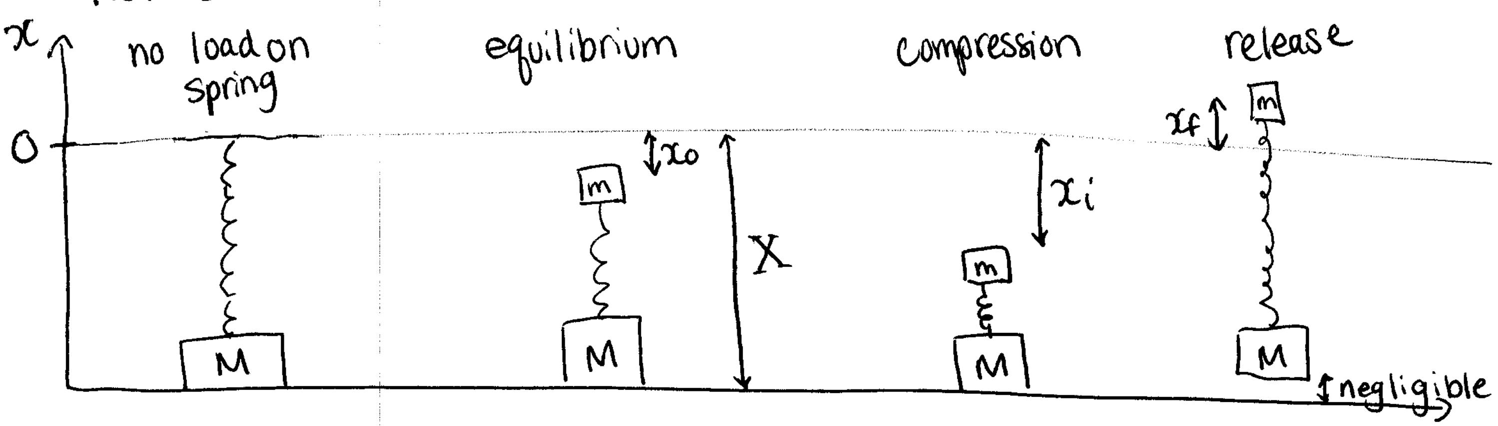
Q1. Serway 8.68

A block of mass M rests on a table. It is fastened to the lower end of a light vertical spring. The upper end of the spring is fastened to a block of mass m. The upper block is pushed down by an additional force 3mg, so that the spring compression is $\frac{4mg}{k}$. In this configuration the upper block is released from rest. The spring lifts the lower block off the table. In terms of m, what is the greatest possible value for M?

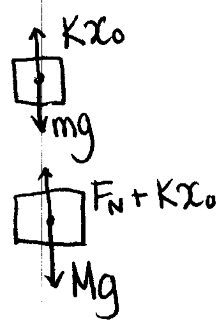
Solution.

The max. mass of M has the block just lifting off the table - enough to balance gravity, but not enough to move it up any noticeable distance.



FBDs for the above cases:

equilibrium:



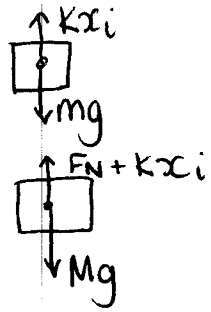
since we're in equilibrium:

$$\Sigma F = 0 \quad \text{(on mass m)}$$

$$0 = kx_0 - mg$$

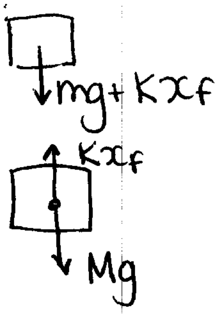
$$x_0 = \frac{mg}{k}.$$

compression:



$x_i = \frac{4mg}{k}$ (given)

final release:



since the large block doesn't move:

$$\Sigma F = 0$$
$$0 = kx_f - Mg$$
$$x_f = \frac{Mg}{k}$$

Now use conservation of energy:

compression: $E_{total,i} = K_i + U_{gi} + U_{spring\,i}$

$$= 0 + mg(-x_i) + Mg(-X) + \tfrac{1}{2}k(-x_i)^2$$

$$E_{total,i} = -mgx_i - MgX + \tfrac{1}{2}kx_i^2$$

final/release: $E_{total,f} = K_f + U_{gf} + U_{spring\,f}$

$$= 0 + mg(x_f) + Mg(-X) + \tfrac{1}{2}kx_f^2$$

$$E_{total,f} = mgx_f - MgX + \tfrac{1}{2}kx_f^2$$

$$E_i = E_f$$

$$-mgx_i - \cancel{MgX} + \tfrac{1}{2}kx_i^2 = mgx_f - \cancel{MgX} + \tfrac{1}{2}kx_f^2$$

sub in values for x_i & x_f:

$$-mg\left(\frac{4mg}{k}\right) + \frac{1}{2}k\left(\frac{4mg}{k}\right)^2 = mg\left(\frac{Mg}{k}\right) + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2$$

$$-\frac{4m^2g^2}{\cancel{k}} + \frac{\cancel{k}}{\cancel{2}}\frac{\overset{8}{\cancel{16}}m^2g^2}{\cancel{k^2}} = \frac{mMg^2}{\cancel{k}} + \frac{\cancel{k}}{2}\frac{M^2g^2}{\cancel{k^2}}$$

$$-4m^2 + 8m^2 = mM + \frac{M^2}{2}$$

$$4m^2 = mM + \frac{M^2}{2}$$

$$0 = M^2 + 2mM - 8m^2$$

$$M = \frac{-2m \pm \sqrt{4m^2 - 4(-8m^2)}}{2}$$

$$= 2m \text{ or } -4m$$

unphysical!

$$\boxed{\therefore M = 2m}$$

Q2. Serway 7.56

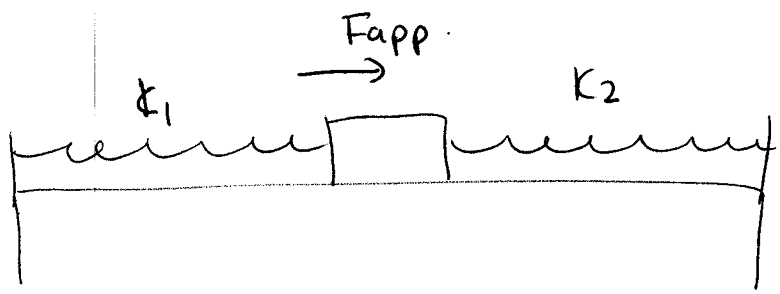
Two springs with negligible masses, one with spring constant k_1 and the other with spring constant k_2, are attached to the endstops of a level airtrack. A glider attached to both springs is located between them. When the glider is in equilibrium, spring 1 is stretched by extension $x_{i,1}$ to the right of its unstretched length and spring 2 is stretched by $x_{i,2}$ to the left. Now a horizontal force F_{app} is applied to the glider to move it a distance x_a to the right from its equilibrium position. Show that in this process

(a) the work done on spring 1 is $\frac{1}{2}k_1(x_a^2 + 2x_a x_{i,1})$

(b) the work done on spring 2 is $\frac{1}{2}k_2(x_a^2 - 2x_a x_{i,2})$

(c) $x_{i,2}$ is related to $x_{i,1}$ by $x_{i,2} = \frac{k_1 x_{i,1}}{k_2}$

(d) the total work done by the force F_{app} is $\frac{1}{2}(k_1 + k_2)x_a^2$.



The spring is stretched to the right at equilibrium, so when the glider is moved to the right, the full stretch of the spring is $(x_{i,1}+x_a)$.

The work done by F_{app} is negative the work done by the spring:

$$W_1 = -\int_{x_{i,1}}^{x_{i,1}+x_a} F\,dx$$

$$= -\int_{x_{i,1}}^{x_{i,1}+x_a} (-k_1 x)\,dx$$

$$= k_1 \int_{x_{i,1}}^{x_{i,1}+x_a} x\,dx$$

$$= k_1 \frac{x^2}{2}\Big|_{x_{i,1}}^{x_{i,1}+x_a}$$

$$= \frac{k_1}{2}\left[(x_{i,1}+x_a)^2 - x_{i,1}^2\right]$$

$$= \frac{k_1}{2}\left[\cancel{x_{i,1}^2} + 2x_a x_{i,1} + x_a^2 - \cancel{x_{i,1}^2}\right]$$

$$\boxed{W_1 = \frac{k_1}{2}\left(x_a^2 + 2x_a x_{i,1}\right)}$$

(b) The spring is stretched to the left at equilibrium, so when the glider is moved to the right, the full stretch is $(x_{i,2}-x_a)$

$$W_2 = k_2 \int_{x_{i,2}}^{x_{i,2}-x_a} x\,dx$$

$$= k_2 \frac{x^2}{2}\Big|_{x_{i,2}}^{x_{i,2}-x_a}$$

$$= \frac{k_2}{2}\left[(x_{i,2}-x_a)^2 - x_{i,2}^2\right]$$

$$\boxed{W_2 = \frac{k_2}{2}\left(x_a^2 - 2x_a x_{i,2}\right)}$$

(c) At equilibrium the spring forces balance:

$$F_1 = F_2$$

$$-k_1 x_{i,1} = -k_2 x_{i,2}$$

$$\boxed{x_{i,2} = \frac{k_1 x_{i,1}}{k_2}}$$

(d) $W_{TOTAL} = W_1 + W_2$

$$= \frac{k_1}{2}(x_a^2 + 2x_a x_{i,1}) + \frac{k_2}{2}(x_a^2 - 2x_a x_{i,2})$$

$$= \frac{k_1}{2}x_a^2 + \cancel{k_1 x_a x_{i,1}} + \frac{k_2}{2}x_a^2 - \cancel{k_2 x_a\left(\frac{k_1 x_{i,1}}{k_2}\right)}$$

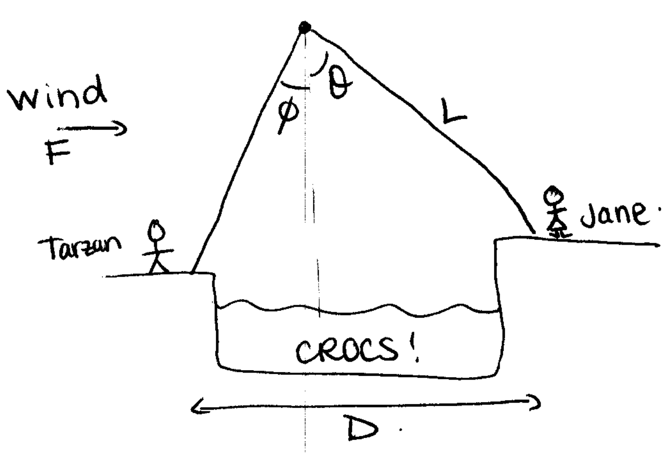
$$\boxed{W_T = \frac{1}{2}x_a^2(k_1 + k_2)}$$

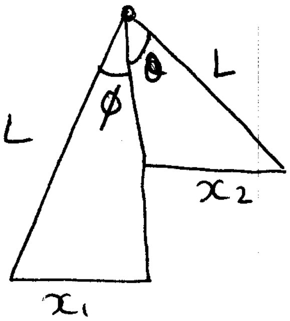
Q3. Serway 8.65

Jane, whose mass is 50.0kg, needs to swing across a river (having width D) filled with man-eating crocodiles to save Tarzan from danger. She must swing into a wind exerting constant horizontal force F, on a vine having length L & initially making an angle θ with the vertical. Taking $D = 50.0$m, $F = 110$N, $L = 40.0$m, and $\theta = 50.0°$,

(a) with what minimum speed must she begin her swing to just make it to the other side?

(b) with what minimum speed must Tarzan & Jane swing back to just make it to the original side? ($m_{Tarzan} = 80.0$ kg).





$$x_1 = L\sin\phi$$
$$x_2 = L\sin\theta$$
$$\Rightarrow D = L(\sin\theta + \sin\phi)$$
$$\sin\phi = \frac{D}{L} - \sin\theta$$
$$\sin\phi = \frac{50.0\text{m}}{40.0\text{m}} - \sin 50^\circ$$
$$\phi = 28.9^\circ$$

(a) set $PE = 0$ at top of vine.

$$h_{J,i} = -L\cos\theta$$
$$h_{J,f} = -L\cos\phi$$

$$E_i + W_{\text{external forces}} = E_f$$
$$PE_i + KE_i + \int_0^D F\cdot dx = PE_f + \cancelto{0}{KE_f}$$
$$mg(-L\cos\theta) + \tfrac{1}{2}mv^2 - FD = mg(-L\cos\phi)$$
$$v^2 = \frac{2(mg(L\cos\theta - L\cos\phi) + FD)}{m}$$
$$= \frac{2\left[(50.0\text{kg})(9.8\text{m/s}^2)(40.0\text{m})(\cos 50^\circ - \cos 28.9^\circ) + (110\text{N})(50.0\text{m})\right]}{(50.0\text{kg})}$$

$$\boxed{v = 6.13\text{m/s}}$$

(b) $E_i + W_{ext} = E_f$

$$PE_i + KE_i + \int_0^D F\cdot dx = PE_f + \cancelto{0}{KE_f}$$
$$mg(-L\cos\phi) + \tfrac{1}{2}mv^2 + FD = mg(-L\cos\theta)$$
$$v^2 = \frac{2(mgL(\cos\phi - \cos\theta) - FD)}{m}$$
$$= \frac{2\left[(50.0\text{kg} + 80.0\text{kg})(9.8\text{m/s}^2)(40.0\text{m})(\cos 28.9^\circ - \cos 50^\circ) - (110\text{N})(50.0\text{m})\right]}{(50.0 + 80.0\text{kg})}$$

$$\boxed{v = 9.89\text{m/s}}$$