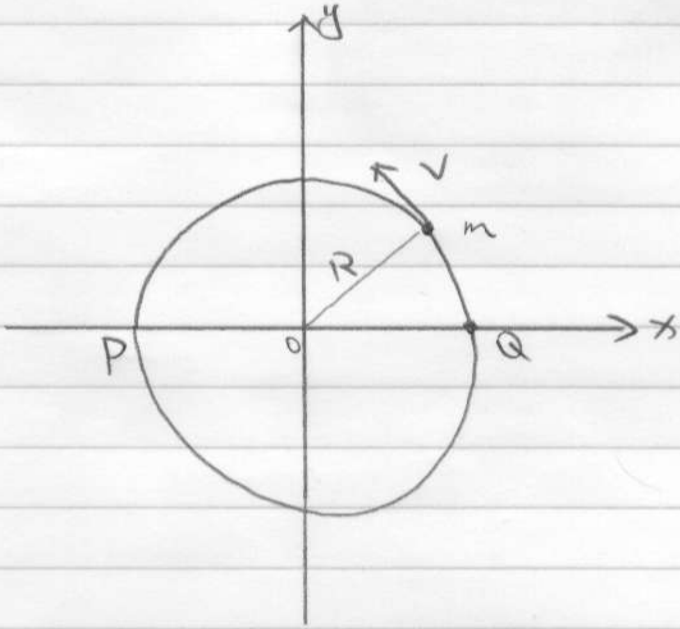
Q1. (Serway 11.15)

A particle of mass m moves in a circle of radius R at a constant speed v, as shown below. If the motion begins at point Q at time $t=0$, determine the angular momentum of the particle about point P as a function of time.



A: About point O, the position of the particle is

$$\vec{x}_0 = (R\cos\theta,\ R\sin\theta,\ 0)$$

where $\theta = \omega t = \frac{vt}{R}$

Hence, about point P,

$$\vec{x}_P = \left(R + R\cos\theta,\ R\sin\theta,\ 0 \right) = \left(R + R\cos\left(\frac{vt}{R}\right),\ R\sin\left(\frac{vt}{R}\right),\ 0\right)$$

The velocity:

$$\vec{v}_p = \frac{d\vec{x}_p}{dt} = \left(-v\sin\left(\frac{vt}{R}\right), \quad +v\cos\left(\frac{vt}{R}\right) \right)$$

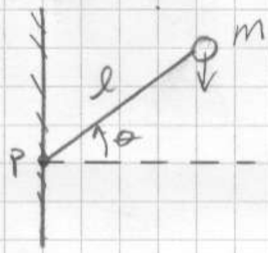
and

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$$

$$= \left(0, \quad 0, \quad mvR\left(1 + \cos\left(\frac{vt}{R}\right) \right) \right)$$

Q2. (Serway 11.19)

A ball having mass m is fastened at the end of a flagpole that is connected to the side of a tall building at point P. The length of the flagpole of length l and it makes an angle θ with the horizontal. If the ball becomes loose and starts to fall, determine the angular momentum (as a function of time) of the ball about point P. Neglect air resistance.



Recall that angular momentum is given by $\bar{L} = \bar{r} \times \bar{p}$

now $\bar{p} = m\bar{v}$ and since gravity is the only force acting on the ball

$$\bar{v} = 0\hat{\imath} - gt\,\hat{\jmath} + 0\hat{k} = v_j\hat{\jmath}$$

From this and Geometry,

$$\bar{r} = l\cos\theta\,\hat{\imath} + (l\sin\theta - \tfrac{1}{2}gt^2)\hat{\jmath} + 0\hat{k} = r_i\hat{\imath} + r_j\hat{\jmath}$$

$$\therefore \bar{L} = m\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ r_i & r_j & 0 \\ 0 & v_j & 0 \end{vmatrix} = 0\hat{\imath} + 0\hat{\jmath} + r_i v_j\hat{k}$$

$$\because r_i v_j = -l\cos\theta\, gt$$

$$\therefore \boxed{L(t) = -mgl\cos\theta\, t\,\hat{k}}$$

Q3. (Serway 11.28)

There are two cylinders on a frictionless axle, with moments of inertia I_1 and I_2. Originally only cylinder 1 is rotating (at ω_1) while cylinder two is suspended above it. Then cylinder 2 drops down and after some time, due to friction, the both rotate at ω_f.

a) Find ω_f

b) show that the kinetic energy of the system is decreased due to the interaction.

a) We use conservation of momentum.

$$L_i = L_f \quad \text{where } L = I\omega$$

$$I_1 \omega_i = (I_1 + I_2)\,\omega_f$$

$$\Rightarrow \boxed{\omega_f = \frac{I_1}{I_1 + I_2}\,\omega_i}$$

b) Kinetic (angular) energy is given by

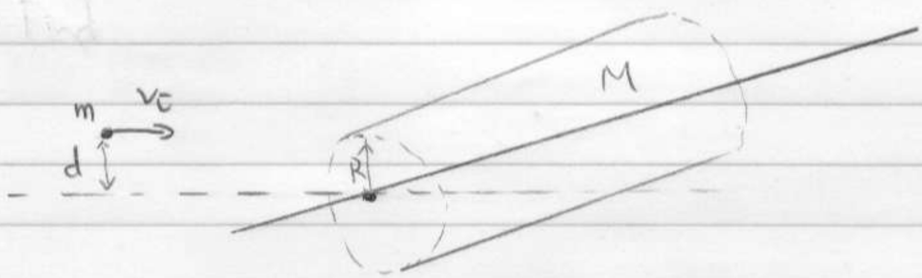
$$K_r = \tfrac{1}{2} I \omega^2$$

$$\therefore K_i = \tfrac{1}{2} I_1 \omega_i^2 \quad ; \quad K_f = \tfrac{1}{2}(I_1 + I_2)\,\omega_f^2 = \tfrac{1}{2}\frac{I_1^2}{I_1 + I_2}\,\omega_i^2$$

$$\therefore \boxed{\frac{K_f}{K_i} = \frac{I_1}{I_1 + I_2} < 1 \quad \therefore \text{Kinetic energy must be reduced by interaction}}$$

Q4.

A wad of sticky clay with mass m and velocity v_i is fired at a solid cylinder of mass M and radius R. See below. The cylinder is initially at rest and is mounted on a fixed horizontal axle that runs through its center of mass. The line of motion of the projectile is perpendicular to the axle and at a distance $d < R$ from the center.



(a) Find the angular speed of the system just after the clay strikes and sticks to the surface of the cylinder.

A: conservation of angular momentum:

$$L_f = L_i$$

$$I\omega = m v_i d \qquad (*)$$

$$\left[\frac{1}{2}MR^2 + mR^2\right]\omega = m v_i d$$

Hence,

$$\omega = \frac{2 m v_i d}{(M+2m) R^2}$$

(b) Is the mechanical energy of the clay - cylinder system conserved in this process?

A:

consider $\omega \cdot d = \frac{2 m v_i d^2}{(M+2m) R^2}$

since $\frac{2m}{M+2m} < 1$, $\frac{d^2}{R^2} < 1$

$\therefore \omega \cdot d < v_i$

from (*) we have

$I\omega = m v_i d$

Hence $I\omega(\omega d) < m v_i d \cdot v_i$

or $I\omega^2 < m v_i^2$

$\frac{1}{2} I\omega^2 < \frac{1}{2} m v_i^2$

$E_f < E_i$

Hence the energy is not conserved.

Intuitively, this is because some energy is spent to stick the clay to the cylinder (internal energy, which we did not consider).