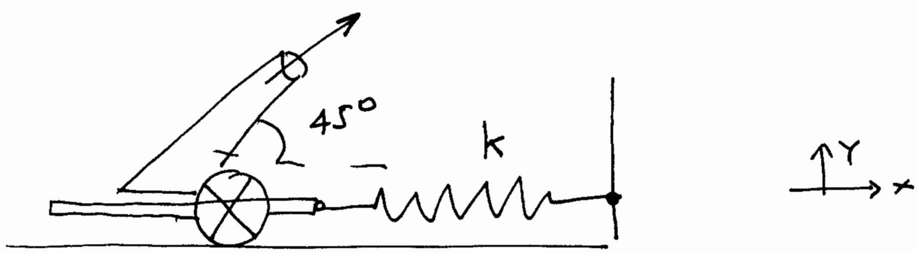
$$\boxed{M_{fuel} = 19.2 \text{ metric tons}}$$

3. A cannon is rigidly attached to a carriage, which can move along horizontal rails but is connected to a post by a large spring, initially unstretched and with force constant $k = 2 \times 10^4$ N/m. (Fig 3.1) The cannon fires a 200 kg projectile at a velocity of 125 m/s directed 45° above the horizontal.

(a) If the mass of the cannon and its carriage is 5000 kg, find the recoil speed of the cannon.

(b) Determine the maximum extension of the spring.

(c) Find the maximum force the spring exerts on the carriage



(a) Conservation of Linear momentum leads to:

$$M_{cannon} V_{recoil} (-\hat{\imath}) + M_{projectile} V_{projectile} \cos\theta (\hat{\imath}) = 0$$

$$V_{recoil} = \frac{M_{projectile} V_{projectile} \cos\theta}{M_{cannon}} \qquad (1)$$

$$V_{recoil} = (5\times10^3)^{-1}(200) \cos 45 \times 125 = \cancel{0.03 \text{ m/s}}.$$

$$\boxed{V_{recoil} = 3.5 \text{ m/s}}$$

(b) Maximum extension of the spring:

$$\frac{1}{2} M_{cannon} V_{recoil}^2 \equiv \frac{1}{2} k \delta^2$$

δ: Maximum extension given by conservation of energy:

$$\delta = \sqrt{\frac{M_{cann}}{k}} V_{recoil} = \sqrt{\frac{5 \times 10^3}{2 \times 10^4}} (3.5) \text{ m}$$

$$\boxed{\delta = 1.75 \text{ m}}$$

c) The maximum force exerted on the carriage is given by

$$F_{spring} = k\delta = (2 \times 10^4) \times (1.75)$$

$$\boxed{F_{spring} = 3.5 \text{ N}}$$

4. A 10g bullet is fired horizontally into a 300g wooden block initially at rest on a horizontal surface and becomes embedded in it. The coefficient of friction between block and surface is 0.5. The combined system slides 4.0 m before stopping. ~~What~~ ~~spee~~

~~[illegible]~~ With what speed did the bullet strike the block?

Sol /

The block goes from some speed v_B to zero in the space of 4.0 m. The only net force on it during this time is friction. By finding the work done by friction, we will know how much energy the block had just after being struck by the bullet

$$F_f = \mu N, \quad F_f = \mu(\underset{\substack{\uparrow \\ \text{mass of} \\ \text{the block}}}{M} + \underset{\substack{\uparrow \\ \text{bullet}}}{m})g = (0.5)(0.3+0.01)(9.8)\ N$$

$$F_f = 1.52\ N$$

$$W_f = F_f\, d \cos\theta \quad , \quad \theta = 180°.$$

$$W_f = (1.52)(4)\cos 180 = -6.08\ J$$

. Now we know that the system had 6.08 J of energy as it started to move. We now can find the initial velocity of the block just after being struck by the bullet using the definition of kinetic energy:

$$KE = \frac{1}{2} m v^2 \quad ; \quad 6.08\ J = \frac{1}{2}(0.01+0.3)\, v_B^2$$

$$v_B = 6.26\ m/s.$$

Since momentum is conserved, we find the momentum of the system block-bullet just after collision and just before collision:

$$P_B = m_B v_B = (0.3+0.01)(6.26) = 1.94 \ \text{kg}\frac{\text{m}}{\text{s}}$$

$$1.94 \ \text{kg}\frac{\text{m}}{\text{s}} = (0.01 \ \text{kg})\ V_0$$

$$\boxed{V_0 = 194 \text{ m/s}}$$

5. The assembly in fig. 5.1 is made with massless strings attached to balls. Ball A ($m = 0.3$ kg) is released from height 0.2 m above ball B ($m = 0.5$ kg) also at rest. The two undergo an elastic collision.

What max height does each ball achieve after collision?

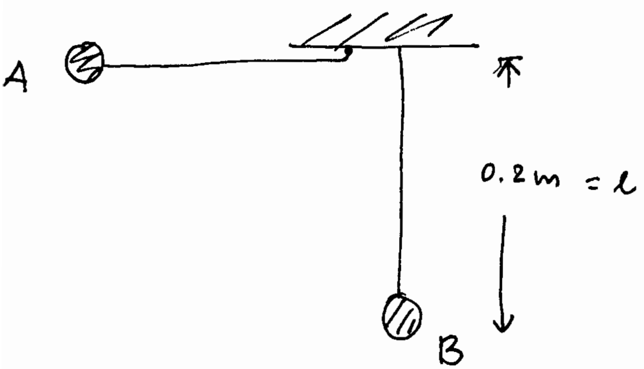


Fig 5.1

Initial conditions

Ball A ⇒ $KE = 0$

$PE = m_A g l$

$= 0.588 \text{ J}$

Ball B ⇒ $KE = 0$

$PE = 0.$

There is 0.588 J of energy in this system that must remain constant a every step. Just before collision:

A : $\frac{1}{2} m_A v_A^{o2} = m_A g l$; $v_A^o = 1.98$ m/s

Just after collision; momentum and energy are conserved:

$$m_A V_A + m_B V_B = p = m_A V_A^o = 0.59 \text{ kg} \frac{m}{s} \quad (1)$$

$$\frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2 = 0.588 = KE. \quad (2)$$

Now we solve for V_B in (1) and plug it into (2) and solve for V_A

$$\boxed{V_B = \frac{p - m_A V_A}{m_B}}$$

$$\frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B \left(\frac{p - m_A V_A}{m_B} \right)^2 = KE$$

Numerically : $0.195 V_A^2 - 0.36 V_A - 0.235 = 0.$

or V_A : $\{ 2.34, -0.51 \}$.

The negative velocity is correct because it is clear that the ball will not rebound with a greater velocity than it started with in the same direction

$$\boxed{V_A = -0.51 \text{ m/s}, \quad V_B = 1.50 \text{ m/s}.}$$

Just after collision

$$KE_A = \frac{1}{2} m_A V_A^2 = 0.0397 \text{ J}$$

$$KE_B = \frac{1}{2} m_B V_B^2 = 0.563 \text{ J}$$

At max height all KE is PE = mgh

$$\boxed{h_A = \frac{PE_A}{m_A g} = \frac{V_A^2}{2g} = 0.013 \text{ m}}, \quad \boxed{h_B = \frac{V_B^2}{2g} = 0.175 \text{ m}}$$

[9].