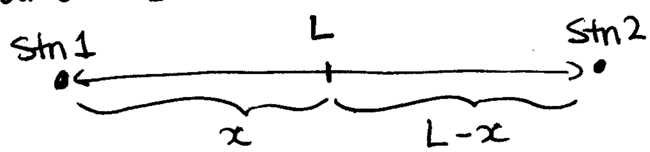
Tutorial Two: Sept. 18/22, 2006

1. Linear motion.

A subway train travels between two downtown stations. Because these stations are only 1.00 Km apart, the train never reaches its maximum possible cruising speed. The engineer minimizes the time t between the stations by accelerating at a rate of $a_1 = 0.100\, m/s^2$ for time t_1 and then by braking with acceleration $a_2 = -0.500\, m/s^2$ for time t_2. Find the minimum travel time t and the times t_1 and t_2.

Sol'n:



Let x be the distance traveled while the train is accelerating.

$L = 1.00\, Km$

$a_1 = 0.100\, m/s^2$

$a_2 = -0.500\, m/s^2$

$V_{i,1} = 0\, m/s$ speed at start (Station 1)

$V_{f,2} = 0\, m/s$ speed at end (station 2)

$V_{max} = V_{f,1} = V_{i,2}$ speed when train begins to deccelerate.

$t = t_1 + t_2$.

For the first part of the trip:

$$x = \cancel{V_{i,1}}^{0} t_1 + \frac{1}{2} a_1 t_1^2$$

$$x = \frac{1}{2} a_1 t_1^2 \quad ①$$

The second part:

$$L - x = V_{i,2} t_2 + \frac{1}{2} a_2 t_2^2$$

$V_{i,2}$: $\cancel{V_{f,2}}^{0} = V_{i,2} + a_2 t_2$.

$V_{i,2} = -a_2 t_2$

$$L - x = -a_2 t_2^2 + \frac{1}{2} a_2 t_2^2$$

$$L - x = -\frac{1}{2} a_2 t_2^2 \quad ②$$

Add ① & ②:

$$x = \frac{1}{2} a_1 t_1^2$$

$$L - x = -\frac{1}{2} a_2 t_2^2$$

$$L = \frac{1}{2} a_1 t_1^2 - \frac{1}{2} a_2 t_2^2$$

Now put t_1 in terms of t_2:

we know $v_{i,2} = -a_2 t_2 = v_{f,1}$

$$v_{f,1} = \cancel{v_{i,1}}^{0} + a_1 t_1$$

$$v_{f,1} = a_1 t_1$$

so $v_{i,2} = v_{f,1}$

$$-a_2 t_2 = a_1 t_1$$

$$t_1 = \frac{-a_2 t_2}{a_1}$$

Sub. into equation for L:

$$L = \frac{1}{2} a_1 \left(\frac{-a_2 t_2}{a_1}\right)^2 - \frac{1}{2} a_2 t_2^2$$

$$2L = \cancel{a_1} \frac{a_2^2 t_2^2}{a_1^{\cancel{2}}} - a_2 t_2^2$$

$$2L = \frac{a_2^2}{a_1} t_2^2 - a_2 t_2^2$$

$$2L = \left(\frac{a_2^2}{a_1} - a_2\right) t_2^2$$

$$t_2^2 = \frac{2L}{(a_2^2/a_1 - a_2)}$$

$$t_2 = \sqrt{\frac{2(1.00\text{Km})}{\left(\frac{(-0.500\text{m/s}^2)^2}{0.100\text{m/s}^2}\right) - (-0.500\text{m/s}^2)}}$$

$$\boxed{t_2 = 25.8\text{s}}$$

Now find t_1:

$$t_1 = -\frac{a_2 t_2}{a_1}$$

$$= -\frac{(-0.500\text{m/s}^2)(25.8\text{s})}{(0.100\text{m/s}^2)}$$

$$\boxed{t_1 = 129\text{s}}$$

And finally t:

$$t = t_1 + t_2$$

$$= 129\text{s} + 25.8\text{s}$$

$$\boxed{t = 154.8\text{s}}$$

2. Free Fall

A first year lab student decides to calculate the height of the physics building by throwing a rock off the roof. He drops the rock and counts 3 seconds before he hears the rock hit the ground. Ignoring air resistance, but not the speed of sound ($v_s =$ ~~350~~ 336 m/s in air), how tall is the building?

Sol'n: total time: $t = t_1 + t_2 = 3$ where t_1 = time for rock to fall

$t_2 = 3 - t_1$; t_2 = time for sound to rise.

the height of the building, h:

$$h = v_s t_2 = \cancel{v_0}^{0} t_1 + \frac{1}{2} g t_1^2$$

$$h = v_s t_2 = \frac{1}{2} g t_1^2$$

$$\rightarrow \quad 2 v_s t_2 = g t_1^2$$

$$2 v_s (3 - t_1) = g t_1^2$$

$$0 = g t_1^2 + 2 v_s t_1 - 6 v_s$$

$$t_1 = \frac{-\cancel{2} v_s \pm \sqrt{4 v_s^2 - 4 g(-6 v_s)}}{\cancel{2} g}$$

$$= \frac{-v_s \pm \sqrt{v_s^2 + 6 g v_s}}{g}$$

$$t_1 = 2.9\,\text{s} \quad (\text{or } -71\,\text{s} \ldots)$$

Sub. into the equation for h:

$$h = v_s t_2$$

$$= v_s (3 - t_1)$$

$$= (336\,\text{m/s})(3 - 2.9\,\text{s})$$

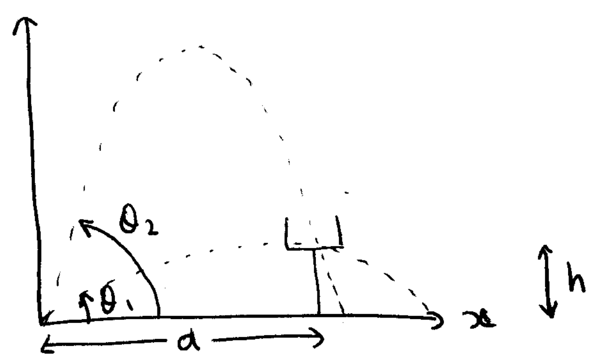
$$= 33.6\,\text{m}$$

$$\boxed{h = 3 \times 10\,\text{m}}$$ (significant digits).

3. Projectile motion

A place kicker is about to kick a field goal. The ball is 27.4m from the goalpost. The ball is kicked with an initial velocity of 19.8 m/s at an angle θ above the ground. Between what two angles, θ_1 and θ_2, will the ball clear the 2.74 m high crossbar?

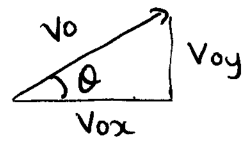
Sol'n:



$d = 27.4\text{m}$
$h = 2.74\text{m}$
$V_0 = 19.8\text{m/s}$.

Divide V_0 into horizontal & vertical motion:

$$V_{0x} = V_0 \cos\theta$$
$$V_{0y} = V_0 \sin\theta$$



Horizontal motion is constant:

$$d = V_{0x} t = V_0 \cos\theta \, t$$
$$t = \frac{d}{V_0 \cos\theta}$$

Vertical motion has constant acceleration:

negative since g pulls down.

$$h = V_{0y} t - \frac{1}{2} g t^2$$
$$h = \cancel{V_0} \sin\theta \left(\frac{d}{\cancel{V_0} \cos\theta}\right) - \frac{g}{2}\left(\frac{d}{V_0 \cos\theta}\right)^2$$
$$h = d\tan\theta - \frac{g d^2}{2 V_0^2 \cos^2\theta}$$

Recall: $\sec\theta = 1/\cos\theta$.

$$h = d\tan\theta - \frac{g d^2}{2 V_0^2} \sec^2\theta$$

Recall: $1 + \tan^2\theta = \sec^2\theta$

$$h = d\tan\theta - \frac{g d^2}{2 V_0^2}(\tan^2\theta + 1)$$

$$0 = \frac{gd^2}{2v_0^2}\tan^2\theta - d\tan\theta + \left(h + \frac{gd^2}{2v_0^2}\right)$$

$$\tan\theta = \frac{d \pm \sqrt{d^2 - \cancel{4}^{\,2}\left(\frac{gd^2}{\cancel{2}v_0^2}\right)\left(h + \frac{gd^2}{\cancel{2}v_0^2}\right)}}{\cancel{2}\left(\frac{gd^2}{\cancel{2}v_0^2}\right)}$$

$$= \frac{v_0^2}{gd} \pm \frac{v_0^2}{gd^2}\sqrt{d^2 - \frac{2gd^2}{v_0^2}\left(h + \frac{gd^2}{2v_0^2}\right)}$$

$$= \frac{v_0^2}{gd} \pm \frac{v_0^2}{gd}\sqrt{1 - \frac{2g}{v_0^2}\left(h + \frac{gd^2}{2v_0^2}\right)}$$

$$\tan\theta = \frac{v_0^2}{gd}\left[1 \pm \sqrt{1 - \frac{2g}{v_0^2}\left(h + \frac{gd^2}{2v_0^2}\right)}\right].$$

$$= \frac{(19.8\,m/s)^2}{(9.8\,m/s^2)(27.4\,m)}\left[1 \pm \sqrt{1 - \frac{2(9.8\,m/s^2)}{(19.8\,m/s)^2}\left(2.74\,m + \frac{(9.8\,m/s^2)(27.4\,m)^2}{2(19.8\,m/s)^2}\right)}\right]$$

$$\tan\theta = 2.375 \text{ or } 0.5437$$

case 1: $\tan\theta_1 = 0.5437$

$\theta_1 = 67.2°$

case 2: $\tan\theta_2 = 2.375$

$\theta_2 = 28.5°$

$\Rightarrow$ The ball will clear the crossbar if $\boxed{28.5° \leq \theta \leq 67.2°}$.

4. Polar coordinates. (Ch. 3 #6)

If the polar coordinates of the point (x, y) are (r, θ), determinte the polar coordinates for the points:

(a) $(-x, y)$

(b) $(-2x, -2y)$

(c) $(3x, -3y)$.

Sol'n:

(a) $r = \sqrt{(-x)^2 + y^2}$

$= \sqrt{x^2 + y^2}$

$\boxed{r = r}$

$\tan\theta = \frac{y}{(-x)}$

$= -\frac{y}{x}$

$\theta = \arctan(-y/x) = \tan^{-1}(-y/x)$

$\boxed{\theta = \pi - \theta}$

$\therefore$ in polar coordinates: $(r, \pi - \theta)$

(b) $(-2x, -2y)$

$r = \sqrt{4x^2 + 4y^2}$

$r = 2r$

$\tan\theta = \left(\frac{-2y}{-2x}\right)$

$\theta = \pi + \theta$ Q: why not θ?

$\therefore$ in polar c: $(2r, \pi + \theta)$.

(c) $(3x, -3y)$

$r = \sqrt{9x^2 + 9y^2}$

$r = 3r$

$\tan\theta = \left(\frac{-3y}{3x}\right)$

$\theta = -\theta$.

$\therefore$ in polar c: $(3r, -\theta)$.