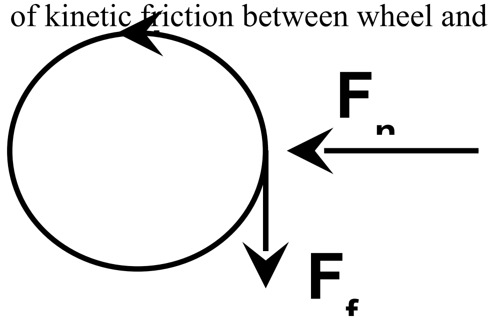
Tutorial for week of Nov. 13 -17, 2006

Serway 10.38

A potter's wheel – a thick stone disk of radius 0.500 m and mass 100 kg – is freely rotating at 50.0 rev /min. The potter can stop the wheel in 6.00 s by pressing a wet rag against the rim and exerting a radially inward force of 70 N. Find the effective coefficient of kinetic friction between wheel and rag.



Giving:

$m=100$ kg

$R=0.5$ m

$F_n=70$ N

$\omega_i=50$ rev/min = (50 rev/min)*(2π rad/rev) /(60 s/min) = 5.24 rad/s

$t=6.0$ s

The friction force (F_f) is equal to:

$F_f=\mu_k F_n$

$\therefore \mu_k = F_f / F_n$ (1)

From the relationship between τ (its magnitude) and F_f:

$\tau=F_f R$

We can get:

$F_f = \tau/R$ (2)

τ can be calculated from:

$\tau=I\alpha$ (3)

where

$I=mR^2/2$ (see Table 10.2)

$\alpha= (\omega_f - \omega_i)/t$

$\therefore I= mR^2/2=(100\text{ kg})*(0.5\text{ m})^2/2 =12.5\text{ kg m}^2$

$\therefore \alpha= (\omega_f - \omega_i)/t = (0-5.24\text{ rad/s})/(6.0\text{ s}) = -0.873\text{ rad/s}^2$

Taking I and α to (3):

$\tau=I\alpha = (12.5\text{ kg m}^2)*(-0.873\text{ rad/s}^2) = -10.9$ Nm

Taking the magnitude of τ to Equation (2):

$F_f = \tau/R=(10.9\text{ Nm})/(0.5\text{ m}) = 21.8$ N

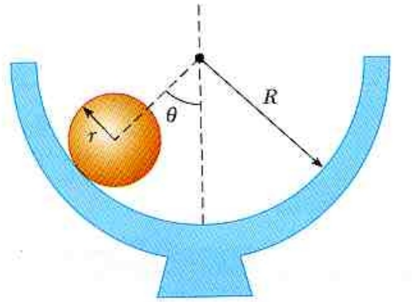
Taking F_f to Equation (1):

$\mu_k = F_f / F_n = (21.8\ N)/(70\ N) = 0.312$

$\therefore$The kinetic friction coefficient between the wheel and rag is 0.312.

Serway 10.76

A uniform solid sphere of radius r is placed on the inside surface of a hemispherical bowl with much larger radius R. The sphere is released from rest at an angle θ to the vertical and rolls without slipping. Determine the angular speed of the sphere when it reaches the bottom of the bowl.



Giving:
R and r
$v_i = 0$
$\omega_i = 0$
I of the sphere = $(2/5)mr^2$ (see Table 10.2)

For the potential energy, taking the potential energy as zero at the centre of the bowl:
$U_i = -mg(R-r)\cos\theta$
$U_f = -mg(R-r)$

For the translational kinetic energy:
$K_{trans,i} = 0$
$K_{trans,f} = mv_f^2/2$

For the rotational kinetic energy:
$K_{rot,i} = 0$
$K_{rot,f} = I\omega_f^2/2$

Assuming the friction between the sphere and the bowl is negligible, energy is conserved and $\Delta U + \Delta K_{trans} + \Delta K_{rot} = 0$
$\{-mg(R-r) + mg(R-r)\cos\theta\} + \{ mv_f^2/2 - 0\} + \{ (2/5)mr^2\omega_f^2/2 - 0\} = 0$
$\therefore mg(R-r)(\cos\theta - 1) + mv_f^2/2 + (2/10)mr^2\omega_f^2 = 0$

As $v_f = r\omega_f$,
$\therefore g(R-r)(\cos\theta - 1) + r^2\omega_f^2/2 + (2/10)r^2\omega_f^2 = 0$
$\therefore (7/10)r^2\omega_f^2 = g(R-r)(1-\cos\theta)$
$\therefore \omega_f^2 = (10/7)g(R-r)(1-\cos\theta)/r^2$

$$\omega_f = \sqrt{\frac{10g(R-r)(1-\cos\theta)}{7r^2}}$$

If R>>r

$$\omega_f = \sqrt{\frac{10gR(1-\cos\theta)}{7r^2}}$$

Serway 9.52

Rocket Science. A rocket has total mass $M_i = 360$ kg, including 33o kg of fuel and oxidizer. In interstellar space it starts from rest, turns on its engine at time $t = 0$, and puts out exhaust with relative speed $v_e = 1500$ m/s at the constant rate $k = 2.50$ kg /s. The fuel will last for an actual burn time of 330 kg/(2.5 kg/s) = 132 s, but define a "projected depletion time" as $T_p = M_i/k = 144$.s.

(a) Show that during the burn the velocity of the rocket is given as a function of time by

$$v(t) = -v_e ln[1 - (t/T_p)].$$

(b) Make a graph of the velocity of the rocket as a function of time for times running from 0 to 132s.

(c) Show that the acceleration of the rocket is

$$a(t) = v_e/(T_p - t).$$

(d) Graph the acceleration as a function of time.

(e) Show the position of the rocket is

$$x(t) = v_e(T_p - t)ln[1 - (t/T_p)] + v_e t.$$

Giving:

$v_i = 0$

$v_e = 1500$ m/s

$T_p = M_i/k = 144$ s

(a) From Equation (9.41) for rocket propulsion, let $v_f = v$:

$$v - v_i = v_e \ln\left(\frac{M_i}{M_f}\right) = -v_e \ln\left(\frac{M_f}{M_i}\right) \qquad (1)$$

As $M_f = M_i - kt$ and $v_i = 0,$ Equation (1) becomes

$$v = -v_e \ln\left(\frac{M_i - kt}{M_i}\right) = -v_e \ln\left(1 - \frac{k}{M_i}t\right). \qquad (2)$$

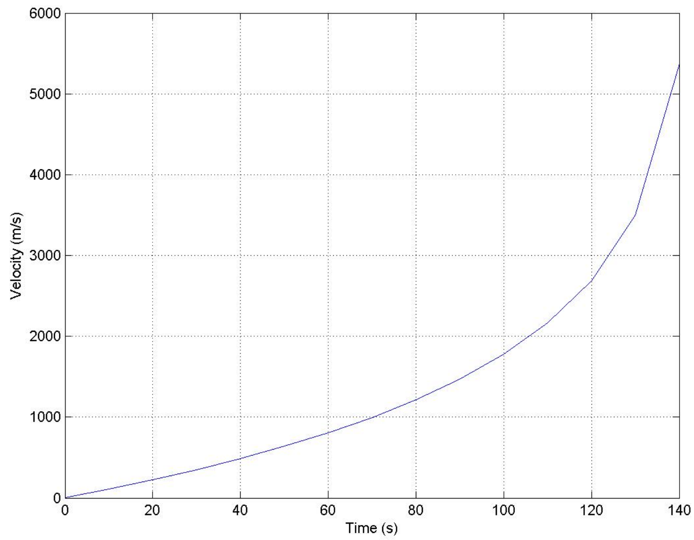
As $T_p = M_i/k,$ Equation (2) becomes

$$v(t) = -v_e \ln\left(1 - \frac{t}{T_p}\right). \qquad (3)$$

(b) With $v_e = 1500$ m/s and $T_p = 144$ s,

$$v(t) = -(1500 m/s)\ln\left(1 - \frac{t}{144s}\right) \text{ (in m/s)} \qquad (4).$$

A graph for (4) is as:

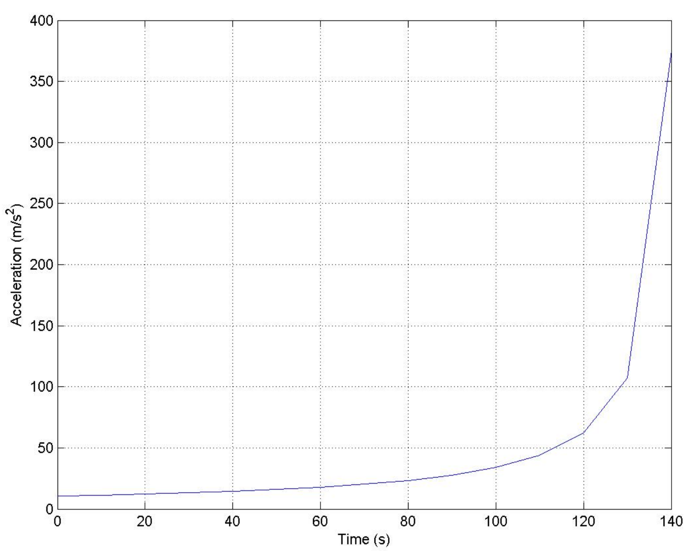


(c) The expression for the acceleration can be derived by taking a derivative of the velocity (Equation 3).

$$a(t) = \frac{dv}{dt} = -v_e\left(\frac{1}{1 - t/T_p}\right)\left(-\frac{1}{T_p}\right) = v_e\left(\frac{1}{T_p - t}\right)$$

(d) With v_e = 1500 m/s and T_p = 144 s,
a(t) = (1500 m/s)/(T_p – 144 s) (in m/s^2) (5).

A graph for (5) is as:



(e) The expression for the potion can be derived by integrating the velocity (Equation 3).

$$x(t) = 0 + \int_0^t [-v_e \ln(1 - \frac{t}{T_p})]dt = v_e T_p \int_0^t \ln(1 - \frac{t}{T_p})(-\frac{dt}{T_p}) = v_e T_p \int_0^t \ln(1 - \frac{t}{T_p})(d(1 - \frac{t}{T_p})$$

From Table B.5:

$$x(t) = v_e T_p [(1 - \frac{t}{T_p}) \ln(1 - \frac{t}{T_p}) - (1 - \frac{t}{T_p})]_0^t$$

$$x(t) = (T_p - t) \ln(1 - \frac{t}{T_p}) + v_e t \qquad (6)$$

(f) With v_e = 1500 m/s = 1.5 km and T_p = 144 s,

x(t) = 1.5(144 – t)ln[1 - t/144] + 1.5 t *(in km)*

A graph for (5) is as: